On the psychology of a hijacker.

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1. *Introduction.*

We shall use the mathematical tools of game theory [3], decision theory [6,12], and catastrophe theory [7,10] to model the psychology of a hijacker. A similar model can be applied to any conflict situation involving negotiations, be it military, political, industrial or social. We shall confine ourselves to the case of a hijacker, however, in order to simplify the description and highlight the essential features of the model.

The hijack situation can be modelled initially by game theory [3]. Although game theory explains why both sides will want to negotiate it does not describe the subsequent evolution of negotiations, and to understand the latter it is necessary to analyse the conflict at a slightly deeper level. For this we use Bayesian decision theory to model the choice between the options open to the hijacker at any given moment. His beliefs about the various possible outcomes of the hijack are integrated against his preferences, giving a risk function that can then be minimised to determine his attitude.

At a still deeper level the beliefs and preferences of the hijacker may depend upon several parameters. For instance examples of parameters that vary with time, and which may cause changes in his attitude, include increasing frustration, increasing pressure, increasing exhaustion, and increasing rapport with the security forces. Introducing parameters into a risk function automatically gives rise to a catastrophe model, which shows how the attitude depends upon the parameters, and indicates the possible slow evolutions and sudden switches of attitude that may occur as the parameters vary.

For example gradually increasing the pressure may cause the hijacker to suddenly either *(a)* blow up the plane or *(b)* surrender. More subtly, the catastrophe model may reveal hidden potential changes taking place behind the scenes. For example gradually increasing exhausion may bias the hijacker from *(a)* towards *(b)*. If this were the case, and if the latter could be detected by appropriate analysis of the dialogue between the hijacker and the security forces, then it behoves the latter to delay increasing the pressure on him until after the bias had taken place, in order to avoid blowing up the hostages.

2. Game theory.

We begin by recommending to the reader Michael Laver's penetrating and entertaining book *The Crime Game* [3], in which he describes many game theoretic negotiations between criminals and their victims. In particular in Chapter 6 he deals with the hijack situation, as follows.

The hijacker (or a team of hijackers) is assumed to have hijacked a plane full of hostages, and is holding the victims to ransom by threatening to blow up the plane with himself and all the hostages inside. Here the victim is the government (or the airline or the security forces) and the ransom is usually a complicated package involving money, escape, and political objectives such as the release of some political prisoners who are comrades of the hijacker. As a first approximation to the situation Laver proposes the following simple 2×2 game :

		Hijacker: No blow	Hijacker: Blow
Victim	Pay	1^{st} 2^{nd}	4^{th} 4^{th}
	No Pay	3^{rd} 1^{st}	2^{nd} 3^{rd}

Figure 1. Laver's hijacker game [3].

Here the hijacker has two options, either blow up the plane or not, and the victim also has two options, either pay the ransom or not, and so the game has four possible outcomes represented by the four boxes shown in

Figure 1. The hijacker's order of preference is shown at the top right corner of each box, while the victim's order of preference is shown at the bottom left corner of each box. The hijacker's first choice is *pay/no blow*, and his second choice is *no pay/blow*, which he threatens to do if the victim refuses to pay. His third choice is the total failure of the hijack, *no pay/no blow*, and his last choice is *pay/blow* because this would involve an unnecessary sacrifice of his own life after having achieved the objectives of the hijack.

Meanwhile the victim's first choice is *no pay/no blow*, and his second choice will be *pay/no blow* provided that he is willing to pay the ransom if necessary to save the hostages. His third choice must be *no pay/blow*, because his last choice of *pay/blow* would involve the double disaster of losing both ransom and hostages.

Now what does the game theory tell us? Notice first that the hijacker has no dominant strategy: if the victim pays then he prefers *no blow*, but if the victim refuses to pay then he prefers *blow*. Therefore the hijacker does not want to make the first move, but would rather wait and see what the victim is going to do. In other words the hijacker wants to negotiate.

By contrast the victim does have a dominant strategy, *no pay*, because his 1^{st} choice is preferable to his 2^{nd}, and his 3^{rd} choice is preferable to his 4^{th}. However, were he to make the first move by playing his dominant strategy and informing the hijacker that he is definitely not going to pay, then this would automatically lead to his 3^{rd} choice of *no pay/blow*. Since he prefers his 1^{st} and 2^{nd} choices he decides not to play his dominant strategy after all, and tries negotiating instead. Therefore the game theory tells us that both sides will want to negotiate.

Laver admits that not all hijackers will necessarily have this order of preference, and some may be so attached to securing the ransom at all costs, that they will put *pay/blow* ahead of *no pay/no blow*, or possibly even as high as their 2^{nd} choice. But this does not affect the main conclusion that they have no dominant strategy, and therefore will want to negotiate. Similarly the victim may be sufficiently tough-minded as to resist paying the ransom at all costs, preferring *no pay/blow* to *pay/no blow* in order to support long term deterrence against hijacking. But as before playing his dominant strategy will immediately deprive him of his

1st choice, and so again he tries negotiating instead.

How do we model the negotiations? One can elaborate the 2×2 game into an algorithm of metagames, by allowing each player to make a series of hypothetical moves in response to the other, but such elaboration is less convincing than the simplicity of the original game. As Thom [8] points out any mathematical model of a piece of nature has an area of contact with reality, within which it is valid, and within which it may, subject to its own limitations, be convincing; but if it strays too far away from that area of contact then the model becomes unglued from reality, and begins to develop a life of its own, interesting possibly to the specialist, but more related to fantasy than to the application. Laver's 2×2 game above is convincing because of its simplicity, but it is too oversimplified to warrant the additional complexity of the induced metagames, without introducing further data.

Returning to our hijacker, when the negotations begin to get serious the victim does not call in a game theorist to advise him; he calls in a psychologist. Similarly, if we wish to understand the negotiations at a deeper level we must get under the hijackers skin and model the psychology of his attitudes and decision making processes.

3. ***Decision theory.*** [6,12]

At any given moment the hijacker will have a great many things on his mind. His head will be buzzing with hopes and fears and aggression, and the extra adrenalin sloshing around in his bloodstream will put him on edge, and make it more difficult to think clearly about tactics and strategies. There will be a tendency to adopt instinctive attitudes, to to react spontaneously, and possibly to think emotionally rather than rationally. Besides trying to achieve his main objectives and carry out preconceived plans, he will also have to continually adapt to unexpected contingencies and be constantly on the watch for countermoves by the victim. In between moments of decision and action there will be anxious and frustrating periods of waiting.

