The London Mathematical Society

LONDON MATHEMATICAL SOCIETY

Mathematics degrees, their teaching and assessment

Following requests from many colleagues, the LMS has produced this document to support everyone teaching mathematics in Higher Education. It aims to explain to non-mathematical colleagues some of the distinctive features of the assessment and teaching of mathematics, both as a subject in its own right and in a service teaching context. The document highlights, amplifies and interprets material, *quoted in italics*, from the QAA's subject benchmark statement for undergraduate programmes in mathematics, statistics and operational research (2007); to which the superscripts refer.

Recommendation 1 A student who fails a small number of individual modules, but has an overall satisfactory average, should not be deemed to have failed a degree programme in mathematics.

> *(mathematical) ideas can take considerable time to be assimilated: learners often do not fully understand something until some time after they have learned it. [Yet] assessment usually comes very soon after a topic has been studied... [this] should be recognized in assessment procedures and by those inspecting programmes.*[2.4]

> *Even the best learners frequently find some particular area(s) of [mathematics] difficult to grasp, and this may lead to [some] quite low marks in a profile that is clearly of overall excellence... In similar vein, learners of modest but nevertheless worthwhile attainment frequently present profiles where a number of modules are failed... The examiners might well judge that such a student, considered overall, has demonstrated a... positive achievement... It is entirely acceptable that this should be so.*[5.9- 5.10]

> *Institutions should accept that the patterns of marks achieved in [mathematics] assessments are likely to differ substantially from those achieved in other subjects, and therefore that global assessment regulations might be less applicable in (mathematics) than elsewhere.*[5.8]

Despite, in many cases, their best efforts, some students will utterly fail to grasp a particular topic by the time they are assessed on it, and a few never will. It is quite possible, and not uncommon, for them to write many pages of what (to the inexpert eye) appears to be mathematics, all of it devoid of meaning and value, and impossible to deem worthy of examination marks. Of course, a good written examination will incorporate some of the easiest, most elementary material – and yet, in a raw assessment of the proportion of the material understood, especially immediately or soon after their first exposure to it, some students will achieve little. Neither will any conversion from raw marks to a university scale, defined by qualitative descriptors[*], be able to turn such a performance into a passing one. Yet this may be, for these students, an aberration: they may well achieve a passing mark overall in their degree programme. The problem is that

[*] We note that, on such scales, in many humanities departments, the majority of marks usually fall into a narrow middle range from 50 to 75%. The situation in mathematics (and some related subjects) is very different; raw marks over 90% and under 20% are not at all uncommon.

the competence-based assessment of a fine, modular sub-division of the programme material, while appropriate for a vocational course, is not appropriate for a difficult academic discipline. A mathematics student has to engage bravely with substantial, interconnected, daunting constructions. To an academic mathematician, it is the student's having done so as a whole during their course, the *whole degree programme* learning outcome, which is important. If asked under what circumstances a student's failing one or a few modules should make him/her unworthy of a degree, they will typically reply – 'never'. The same is clearly not true of, say, a medical practitioner's ability to calculate drug doses correctly. Credit-framework models, however appropriate for some programmes, will nearly always be viewed as inappropriate for the assessment of mathematics.

We are deeply concerned about the tendency in some universities to impose on mathematics and other programmes the requirement that students pass essentially all modules, regardless of their overall average score. This changes the nature of the study of mathematics in our country; it encourages the removal of all challenging material from modules, and encourages the students to 'play safe' in their choice of subjects.

Recommendation 2 On occasion, a specific module should be available to more than one year of a mathematics degree.

> *Sequential subjects... require sequential layers of foundations, [continuing] right through a programme... The earlier years are not necessarily exclusively concerned with laying foundations; in many programmes it may be entirely proper for more advanced work... to start at an early stage, provided always that... prerequisite knowledge is in place.*[3.3]

Because of the cumulative, sequential nature of mathematics, it can sometimes be difficult to assign a clear level – a natural year of study in the undergraduate programme – to a particular topic. In a sense a mathematical topic fits quite naturally into a modular system: it has its pre-requisites, and in turn many more modules may require it. But it may be inappropriate to fix its horizontal level in this web. A module may quite naturally be studied in different years for different students, and yet still make appropriate contributions to the two students' differing sub-webs of connected modules.

