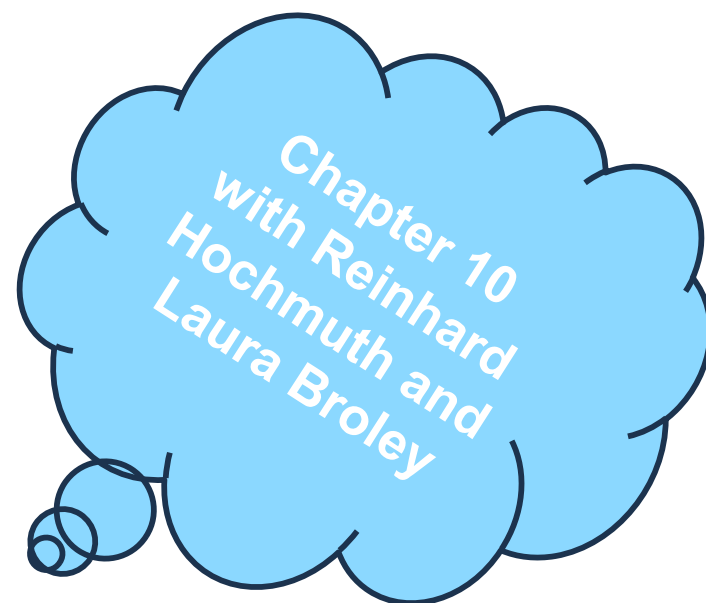


Thank you

for the invitation, your hospitality and your presence today!



Research in Mathematics Education (RME) Group at UEA
www.uea.ac.uk/education/research/areas/mathematics-education



INTRODUCTION

Part I: three types of transition

- from school to university mathematics
- within university courses
- from university to the workplace (where the workplace includes also the mathematics classroom)



epistemological, pedagogical, social, cultural and affective issues
[theories]

Part II: Example of intervention study on transition from school to university



Gueudet, Bosch, diSessa, Kwon, Verschaffel, Biza et al.



Research in Mathematics Education (RME) Group at UEA
www.uea.ac.uk/education/research/areas/mathematics-education



INTRODUCTION



A challenge:

what counts as school and university mathematics varies
significantly across contexts

Examples

Type I: limits of sequences

(school: informal, algebraically calculated; university: formal definitions of limits in \mathbb{R} , \mathbb{C} , metric- or Banach-spaces, axiomatically introduced, fundamental for understanding continuity and differentiability; sometimes even a Type II transition!)

Klein's (1908/1939) double discontinuity



Research in Mathematics Education (RME) Group at UEA
www.uea.ac.uk/education/research/areas/mathematics-education



INTRODUCTION

A challenge:

what counts as school and university mathematics varies significantly across
contexts

Examples

Type II: continuity

(beginner courses: based on definition of limit; topology courses: for
functions between topological spaces)

vertical transitions but also **horizontal transitions**, e.g. Riemann – or
Lebesgue – integral in courses about partial differential equations or
stochastics

mixture of horizontal and vertical interrelations / transitions: from
mathematics service courses to non-mathematics major courses, Fourier
series for engineers but also in electrical engineering signal theory

Klein's (1908/1939) double discontinuity



INTRODUCTION

A challenge:

what counts as school and university mathematics varies significantly
across contexts

Examples

Type III: from university to the workplace (e.g. teaching)

(we may know well the mathematics we teach; but knowing
something and knowing how to teach something is not the same!)

didactic transposition, PCK: addressing students' learning
needs, choosing appropriate ways to introduce new contents,
design and carry out assessment etc

Klein's (1908/1939) double discontinuity



Research in Mathematics Education (RME) Group at UEA
www.uea.ac.uk/education/research/areas/mathematics-education



INTRODUCTION

Part I: three types of transition

- from school to university mathematics
- within university courses
- from university to the workplace (where the workplace includes also the mathematics classroom)



**epistemological, pedagogical, social, cultural and affective issues
[theories]**

Gueudet, Bosch, diSessa, Kwon, Verschaffel, Biza et al.