What sort of decisions does the hijacker have to make? Most of his decisions will be about what to do or say to the hostages and the victim in order to persuade them to do what he wants, such as refuelling the plane, mending a burst tyre perhaps, or getting the released prisoners to the plane by such and such time, etc. His actions and words will be coloured by the prevailing level of aggression in his attitude.

So how does one set about modelling his decision making? We suggest that it is important to focus attention upon his attitude, or more precisely on the level of aggression in his attitude. As Allport [1] says, attitude is a mental state that predisposes behaviour. Attitude is not only relatively simple to describe, but also fundamental in the sense that it is determined by the input complexity, and determines the output complexity. Here by the input complexity we mean the hijacker's awareness of all the possible outcomes of the hijack, his beliefs about what is most likely to happen and his order of preferences for what he would like to happen, all of which must be contributory factors towards the formation of his attitude. By the output complexity we mean the details of his behaviour, all his actions, words, threats and ultimatums in trying to persuade other people to do what he wants. Attitude is the central simplicity sandwiched in between the input and output complexities. If we explicitly model that simplicity, while implicitly allowing for the complexities, then the model will be psychologically realistic.

We construct the model as follows [as in 6,11,12,13]. Let x denote the hijacker's *attitude*, or more precisely the level of agression in his attitude at any given moment. We assume that x lies in a one-dimensional spectrum X, and describe the attitude at various points of X by indicating a typical action that might result from that attitude, as follows.

aggression X (↑):

- blow up the plane
- kill a hostage
- issue an ultimatum
- threaten
- negotiate
- withdraw an ultimatum
- release some hostages
- surrender

We now construct a risk function R on X to represent the input complexity. Let Y denote the *set of possible future outcomes* of the hijack. In the game described in the last section we assumed that Y consisted of 4 possible outcomes, represented by the 4 boxes in Figure 1, but here Y is allowed to be as complicated as necessary. For instance in some possible outcomes only part of the ransom might be paid. It might become clear to the hijacker that the victim was not prepared to budge on the matter of political prisoners, but might be willing to settle for money, an escape route, and an undertaking to publicise the hijacker's cause in return for the release of hostages. Other possible outcomes might involve the killing of a few hostages or a shoot-out with the security forces. We shall allow Y to be as complicated a set of possible future outcomes as may exist in the imagination of the hijacker at the particular moment in question.

We now introduce a probability distribution P to represent his *beliefs* about Y. Let P(x,y) denote the probability of future outcome y given that he adopts attitude x now (according to the hijacker's belief). Since P is a probability distribution,

$$\sum_{y \in Y} P(x,y) = 1, \qquad \text{for each } x \in X.$$

We next introduce a loss function L to represent the hijacker's *preferences* for the various outcomes. Let L(x,y) denote the loss he will incur if he adopts attitude x now and y is the subsequent outcome. By "loss" we do not mean financial loss, but rather a valuation on some scale that indicates the ordering of his preferences. In the game in the last section we only used an ordering, but here we use a valuation, which is stronger than an ordering in the sense that it can indicate which choices are strongly preferred, or strongly rejected, and which are much of a muchness.

We can now calculate the *risk* R(x) of adopting an attitude x. Define

$$R(x) = \sum_{y \in Y} L(x,y)P(x,y).$$

In other words the risk is the sum of possible losses, each weighted according to its probability. Define the *Bayesian decision* to be that attitude x carrying the least risk.