Recommendation 3 Masters degrees in mathematics should not necessarily be obliged to reach the frontiers of knowledge.

> *[Mathematics] is a very advanced and highly developed subject and is being continually expanded by further advances in research. Many master's level programmes will nonetheless be informed by current research activity, [and] will provide a basis for originality in developing and applying ideas.*[A15]

Given the sequential nature of mathematics and its development over a very long timespan, it is unsurprising that in some areas of the subject the frontier could require five or more years of rigorous study to reach. There are no royal roads, no short-cuts: to study the higher levels students must have the experience and expertise that come with having re-constructed each lower level for themselves. Thus, while MMath programmes will have developed the students' mathematical maturity, and given them some sense of the nature of the research frontier, they will certainly not have reached it along a broad front. Some may, in a few modules, in specific examples in a lecture, or in a final-year project, have reached it in isolated thrusts, but it is unrealistic to expect more than this.

We now have some comments related to teaching styles most appropriate to HE mathematics.

Recommendation 4 Despite the agreed importance of modern, computer-based teaching and learning, lectures delivered using clearly visible boards should continue to play an important role.

The current widespread availability of high-performance PCs equipped with first-class software has led to tremendous advances in the teaching and learning of mathematics. As well as facilitating calculation (and introducing students to concepts in programming), MATLAB, for example, enables both the visualisation of concepts and results and the computation of the solution of complex equations, giving insight into both the solution and application of these equations in a way scarcely imaginable in the past. MAPLE and MATHEMATICA can aid in the practice of mathematics by accurately performing highly complex symbolic calculations and manipulations. Statistics packages have also made a big impact, allowing real-world data to be examined and analysed. These software packages are all practical aids to learning mathematics. Moreover they allow a process of exploring conjectures coupled with subsequent proof or counterexample.

Nevertheless, it is crucial to recognise that the teaching of the underlying mathematics requires a substantially different approach. In the teaching/learning process it must always be remembered that

> *Mathematics [involves] strict logical deduction with conclusions that follow with certainty and confidence from clear starting points.*[1.13]
>
> *By the nature of the subject area... the traditional board-based lecture continues to have substantial merit.*[4.6]

This rigorous process is the basis of understanding mathematics, and must be emphasized continually throughout the course. It is not sufficient to leave it to chance by requiring that theorems are studied passively by working through printed notes or a textbook, which may be altogether too hard a task. Rather one needs to see someone else, the lecturer, working through and *creating* the results. During all years of study the good lecturer will develop theorems by constructing the argument for the students, incrementally and in real time, and will describe at various levels what is being achieved. Like a good general, (s)he will explain not only the details but also the tactics and the overarching strategy, and thus will help the students to gain a fuller understanding. The lecturer must be able to create, to write out, during the lecture itself, a large body of argument, and many lecturers prefer to move around the theatre, sharing the students' view of the material. Ideally, most of what is written during a fifty-minute lecture will still be visible at the end: the lecturer will often be referring back to earlier material, not only to the detail but also to the thrust and tenor of it. Such a board-based teaching style naturally results in an appropriate pace, suitable for the students to take their own notes and aiding engagement with the material. To deliver this style of lecturing the lecturer will require several boards, with a large total area, on which writing is clearly visible to all of the students. In large lecture theatres especially, this may require chalkboards.

Nevertheless it is important that students also have easy access to summaries of lecture notes or appropriate text books, where the results of the lecture are clearly and correctly stated (noting that students often make mistakes when copying from the board). However, we do not encourage a general policy of provision to the students of full and detailed printed versions of the lectures in advance of the class (for example, by making them available on the internet or a virtual learning environment): this can give some students the impression that a quick glance at these notes is a satisfactory alternative to attending the lecture and engaging with the process of creating the material.

We learn mathematics by doing mathematics, and the lecture material will therefore be accompanied by problems and examples which tease out the context of the argument - why it works, where it does not, its generalizations and specializations. In the end the lecturer will have created a set of course materials which embody a perfect argument, and the students will have seen and learned about the construction of that argument, and its uses and implications.

Our conclusion is that, when used in conjunction with the technological and other developments associated with learning mathematics and for the reasons outlined above, lecture boards remain an important technology for teaching mathematics in an exciting and interactive way, leading to a good understanding of the subject.

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