Research in Mathematics Education (RME) Group at UEA
www.uea.ac.uk/education/research/areas/mathematics-education



THEORETICAL LENSES THAT CAPTURE TRANSITION

- a first period of transition studies: **cognitive approaches**, identifying transition as a need for conceptual changes
- “advanced mathematical thinking” (AMT book, Tall’s 3 worlds), Fischbein’s tacit models
- major differences between school and university mathematics: informal vs. formal, concrete vs. abstract, calculation- vs. structure-oriented
- most studies are on Type I transitions
- As we move to Type II and Type III, **a need for theories that attend to social, interactional and intersubjective issues** emerges...
- We stand on the shoulders of these cognitively focused findings and move forward...

Three key sets of theories

Concept image - concept definition and APOS

Anthropological Theory of the Didactic (ATD)

Theory of Commognition (ToC)

Artigue, Gueudet et al., Nardi



Research in Mathematics Education (RME) Group at UEA
www.uea.ac.uk/education/research/areas/mathematics-education



THEORETICAL LENSES THAT CAPTURE TRANSITION



Concept image - concept definition and APOS

Shlomo Vinner in the 1970s, Tall & Vinner (1981)
distinction between formal definitions and
individual accounts of these definitions

APOS (action-process-object-schema)
(Dubinsky, 1991; Arnon et al., 2013)

based on Piaget's genetic epistemology (reflective abstraction)

Artigue, Gueudet et al., Nardi



Research in Mathematics Education (RME) Group at UEA
www.uea.ac.uk/education/research/areas/mathematics-education



THEORETICAL LENSES THAT CAPTURE TRANSITION



Anthropological Theory of the Didactic (ATD)

knowledge as something that lives in institutions
institutionalisation of knowledge as the result of complex
transformation processes
ecological and hierarchical scale of **“levels of codetermination”**:
civilisation, society, school, pedagogy, discipline, domain, ...
praxeology (logos about praxis):
4T model: task, technique, technology, theory
“study and research paths” approach: a variation of inquiry-
oriented teaching efforts

Chevallard, Bosch, Hausberger, Hochmuth, Winsløw



Research in Mathematics Education (RME) Group at UEA
www.uea.ac.uk/education/research/areas/mathematics-education



THEORETICAL LENSES THAT CAPTURE TRANSITION

Theory of Commognition (ToC)

Communication + Cognition = Commognition
knowledge emerging through and during communication

commognitive conflict

focus on **discursive shifts** within and between different communities

word use, **visual mediators**, **endorsed narratives** and **routines**
routines can be rituals and explorations

a **non-binary**, **non-deficit**, **nuanced** lens

Sfard, Viirman, Biza, Nardi



Research in Mathematics Education (RME) Group at UEA
www.uea.ac.uk/education/research/areas/mathematics-education



THEORETICAL LENSES THAT CAPTURE TRANSITION

Not only these theories...

Psychological underpinnings of transition anxiety
questionnaires...

Realistic Mathematics Education

Theory of Didactic Situations

And so on...

Artigue, Gueudet et al., Nardi

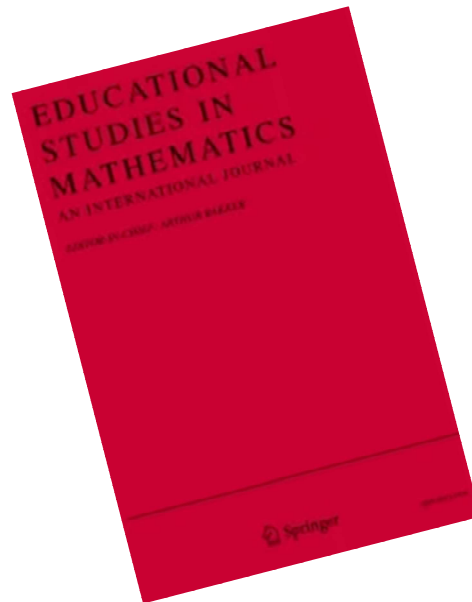


Research in Mathematics Education (RME) Group at UEA
www.uea.ac.uk/education/research/areas/mathematics-education



three types of transition / Type I

- from school to university mathematics



Volume 113, Issue 1
May 2023
**Transition from
school into university
mathematics:
experiences across
educational contexts**
Editors:
Pietro Di Martino
Francesca Gregorio
Paola Iannone