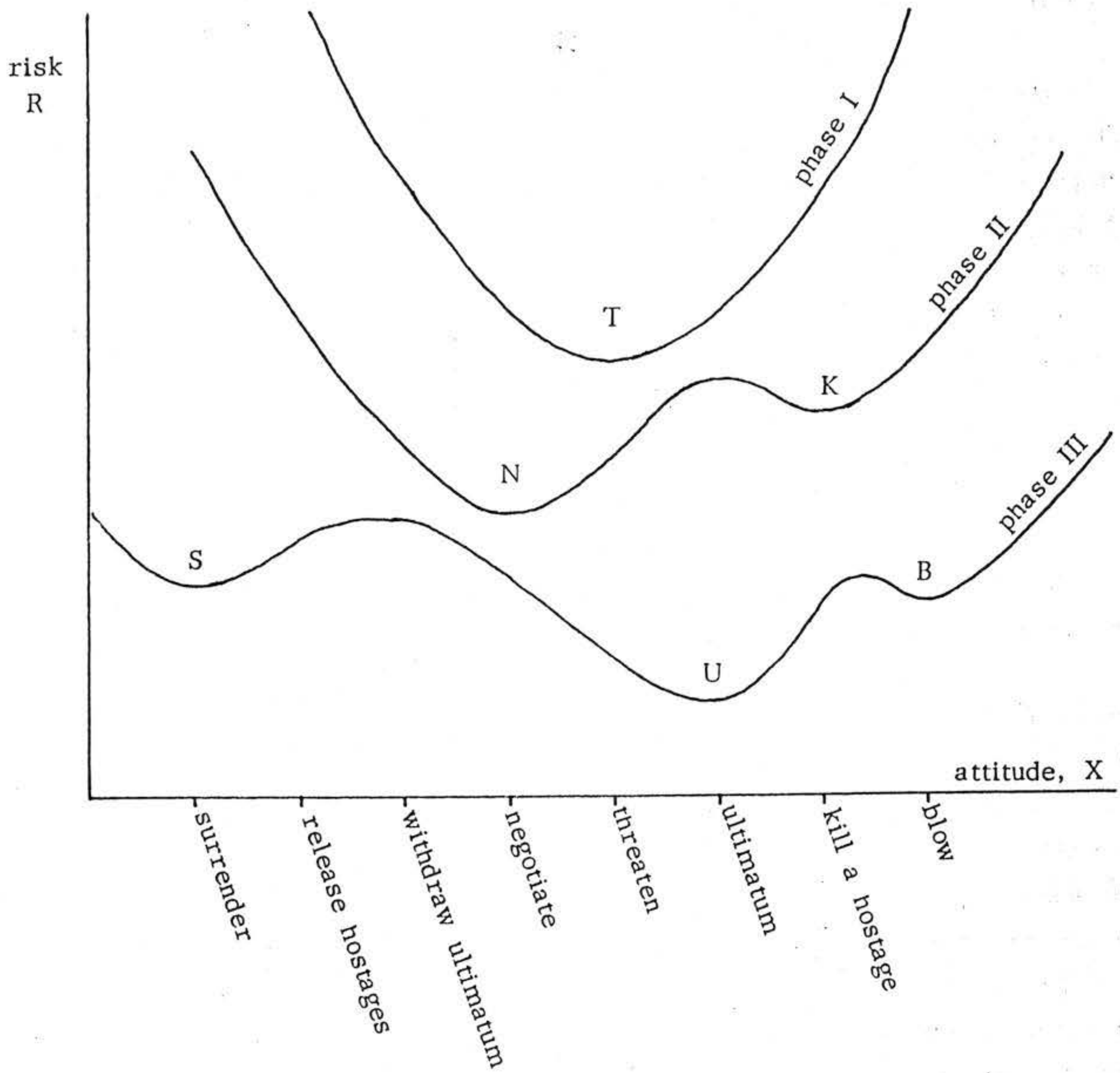


Figure 2. *Three phases of the risk function. For visual convenience we have drawn the three graphs below one another. The most important features of each graph are the positions and relative levels of the minima.*

Having set up the model, there are now two possible ways to use it. Firstly we could try and estimate P(x,y) and L(x,y) for each value of x and y, and hence compute the risk function R, and identify its minimum. This is probably what is going on subconsciously inside the hijacker's brain all the time, and may be the underlying mechanism responsible for his "instinctive" attitude at any given moment. However, it would be well nigh impossible for an outside observer to make all the estimates sufficiently accurately as to be able to compute where the minimum is with any reliability, and so it is probably better to leave most of this input complexity implicit rather than explicit.

The second approach, which is the one that we shall adopt, is to make a direct hypothesis about the shape of R that is compatible with the hijacker's known beliefs and preferences, and to predict the attitude and behaviour resulting from that shape. In other words, we make a hypothesis about the central simplicity in order to gain insight into the surrounding complexity. Our hypotheses are given in Figure 2, and show the changing shape of R during three successive phases of the hijack, as we shall now explain.

Phase I.

During the initial phase R has a unique minimum T representing an attitude in which the hijacker is prepared to threaten the victim. We justify this hypothesis as follows. At this stage the hijacker's main preference is to secure the ransom in exchange for the hostages, and he believes there is a good chance of success provided he can threaten convincingly. For his threats to be effective it is important to establish credibility; it may be risky to issue an ultimatum too soon, lest he has to withdraw it later and thereby lose credibility. Killing a hostage at this stage is even more risky, because it reduces the number of hostages that can be exchanged for the ransom, and may push the victim towards opting for a shoot-out instead of paying the ransom. Going the other way is also risky, because if he starts negotiating too soon this might reduce the credibility of his threats. The more reasonable and placatory the hijacker appears to be initially, the more confident the victim grows about not having to pay the ransom and the less likely will be the hijacker's preferred outcome; consequently the more risky will be his attitude.

Summarising, in phase I the risk function R is unimodal, and the minimum T is compatible with the input complexity. Consequently the hijacker will begin his communications with the victim by issuing simple clearly defined threats, such as blowing up the plane unless the ransom is paid.

Phase II.

As explained by the game in the last section both sides will want to negotiate once communication has been established. Since the hijacker has no dominant strategy he wants to find out what the victim is going to do, and if the latter agrees to pay the ransom then there will be many logistic details to be settled. Meanwhile the victim will want to find out as much as possible about the hijacker, in order to assess the credibility of his threats. It is good policy for the victim to play it cool initially in order not to precipitate any killing of hostages; he will also gain time to assemble his security forces and plan appropriate countermeasures.

From the hijacker's point of view the minimum of his risk function R will gradually move to the left from the point T in the phase I graph in Figure 2 to the point N in the phase II graph, representing an attitude in which he is prepared to negotiate. We suggest that a consequence of this move to the left will be the emergence of another higher minimum K on the right, counterbalancing the move to the left. It is not as if there will be any reduction in the total amount of aggression felt by the hijacker, but rather a splitting of it into the bimodality of N plus K. It is diplomatic for him to move from threatening to negotiating in phase II, and less risky in the precise sense of the model, in that the negotiating attitude is more likely to ultimately achieve his preferred outcome. At the same time, however, there will be a hardening at the back of his mind in case the negotiations are unsuccessful, and the new minimum K represents an attitude in which he is quite prepared to kill a hostage if necessary. If the victim exploits the hijacker's apparent softening by prevaricating, then the hijacker may well punish him by killing a hostage in order to concentrate the victim's mind. For instance in the 1977 Mogadishu hijacking [3], the hijacker shot the pilot in cold blood as a punishment and warning to the security forces against further prevarication.

We explain the nature of the change of attitude as follows. As the victim's intransigence gradually dawns upon the hijacker, the latter's attitude towards negotiating becomes more risky in the strict sense of the model, because his belief that this will eventually lead to the paying of the ransom begins to dwindle. Meanwhile his attitude towards killing a hostage becomes relatively less risky, because he begins to believe that it might in fact increase the probability of the preferred outcome. This is illustrated by the sequence of graphs in Figure 3.

