Senior Berwick Prize: citation for John Greenlees and Brooke Shipley

Short citation:

Professors John Greenlees (Warwick University) and Brooke Shipley (University of Illinois Chicago) are awarded the Senior Berwick Prize for their paper 'An algebraic model for rational torus-equivariant spectra', published in the *Journal of Topology* in 2018.

Long citation:

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This paper is a fundamental breakthrough in equivariant stable homotopy theory. Stable homotopy theory begins with the study of generalized cohomology theories. Such a theory assigns to any space a sequence of abelian groups and we require this assignment to have good local-to-global properties; that is, we can piece together the cohomology of a space from an open cover. These are the basic invariants of algebraic topology. Cohomology theories have representing objects, which are collections of topological spaces with connecting data. These are known as spectra, and stable homotopy theory is the homotopy theory of spectra. This is basic to all of algebraic topology. Equivariant stable homotopy theory adds group actions, accounting for and studying the inherent symmetries in many topological questions. A great deal is known when the group is finite; however, for many geometric problems the real interest is with compact Lie groups. Far less was known in this generality.

The Whitehead Theorem says that a map of simply-connected simplicial complexes is a homotopy equivalence if and only if it induces an isomorphism on classical simplicial homology. Rational homotopy theory asks only that such a map induces an isomorphism on homology with coefficients in the rational numbers. Work of Serre from the 1950s implies that rational stable homotopy is equivalent to the study of integer graded chain complexes over the rationals. Thus, we have a simple algebraic model in this case. Working over a finite group, Greenlees and May showed that rational equivariant homotopy theory is equivalent to differential graded objects in Mackey functors, an efficient algebraic device for keeping track of the input of subgroups. This work appeared in the 1980s, now over thirty-five years ago. For many years there wasn't even a proposal for a generalization.

The Greenlees-Shipley paper gives the first systematic answer for compact Lie groups. The essential difficulty is that Lie groups have themselves a rich topological structure and this must be accounted for in any algebraic model. The main result of this paper is to show that rational torus-equivariant stable homotopy theory is equivalent to a category of chain complexes of sheaves. More precisely, Greenlees and Shipley use the subgroup structure of the torus to create a diagram of rings that functions much like the structure sheaf of a projective scheme, and the torus-equivariant homotopy theory is equivalent to chain complexes of quasi-coherent modules over this structure sheaf.

This is the culmination of many years of work, but it is by no means the end: there has been considerable follow-on research. This paper shows where to look for answers.