Di Martino, Gregorio, Iannone



Research in Mathematics Education (RME) Group at UEA
www.uea.ac.uk/education/research/areas/mathematics-education



three types of transition / Type I, from school to university mathematics



an **exciting** and often **complex** experience

many facets: mathematical, curricular, academic and social

new discourse governed by new rules that may intrigue / alienate newcomers

obvious contextual differences but **key features** in common

academic and social differences in learning environment (changes in living arrangements, location, friendship groups, relationships with peers and teachers)

most emphatically: a **shift in thinking mathematically**

increased requirements for **rigour and precision**

moving from instrumental to more relational approaches

developing more **learner autonomy**

deficit discourses on mathematical under-preparedness

Gueudet et al, Solomon, Nardi et al, Bampili et al, Thomas et al



Research in Mathematics Education (RME) Group at UEA
www.uea.ac.uk/education/research/areas/mathematics-education



three types of transition / Type I, from school to university mathematics



foregrounding

mathematical content
students' approaches to study
institutional support

Gueudet et al, Solomon, Nardi et al, Bampili et al, Thomas et al



Research in Mathematics Education (RME) Group at UEA
www.uea.ac.uk/education/research/areas/mathematics-education



three types of transition / Type I, from school to university mathematics

foregrounding

mathematical content

- transition from **informal to formal** calculus
- what **means of justification** students bring to their mathematical work
- what forms of **scientific debate** may help students' arguments grow more robust
- what **guided intervention** processes may assist students' collective construction of a definition (convergence)
- classification of arriving students' **concept images** of decimals and irrationals
- **epistemological analyses** of the transition to a formal definition of real numbers
- increased complexity of **signs and reasoning** processes
- challenges in learning new **techniques** and diagrammatic reasoning
- problematising **compartmentalisation** of the curriculum (probability, continuous distributions; calculus, integrals)

Lecorre, Ghedamsi, Schüler-Meyer, Kidron, Bloch, Gibel, Derouet



Research in Mathematics Education (RME) Group at UEA
www.uea.ac.uk/education/research/areas/mathematics-education



three types of transition / Type I, from school to university mathematics

foregrounding

students' approaches to study

- how students' **independent work** changes as they enter university
- how they use **resources** (greater variety at university)
- what concerns, and sometimes **hesitation**, students have about how to deploy resources available to them (and deficits of strategy thereof)
- how university mathematics teachers expect students to use resources and how students actually use them (**epistemic versus pragmatic mediation**)
- **diminished role of the teacher** as the main resource
- associations between students' use of resources and **course organisation**
- poor alignment between how mathematical content is presented in **textbooks** and in lectures

Gueudet, Pepin, Kock, Quéré



Research in Mathematics Education (RME) Group at UEA
www.uea.ac.uk/education/research/areas/mathematics-education



three types of transition / Type I, from school to university mathematics



foregrounding

institutional support

- transitions experienced differently by students with **different SES**
- part of concurrent, broader and deeper **identity** formation processes
- student **learning support** systems, personal tutoring and peer-support systems
- programmes that aim to raise **appreciation** of university mathematics
- identifying newly arriving **students' interest** in different aspects of mathematics such as real-world problems, calculations and proofs
- exploring who may be at **risk of dropping out** of university mathematics
- evaluating **tailored support** for students (bridging courses)

Biehler, Eichler, Kuklinski, Landgärds, Törner, Rach, Bracke



Research in Mathematics Education (RME) Group at UEA
www.uea.ac.uk/education/research/areas/mathematics-education



three types of transition / Type I, from school to university mathematics

foregrounding

institutional support

- analysis of **affective variables** such as interest, mathematical self-concept, basic needs and self-efficacy and goal-fulfilment
- **tracing decline in interest** and shifts in student beliefs e.g. about the “toolbox” nature of mathematics
- the **impact of anxiety and personality** factors
 - Mathematics Anxiety Rating Scale (MARS)
 - “Big Five” inventory (extraversion, agreeableness, conscientiousness, neuroticism and openness)