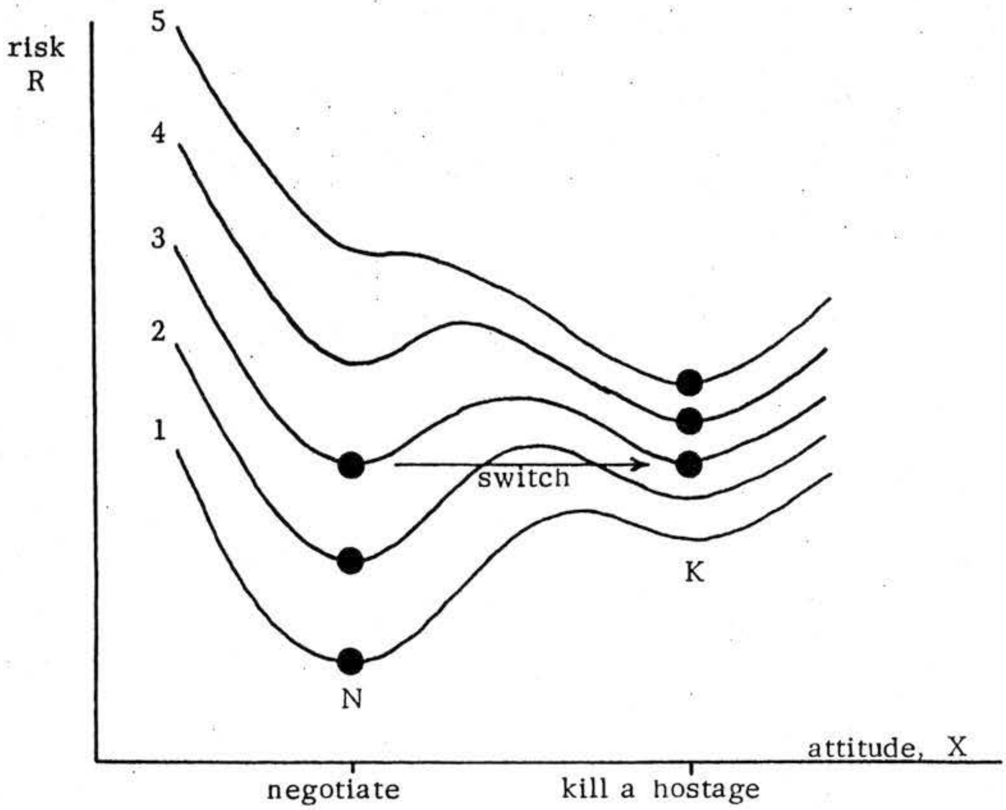


Figure 3. *Switch of attitude. The victim's intransigence causes a gradual change 1,2,3,4,5 in the risk function, and hence a sudden switch of attitude from N to K by the hijacker. For visual convenience we have drawn the graphs above one another. The important feature is the relative levels of N and K.*

The minimum at N is rising, while that at K is falling relative to N. N is below K in graphs 1 and 2, level with K in graph 3, and above K in graphs 4 and 5. In fact N has coalesced with the maximum in graph 5 and is about to disappear. The global minimum of each graph representing the Bayesian decision is marked with a dot, because this is the attitude that will be adopted. At graph 3 the global minimum switches from N to K, and therefore the hijacker will suddenly switch his attitude from negotiating to killing a hostage. His calculations of the risk functions may have been either conscious or subconscious, and hence his switch of attitude may appear either as a rational choice or as an instinctive reaction, similar to a sudden switch of perception [13].

This is the essence of catastrophe theory, a continuous change surprisingly causing a discontinuous effect. The gradually changing risk function causes the sudden catastrophic switch of attitude. The victim may well be caught off guard because the negotiations seem to be proceeding smoothly and apparently going his way when all of a sudden the hijacker turns nasty and kills a hostage. On the other hand, if the hijacker believes that the victim is negotiating seriously then N will remain below K, and there will be no need to kill a hostage.

Phase III.

Even if the negotiations are proceeding successfully, both the hijacker and the victim will be aware that time is on the side of the latter, and so the hijacker will gradually harden his attitude to the point of delivering an ultimatum if necessary. The minimum of his risk function R will therefore gradually move to the right from the point N in the phase II graph in Figure 2 to the point U in the phase III graph, representing an attitude in which he is prepared to issue an ultimatum. We suggest that a consequence of this move to the right will be to push the second minimum K even further to the right into a minimum B, representing an attitude in which he is now prepared to blow up the plane if necessary. For that is exactly what he threatens to do in the ultimatum.

We also suggest that a further consequence of this move to the right will be the emergence of another counterbalancing minimum S on the left, representing an attitude in which he is prepared to release hostages or even surrender. The more closely he has to face the possibility of failure of the ultimatum, leading to his own imminent death when he has blow up plane, the more likely he is to explore the risk of alternative options that might save his life. In effect the amount of aggression behind N will be split bimodally between S and U, and so the total aggression will be split trimodally between S, U and B.

If the victim perceives the emergence of S he may seize the opportunity to put pressure on the hijacker, for instance by renegotiating the deadline, calling his bluff, or threatening a shoot-out. The purpose of putting on the pressure is to raise the level of U until it is higher than S, in the hope that this will cause a sudden switch in the attitude of the hijacker from U to S, resulting in his surrender. The danger of this course of action, however, is that it might precipitate the opposite switch from U to B, resulting in the hijacker blowing up the plane, with himself and all the hostages in it. Therefore it is crucial to know whether S is higher or lower than B before putting on the pressure to raise U. The victim's best strategy may be to appear to concede to the ultimatum so as to keep the hijacker's attitude at U, while at the time entering a dialogue aimed at reducing the risk of S. For example the victim might offer to pay a modified ransom in exchange for the hostages, guaranteeing at least some benefit to the hijacker's cause. He could stress how much the hijacker has already benefitted from the publicity so far, and could offer generous surrender terms emphasising the advantages of life over death. The increasing exhaustion of the hijacker over a number of days may also contribute to the effect of lowering S relative to B, by sapping the hijacker's willpower to blow up the plane, and reducing the risk of S by modifying his beliefs and preferences.

If the victim can estimate the relative levels of S and B by carefully monitoring his dialogue with the hijacker, he may be able to detect when S drops below B, and choose that moment to put the pressure on in order to trigger the switch from U to S.

Notice that in the above discussion the catastrophes, in other words the sudden switches of attitude, have played a much more significant rôle than the gradual changes of attitude. This is surprising, because if you want to change someone's mind it might appear at first sight to be more natural to try and argue them gradually out of their present position. Such arguement, however, can be uphill work, and is indeed literally uphill if you are working against a subconscious risk function. It may be a more effective strategy to persuade them first to lower a distant minimum and second to raise the existing minimum, for then the subconscious will do the work for you by switching the attitude automatically. If the situation is trimodal as in phase III above, the strategy may involve several steps, and to decide the best tactical order in which to make those steps it may be necessary to appeal to the higher dimensional geometry of catastrophe theory, as follows.

4. *Catastrophe theory.*

The sophisticated theorems of catastrophe theory classify universal families of functions [5,7,10]. Given an evolution from a function R_1 to a function R_2 we can ask the question: what is the smallest universal family containing that evolution? The answer is called the *unfolding* of the evolution.

For example consider the evolution of the risk function shown in Figure 2 from phase I to phase II, going from unimodal to bimodal, in which T bifurcates into N plus K. The unfolding of this evolution is the *cusp catastrophe* illustrated in Figure 4.

