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Berwick Prize: citation for Adrien Brochier and David Jordan

Short citation:

Professor Adrien Brochier of Institut de Mathématiques de Jussieu-Paris Rive Gauche and Professor David Jordan of the University of Edinburgh are awarded the Berwick Prize for their formative paper 'Integrating quantum groups over surfaces', published in the Journal of Topology (2018), written jointly with Professor David Ben-Zvi of the University of Texas at Austin.

Long citation:

Professor Adrien Brochier of Institut de Mathématiques de Jussieu-Paris Rive Gauche and Professor David Jordan of the University of Edinburgh are awarded the Berwick Prize for their formative paper 'Integrating quantum groups over surfaces', published in the Journal of Topology (2018), written jointly with Professor David Ben-Zvi of the University of Texas at Austin.

This paper brings together several currents in mathematics, and a bit of physics as well. One inspiration is the geometric Langlands program, which attaches to a surface S a category of sheaves over the space of flat connections on S. This is the spectral side of geometric Langlands in its Betti version. Work of Kapustin and Witten set geometric Langlands in topological field theory, and that provides direct inspiration for this paper. From this point of view the authors construct a 2-dimensional topological field theory, but shifted from the usual field theory: the invariant assigned to a closed surface is a category rather than a number. (In fact, this paper deals with punctured surfaces; a subsequent paper treats closed surfaces.) This is a rigorous version of the Kapustin–Witten theory, which is a topologically twisted variant of a certain 4-dimensional supersymmetric Yang–Mills theory.

The key tool deployed is factorisation homology. Factorisation homology was introduced by Beilinson and Drinfeld in an algebro–geometric context. It was brought over to topology in several works; this paper is one of the first concrete applications of the theory. The authors make several technical advances to carry out this application. For example, they need to work in a robust 2-category of categories. The foundations they develop have proved useful in other contexts. Once these are laid, their starting point for factorisation homology is the braided tensor category RepqG of representations of a quantum group, viewed as an E2-algebra in a 2-category of categories. Another set of results advances the theory of module categories of tensor categories. These were previously developed in more finite settings not adequate for the application here.

All of this is applied to compute the factorisation homology on punctured surfaces equipped with a combinatorial presentation. These are categories of modules over an algebra, and it is this algebra that is computed explicitly in terms of the combinatorial decomposition of the surface. In this way the authors recover familiar algebras in geometric representation theory.