*[as transition from school to university mathematics ease...
Type II...]*

Biehler, Eichler, Kuklinski, Landgärds, Törner, Rach, Bracke



Research in Mathematics Education (RME) Group at UEA
www.uea.ac.uk/education/research/areas/mathematics-education



three types of transition / Type II, within university courses

transitions **between mathematics courses** that occur in
succession

(i.e., one is the prerequisite of the other, mainly in the Analysis
path / single variable Calculus, multivariable Calculus, Real
Analysis, Metric Spaces, ...)

transitions **from foundational mathematics courses to
specialist courses** for disciplines that use mathematics
(e.g. engineering)

Hardy, Broley, Winsløw, Bergé, Hamza, O'Shea, González-Martín, Hernandez-Gomes



Research in Mathematics Education (RME) Group at UEA
www.uea.ac.uk/education/research/areas/mathematics-education



three types of transition / Type II, within university courses

transitions **between mathematics courses** that occur in succession

Example / Bergé: from one course to the next, activities given to students (first find suprema of concrete sets, then use the supremum as a tool in proof tasks). Students first saw the supremum either as not being useful, or as an ordinary upper bound; then could name situations where the supremum functions as a tool.

transitions **from foundational mathematics courses to specialist courses** for disciplines that use mathematics

Example / González-Martín (engineering): comparative analysis of how integrals are presented in Calculus textbooks and in Strength and Materials textbooks. Norm is that engineers learn about them. However, techniques for calculating integrals are not needed to solve Strength and Materials tasks. Engineering students question need to learn integrals...

Hardy, Broley, Winsløw, Bergé, Hamza, O'Shea, González-Martín, Hernandez-Gomes



Research in Mathematics Education (RME) Group at UEA
www.uea.ac.uk/education/research/areas/mathematics-education



three types of transition / Type III, from university to the workplace



to **primary / secondary** school teaching

to **post-secondary** teaching

...transitions to different workplaces increasingly being studied ...

Cooper, Pinto, Biza, Nardi, Hanke, Schäfer, Stender, Corriveau, Florensa, Broley



Research in Mathematics Education (RME) Group at UEA
www.uea.ac.uk/education/research/areas/mathematics-education



three types of transition / Type III, from university to the workplace

to **primary / secondary** school teaching

MathTASK

(student) teachers engage with fictive, albeit realistic classroom situations

data analysed per four characteristics

consistency (between intended and actual practice)

specificity (to the situation)

reification of pedagogical discourses / reification of mathematical discourses

we analyse / facilitate transition process

Student teachers do not naturally **gain the “higher standpoint”** on school mathematics that is intended by curriculum developers; we need to help them

There are **natural connections between university and school mathematics**; let's find and focus on these (e.g. understand the problem, change representation, consider special cases, break the problem into sub-problems...)

Cooper, Pinto, Biza, Nardi, Hanke, Schäfer, Stender, Corriveau, Florensa, Broley

Research in Mathematics Education (RME) Group at UEA

www.uea.ac.uk/education/research/areas/mathematics-education



three types of transition / Type III, from university to the workplace

to *post-secondary teaching*

- **challenge of transferring results** about schoolteachers to the post-secondary level
- transition as a **discursive shift** in one's narratives about mathematics and its pedagogy
- **designing and evaluating** attempts at easing transition
- lecturer / TA courses and the pitfalls of “contentless”,
“pedagogical generalism”
- exploring the **interplay** between mathematical research and pedagogical practice (gaps, opportunities, authenticity, workplaces)

Cooper, Pinto, Biza, Nardi, Hanke, Schäfer, Stender, Corriveau, Florensa, Broley

Research in Mathematics Education (RME) Group at UEA

www.uea.ac.uk/education/research/areas/mathematics-education



three types of transition / Type III, from university to the workplace

*...transitions to **different workplaces** increasingly being studied ...*



**The Learning and Teaching of Calculus
Across Disciplines**

5-9 Jun 2023 Bergen (Norway)

Biology, Chemistry, Economics, Engineering, Physics

Special Issue: coming soon!