The cusp catastrophe is a 2-dimensional family of risk functions parametrised by the 2-dimensional parameter space C represented by the horizontal plane in Figure 4. The attitude spectrum X is represented by the vertical line. For each point $c \in C$ we have a risk function R_c on X. To illustrate all the R_c's would require a 4-dimensional picture, which is difficult to visualise, and so we confine ourselves to the qualitatively most important feature, namely the maxima and minima of the R_c, as follows. Define the attitude surface A to be the surface in the 3-dimensional space $C \times X$ given by

$$A = \{(c,x);\ R_c \text{ has a maximum or minimum at } x\}.$$

Then A is the folded surface shown in Figure 4, of which the shaded area represents maxima, and the rest minima. The sheet at the back of A represents the attitude T (threaten), the bottom left front sheet represents N (negotiate), and the top right front sheet represents K (kill a hostage). The two front sheets merge together at the back, but form separate layers at the front. The fold curve $\overline{Q}$ of A separates the maxima and minima and projects down onto the cusp Q in C.

Q separates C into unimodal and bimodal risk functions as follows. If the parameter point c lies outside Q then the corresponding risk function R_c is unimodal, and so there is a unique point of A above c corresponding to the unique minimum of R_c. If c lies inside Q then R_c is bimodal, and so there are three points of A above c, corresponding to the two minima N and K separated by a maximum. The dashed line L bisecting Q is called the

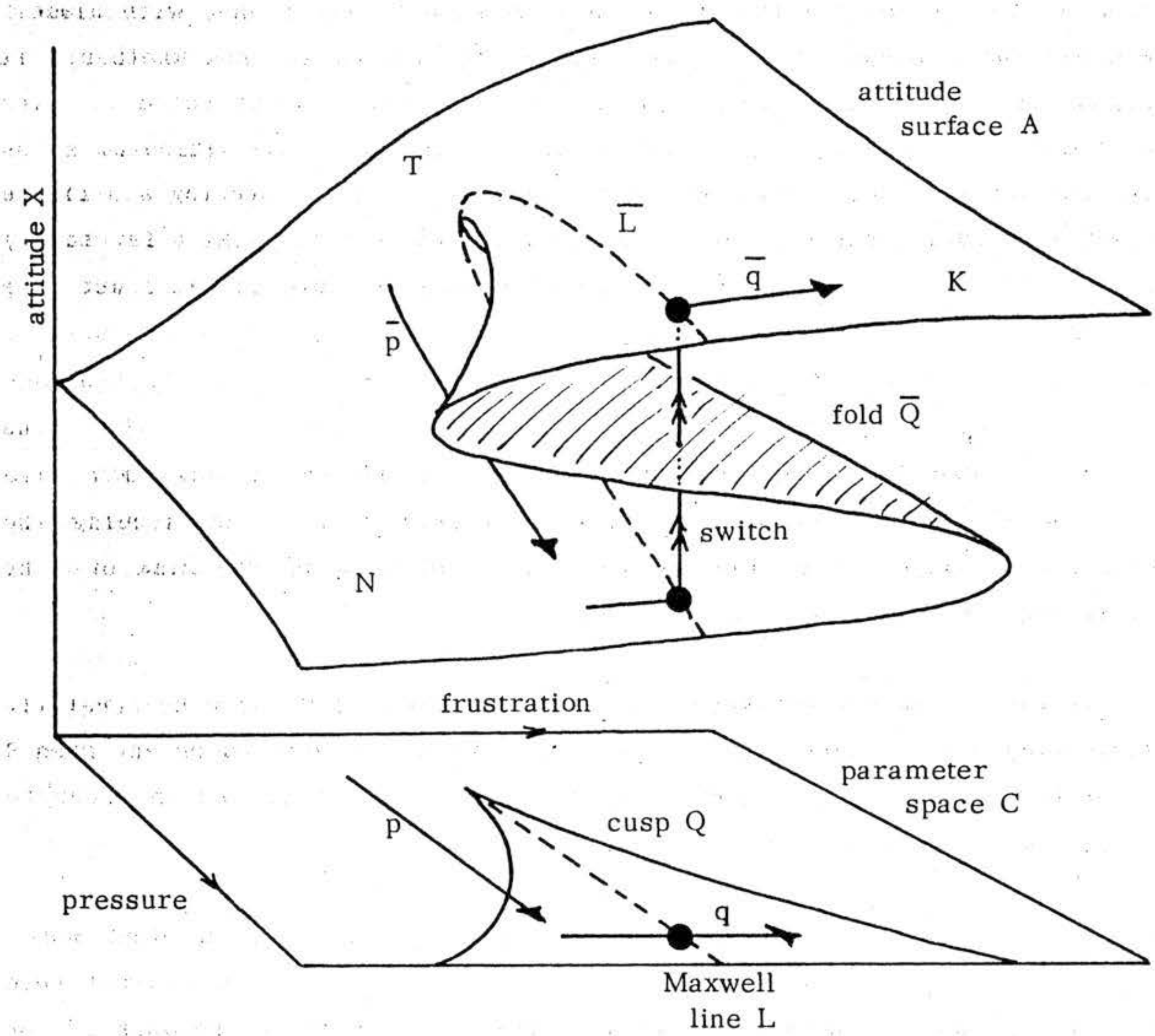


Figure 4. Cusp catastrophe unfolding the evolution from phase I to phase II. Paths $p,\overline{p}$ represent the gradual change in the hijacker's attitude from threatening to negotiating. Paths $q,\overline{q}$ represent the catastrophic switch of attitude from negotiating to killing a hostage as q crosses the Maxwell line L.

Maxwell line [10], and is the set of points c for which the minima N and K are at the same level, as in graph 3 of Figure 3. If c lies to the left of L inside Q then N is lower than K as in graphs 1 and 2 of Figure 3, so N is the attitude that will be adopted; and if c lies to the right of L then vice versa. Therefore L is the frontier across which sudden switches of attitude will take place, and the dashed parabola $\bar{L}$ is the induced frontier on the attitude surface A above. The gradual change of risk function illustrated in Figure 3 is represented in Figure 4 by the path q in the parameter space C, inducing the path $\bar{q}$ in the attitude surface A above, containing the sudden catastrophic switch from N to K as q crosses L.