Cooper, Pinto, Biza, Nardi, Hanke, Schäfer, Stender, Corriveau, Florensa, Broley

Research in Mathematics Education (RME) Group at UEA

www.uea.ac.uk/education/research/areas/mathematics-education



three types of transition / Part I, concluding remarks

Part I: three types of transition

- from school to university mathematics
- within university courses
- from university to the workplace

epistemological, pedagogical, social, cultural and
affective issues
[theories]

three types of transition / Part I, concluding remarks

a **vibrant and broadening** research area

internationality / importance of **contexts**

Type II / Type III: rather **new**

beyond Year 1 or 2: even less!

more Calculus / Analysis than Algebra or Statistics

less **within-discipline uses** of mathematics,
other than engineering

need to **broaden** mathematical **skills studied**
(numeracy, computational thinking, logic, statistical thinking)

three types of transition / Part I, concluding remarks

*From a commognitive (ToC) perspective, transitions are currently studied mostly in terms of **changes in meta-discursive rules**, changes in word use and emerging **commognitive conflicts** in the interactions between students and teachers. There is ample space for data collection and **analyses of a different grain size**, for example for studies of how changes in the **curriculum** and in **institutional factors** play out in the way students' discursive activity changes from one educational level to another, from one course to another, from the world of study to the world of work, etc.*

three types of transition / Part I, concluding remarks



need for research that evaluates **interventions** or
explores **sustainability** issues

challenge tendency for **transition measures** as “**add-
ons**” to existing systems



need for **deeper systemic changes**

need for **cross-community collaboration** ...

Part II Example of cross-community collaboration on a transition initiative

Research-informed and collaborative reform in university mathematics teaching: The case of secondary to tertiary transition

George Witty, Owen Hooper, Elena Nardi, Irene Biza
G.Witty@uea.ac.uk, O.Hooper@uea.ac.uk, E.Nardi@uea.ac.uk, I.Biza@uea.ac.uk
University of East Anglia, Norwich, United Kingdom

Introduction

- Mathematics students are known to face many difficulties during their transition from secondary to tertiary education (Ellis, 2008).
- During the teaching of an introductory course for first-year mathematics students, the first author (Witty) encountered numerous issues which he associated with this transition, particularly related to students' attempts at mathematical communication, as well as their low engagement levels.
- For example, it was common to encounter students who would produce solutions that used new mathematical words and symbols, but lacked the rigour he would expect from something 'mathematical'.
- Witty became interested in learning more about changes he could make to help.
- Witty started working with Mathematics Education doctoral researcher Hooper whose study focuses on active learning in university mathematics teaching (Hooper & Nardi, 2024).
- This poster follows a reflexive autoethnographic approach (Ellis et al., 2010).
- Witty reflects on his own experience of designing and implementing a new set of teaching resources for a Year 1 Maths course, titled 'Mathematical Skills'.
- In this course, students are introduced to core definitions and concepts of pure mathematics, such as properties of sets, numbers and functions, as well as to mathematical proof. When Witty took on this course, he was given materials which lend themselves to a typical 'chalk-and-talk' lecture format.

Past issues when teaching the course

In teaching the course in this way Witty encountered various issues:

- There was a lack of student engagement in terms of participation.
- Students struggled to use new mathematical words and symbols correctly.
- It was difficult to convey passion for mathematical thinking when constrained to a blackboard.
- Problem classes often required repetition of content since students had not engaged with material in between lectures and the classes.

Early attempt to address issues

- Witty attempted to fix some of these problems in his second year of teaching the course by introducing quizzes into lectures and moving some problem solving activity into lecture time.
- Things improved, but the lack of a systematic approach and understanding of whether he was changing things in a pedagogically sound way meant he did not feel he was reaching the full potential of moving to a new lecture format.