The evolution from phase I to phase II illustrated in Figure 2 is represented in Figure 4 by the path p in the parameter space C, inducing the smooth path $\bar{p}$ in the attitude surface A above, representing the smooth evolution from T to N without any sudden switches of attitude.

The two main advantages of considering the unfolding of the evolution in Figure 4 are as follows. Firstly it provides a comprehensive picture of the variety of smooth changes and sudden switches of attitude that are possible under given changes of parameter, and explains how they are all interrelated. Understanding what is possible improves the chances of prediction.

Secondly the knowledge that the unfolding is 2-dimensional tells us to look for two parameters governing the risk function, and hence governing the changes of attitude. The clue to identifying these two parameters is given by the paths p and q. The parameter parallel to p is called the *splitting factor* (because it splits A) and that parallel to q is called the *normal factor* (because it correlates with x).

The path p representing the evolution from phase I to phase II is essentially caused by the pressure of time on the hijacker. He knows that time is on the side of the victim, and that unless he swallows some of his aggression and gets down to the negotiations the situation may well drift out of control. Compare the hijacker with a kidnapper, for instance, who has his hostage stashed away in some secret hideout, and hence is under no such pressure to negotiate. By contrast the hijacker is under increasing pressure, and so we can identify pressure with the splitting factor, whether it be pressure of time or any other kind of pressure exerted by the victim.

The path q, meanwhile, is caused by increasing frustration due to the intransigence of the victim, and so we identify the normal factor with increasing frustration, as shown in Figure 4.

At this stage the reader will see that the model is closely related to the classical cusp catastrophe model of aggression [9,10,11] shown in Figure 5. The fight/flight mechanism is phylogenetically ancient and permanently wired into the brains of most animals, and is therefore a basic template for many types of animal and human behaviour. The similarity of Figures 4 and 5 shows that our hijack model is compatible with basic instinctive human behaviour.

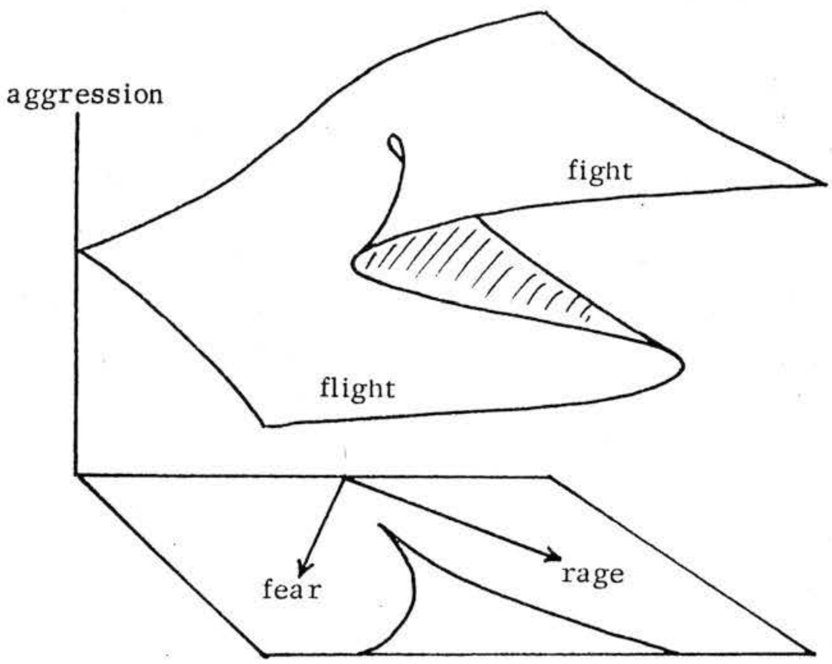


Figure 5. Cusp catastrophe model of aggression.

Phase III.

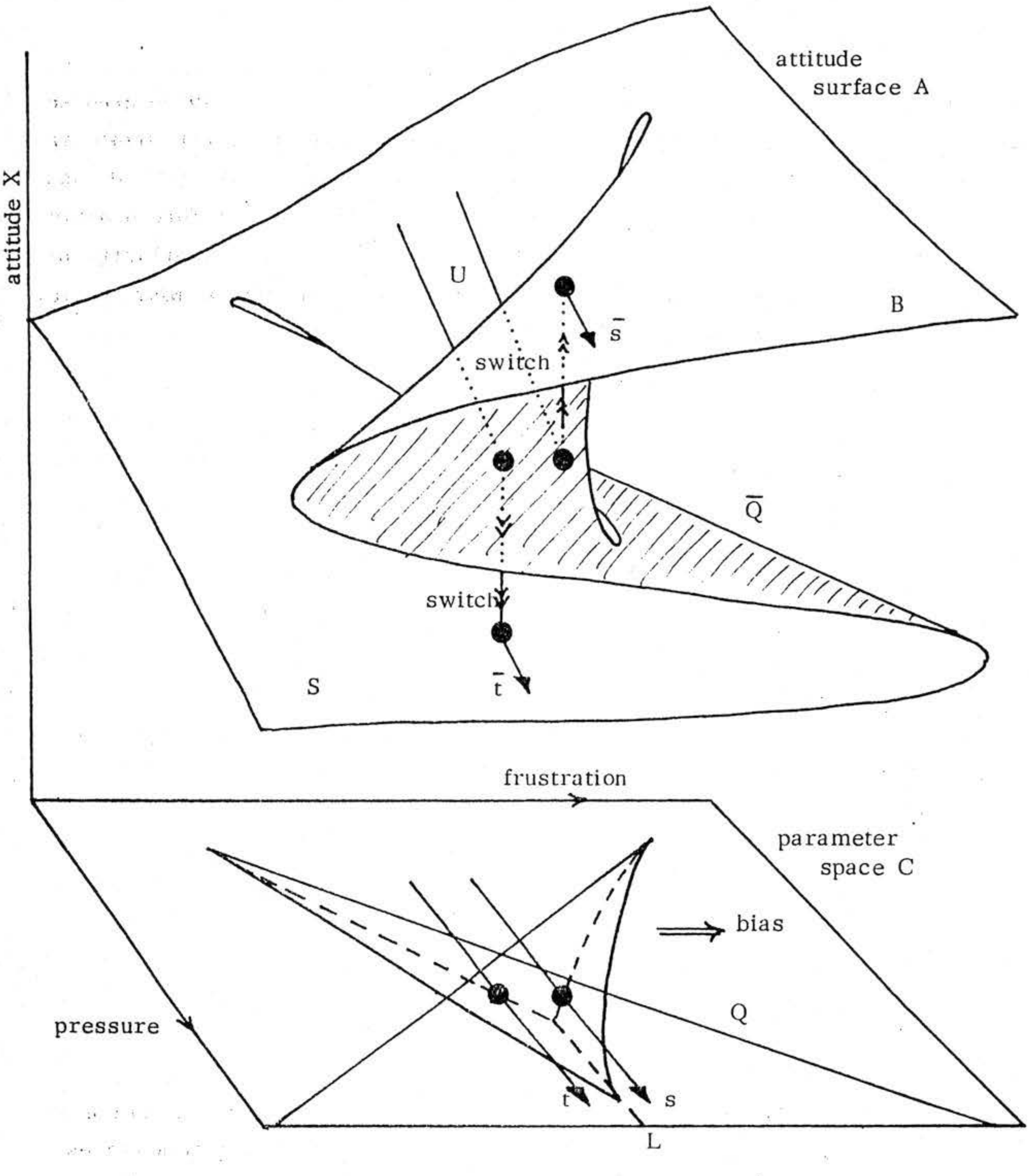
The evolution from phase II to phase III in Figure 2, in which N bifurcates into S plus U, unfolds into another cusp. The question arises: how do these two cusps fit together? The answer is revealed by unfolding the total evolution from phase I to phase III. This gives a butterfly catastrophe [7,10], which has four parameters, namely butterfly and bias factors in addition to the splitting and normal factors that we have already identified. We suggest identifying all four factors as follows:

normal factor : increasing frustration of the hijacker
splitting factor : increasing pressure on the hijacker
bias factor : increasing exhaustion of the hijacker
butterfly factor : increasing rapport with the victim.

We must now justify these suggestions in terms of the geometry of the butterfly catastrophe. The butterfly is a generalisation of the cusp, and the easiest way to understand it is to see the effect of the two new parameters on the cusp. Consider first the butterfly factor.

One of the consequences of the dialogue between the hijacker and the victim over a period of possibly a few days will be an increasing rapport between them. Each learns about the other's beliefs and preferences, strengths and weaknesses. The victim's security forces take care to funnel the dialogue through a single experienced negotiator, whom the hijacker can then get to know personally, and on whose word he can begin to rely. It is this increasing rapport that essentially facilitates the bifurcation of N into S plus U, because it makes it easier for the hijacker to deliver his ultimatum to the victim through the negotiator, together with all the appropriate logistical details. The negotiator has to accept the ultimatum, or at any rate has to promise to convey it faithfully to the victim, otherwise he will lose the hijacker's confidence. At the same time the rapport helps to establish the new minimum S, because the negotiator will work at creating a perception of the surender option in the hijacker's mind [13]. Thus the rapport is responsible for causing the trimodality of phase III. Therefore we can identify increasing rapport as the butterfly factor, which geometrically causes the evolution from Figure 4 to Figure 6, as we now explain.

Corresponding to the trimodality of phase III the attitude surface A in Figure 6 has evolved into three sheets of minima which merge together at the back but form separate layers at the front: the top right front sheet represents B (blow), the bottom left front sheet represents S (surrender), and the middle triangular sheet in between represents U (ultimatum). The cusp Q has evolved into a figure containing three cusps, which form the triangle underneath the triangular sheet U above. The Maxwell line L has evolved into a Y-shape, separating the regions of C dominated by B,S and U respectively. The path s in the parameter space C represents increasing pressure on the hijacker at a high level of frustration, and induces the



<u>Figure 6.</u> Section of the butterfly catastrophe modelling phase III. Paths s, s̄ represent increasing pressure inducing the disastrous switch from ultimatum to blowing up the plane. Paths t, t̄ represent the opposite switch from ultimatum to surrender.

path $\overline{s}$ on the attitude surface A above, containing the disastrous sudden switch of attitude from U to B (blowing up the plane) as s crosses the right branch of L. The increasing pressure is due partly to the pressure of time on the hijacker, but may also be due to a deliberate policy by the victim. The parallel path t represents a similar increase of pressure, but this time at a lower level of frustration, and consequently induces the opposite, and more desirable, switch from U to S (surrender) as t crosses the left branch of L.

The close similarity between these two paths in C reveals the delicacy of the situation, and emphasises the victim's need to know the precise position of the hijacker relative to L before he dares to increase the pressure. This is where the bias factor comes in.

Geometrically the effect of small positive bias is to move Q and L to the right. Therefore, relative to Q, both the hijacker and the frustration axis move to the left, transforming the disastrous path s into the desirable path t, while maintaining the same level of frustration. Thus a prior application of sufficient positive bias makes it safe for the victim to subsequently step up the pressure in order to trigger the hijacker's surrender without fear of blowing up the plane. We suggest that the increasing exhaustion of the hijacker is a positive bias factor, as explained in the last section. This may not be the only bias factor, because for instance there may be an overriding negative bias in the hijacker's personality; a fanatic may prefer martyrdom to surender however exhausted he may be. Estimation of the bias must be one of the main tasks of the consultant psychologist who is monitoring the dialogue for the victim.

As we have said, the effect of small bias is to move Q and L sideways. Geometrically the effect of large bias is to withdraw one of the horns of Q and abolish the corresponding branch of L, as shown in Figure 7 (for proof see [10]).