Working with researchers

- Witty was made aware of an opportunity to work with the university's Mathematics Education research group which intrigued him.
- This would involve redesigning the course to include more interactive elements and different teaching methods.
- Witty became involved and through regular meetings with Hooper, as well as meetings with Nardi and Biza, an approach to planning and designing the new structure was formed.

Initial planning

- The process began with two key tasks:

 - Identifying which parts of the lecture notes related to content such as definitions, notation or results, as well as which elements would benefit from interactive elements such as quizzes or problem solving activity.
 - Portioning the content into small sections to allow topics to be matched to the timetabled lecture slots.

- In performing the latter of these tasks, Witty found himself begin to feel more in control of the content compared to previous years.

Creating lesson plans

- These initial plans were then converted into lesson plans which accompany each lecture.
- These plans not only details each activity with timings, but also include reasons for why each activity is included.
- Justifying the 'why' in writing ensured Witty felt like he was actually picking the best activity for each situation.
- The structured lesson plans meant Witty felt more committed to delivering engaging activities with the students than in previous years.

Padlet activities

- One of the common activities used was student submission in lectures using Padlet (padlet.com).
- Student submissions on Padlet varied from short answers to photos of written proofs or solutions to problems.
- This worked a lot better than Witty initially expected.
- Witty had previously felt compelled to ensure he showed students all of the material and gave full proofs.
- Work posted on Padlet often replaced content that was typically lectured, e.g. students would prove key theorems.
- Before attempting this Witty felt this would take too long or students would struggle, but instead Witty found students could cope and so he decided to keep giving students content that would typically be seen as core lecture content.
- Allowing students to submit their own work meant Witty felt his role shifted from that of an omniscient presenter conveying mathematics to a guide working with the students to explore mathematics.

Woodcap quizzes

- Witty's lecture plans made him realise that whilst previous use of quizzes on Woodcap (woodcap.com) were fun, they were not always aligned to the teaching aims.
- The new lesson plans ensured Witty could justify the use of quizzes and he felt like students would benefit from them.
- Witty found that his most more of his quiz questions led to discussion, for example when the class was split.
- Witty was more invested in such discussions and they were easier to hold, when it was clear there was an issue arising from student answers.

Summary

- Justifying 'why' different teaching activities are included helped Witty establish confidence in the chosen activities.
- Student contributions allowed Witty to gauge students' understanding and directly address issues as they were arising.
- Student contributions actively gave Witty content to deliver in lecture.
- Discussion with researchers allowed Witty to reflect on his teaching regularly.
- The whole process took quite a lot of time. Witty believes when running the course again it wouldn't take so much time since there would not be new activities to create.
- Individual lesson plans with timings will allow for easy adaptation or 'modular' use of content.
- For future teaching, Witty intends to follow a process of creating similar lesson plans and using as much interactivity as possible.

References

- Ellis, C., Adams, T. E., & Bochner, A. P. (2010). Autoethnography: An Overview. Forum: Qualitative Social Research, 12(1). <https://doi.org/10.17169/for-12.1.1589>
- Gossard, G. (2008). Investigating the secondary-tertiary transition. Educational Studies in Mathematics, 67(3), 237-254. <https://doi.org/10.1007/s10649-007-9100-6>
- Hooper, O., & Nardi, E. (2024). Emergency Remote Teaching Inspired a Blended Learning, Flipped Classroom Intervention in a University Mathematics Course. In Proceedings of the Fifth conference of the International Network for Didactic Research in University Mathematics (INDRUM 2024) (pp. 801-810). <https://indrum2024.scienceandart.org/>

<https://tinyurl.com/CERME14-Poster-Witty-et-al>

Thank you

Q&A after Paul's keynote!



**INDRUM2026: Sixth conference of the
International Network for Didactic Research
in University Mathematics**

15-19 Jun 2026 Dubrovnik (Croatia)

**Submit a paper or a poster proposal!
Join us!**