Negative bias leaves intact the right horn of Q, which is the old cusp that originally evolved in phase II. Positive bias leaves intact the left horn, which is the new cusp that evolved in phase III. Figure 7 explains how these two cusps fit together, and how they are both related to Figure 4 by suitably biasing and applying the butterfly factor. Figure 7 also shows

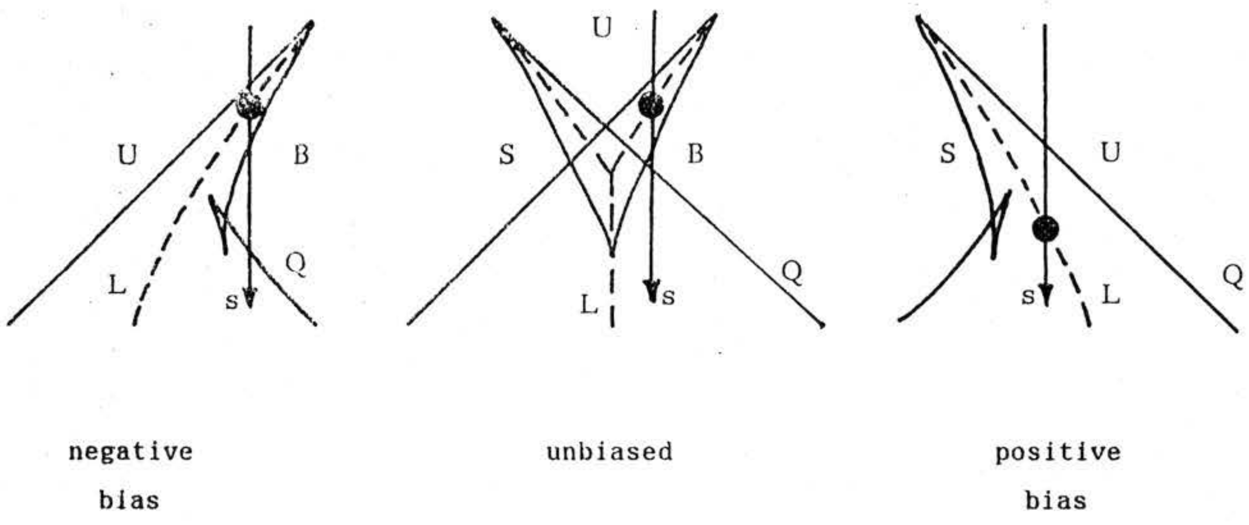


Figure 7. The effect of bias. In the unbiased situation the path s of increasing pressure induces the diastrous switch from ultimatum to blowing up the plane. Negative bias reinforces this switch, but positive bias converts it to the opposite switch of surrendering.

how the disastrous path s of suddenly blowing up the plane is converted by positive bias into surrendering. Notice that positive bias moves the dot on s lower: this means that it will take more pressure to make the hijacker surrender than it would have done to have made him blow up the plane.

For further discussion on the butterfly catastrophe, and the effects of other possible paths in parameter space see [2,5,10].

5. *Emotional delays.*

Before concluding we return to the theme of emotional versus rational decision making. So far in this paper we have assumed that the switch of attitude takes place at the Maxwell point, where one minimum falls below the level of another, as in graph 3 of Figure 3. This is essentially a property of intelligent decision making, and a mathematical consequence of minimising functions. By contrast in non-intelligent decision making the switches are delayed until the bifurcation point [12]. Here a bifurcation point means a point where a minimum coalesces with a maximum, as in graph 5 of Figure 3. In Figures 4 and 6 the set of bifurcation points is Q. A typical example of non-intelligent decision making is Darwinian evolution, since it is governed locally by natural selection [14].

We suggest that a similar delay may occur in switches of attitude, if the latter happen to be determined by the emotions rather than by rational thinking, as illustrated in Figure 8 and as we explain below.

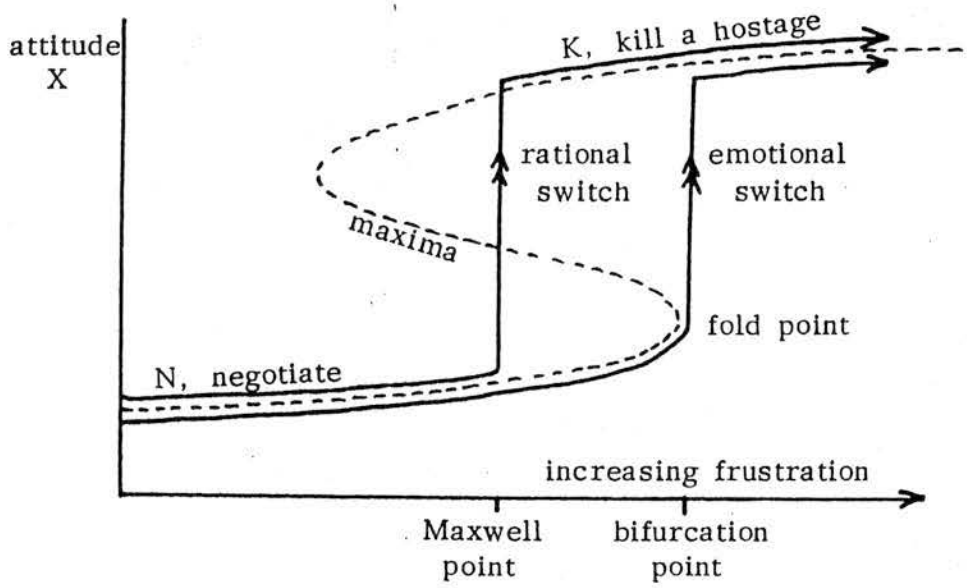


Figure 8. Rational and emotional switches.

Emotions and moods are governed by the limbic brain [4] rather than the cortex, and changes of mood characteristically exhibit delays and hysteresis. Therefore to model the limbic brain we choose a mathematical tool with similar characteristics, such as a dynamical system, whose attractors switch at the bifurcation point. By contrast, the cortex is responsible for the visual and auditory systems, focus of attention, rational thought and intelligent response, none of which exhibit such delays as the limbic brain. Therefore to model the cortex we choose a mathematical tool with the opposite characteristics, such as a risk functions, whose minima switch at the Maxwell point.

If the hijacker's attitude is primarily determined by rational thought, be it conscious or subconscious, calculating or instinctive, then the switches of attitude will take place when crossing the Maxwell line as we have assumed throughout the paper. If, however, his attitude is primarily determined by his emotions, then the switches will be qualitatively the same, but temporarily delayed until crossing the cusp line beyond the Maxwell line. The difference might be observable by monitoring the dialogue with the hijacker.

6. *Further developments.*

What needs to be done is to analyse the recorded dialogues of various hostage situations, to see if they can be interpreted in the light of the model and classified accordingly. Could the model have given a better insight in certain cases, or made better predictions of the hijacker's change of attitude? Could it be used predictively in future hijacks? A further complexity that needs analysing is to combine the model of the hijacker with a model of the victim into a single interacting model in higher dimensions. More generally it would be interesting to apply a similar model with appropriate parameters to other types of conflict situations.

I am indebted to Peter Shapland who first suggested the problem to me, particularly for the case of hostage-takers in prisons, where the inmates sometimes take the warders hostage.

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