Thank you

for the invitation, your hospitality and your presence today!

Chapter 10 with Reinhard Hochmuth and Laura Broley

INTRODUCTION

Part I: three types of transition

- from school to university mathematics
- within university courses
- from university to the workplace (where the workplace includes also the mathematics classroom)

epistemological, pedagogical, social, cultural and affective issues
[theories]

Part II: Example of intervention study on transition from school to university

Gueudet, Bosch, diSessa, Kwon, Verschaffel, Biza et al.

INTRODUCTION

A challenge:
what counts as school and university mathematics varies significantly across contexts
Examples

Type I: limits of sequences

(school: informal, algebraically calculated; university: formal definitions of limits in $\mathbb{R}$, $\mathbb{C}$, metric- or Banach-spaces, axiomatically introduced, fundamental for understanding continuity and differentiability; sometimes even a Type II transition!)

Klein's (1908/1939) double discontinuity

INTRODUCTION

A challenge:

what counts as school and university mathematics varies significantly across contexts

Examples

Type II: continuity

(beginner courses: based on definition of limit; topology courses: for functions between topological spaces)

vertical transitions but also **horizontal transitions**, e.g. Riemann – or Lebesgue – integral in courses about partial differential equations or stochastics

mixture of horizontal and vertical interrelations / transitions: from mathematics service courses to non-mathematics major courses, Fourier series for engineers but also in electrical engineering signal theory

Klein's (1908/1939) double discontinuity

INTRODUCTION

A challenge:

what counts as school and university mathematics varies significantly across contexts

Examples

Type III: from university to the workplace (e.g. teaching)

(we may know well the mathematics we teach; but knowing something and knowing how to teach something is not the same!)

didactic transposition, PCK: addressing students' learning needs, choosing appropriate ways to introduce new contents, design and carry out assessment etc

Klein's (1908/1939) double discontinuity

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Gueudet, Bosch, diSessa, Kwon, Verschaffel, Biza et al.

THEORETICAL LENSES THAT CAPTURE TRANSITION

- a first period of transition studies: **cognitive approaches**, identifying transition as a need for conceptual changes
- "advanced mathematical thinking" (AMT book, Tall's 3 worlds), Fischbein's tacit models
- major differences between school and university mathematics: informal vs. formal, concrete vs. abstract, calculation- vs. structure-oriented
- most studies are on Type I transitions
- As we move to Type II and Type III, **a need for theories that attend to social, interactional and intersubjective issues** emerges…
- We stand on the shoulders of these cognitively focused findings and move forward…

Three key sets of theories

Concept image - concept definition and APOS

Anthropological Theory of the Didactic (ATD)

Theory of Commognition (ToC)

Artigue, Gueudet et al., Nardi

THEORETICAL LENSES THAT CAPTURE TRANSITION

Concept image - concept definition and APOS

Shlomo Vinner in the 1970s, Tall & Vinner (1981) distinction between formal definitions and individual accounts of these definitions

APOS (action-process-object-schema) (Dubinsky, 1991; Arnon et al., 2013)

based on Piaget's genetic epistemology (reflective abstraction)

Artigue, Gueudet et al., Nardi

THEORETICAL LENSES THAT CAPTURE TRANSITION

Anthropological Theory of the Didactic (ATD)

knowledge as something that lives in institutions

institutionalisation of knowledge as the result of complex transformation processes

ecological and hierarchical scale of **"levels of codetermination"**:

civilisation, society, school, pedagogy, discipline, domain, …

praxeology (logos about praxis):

4T model: task, technique, technology, theory

"**study and research paths**" approach: a variation of inquiry-oriented teaching efforts

Chevallard, Bosch, Hausberger, Hochmuth, Winsløw

THEORETICAL LENSES THAT CAPTURE TRANSITION

Theory of Commognition (ToC)

Communication + Cognition = Commognition

knowledge emerging through and during communication

commognitive conflict

focus on **discursive shifts** within and between different communities

word use, **visual mediators**, **endorsed narratives** and **routines**

routines can be rituals and explorations

a **non-binary**, **non-deficit**, **nuanced** lens

Sfard, Viirman, Biza, Nardi

THEORETICAL LENSES THAT CAPTURE TRANSITION

Not only these theories…

Psychological underpinnings of transition anxiety questionnaires…

Realistic Mathematics Education

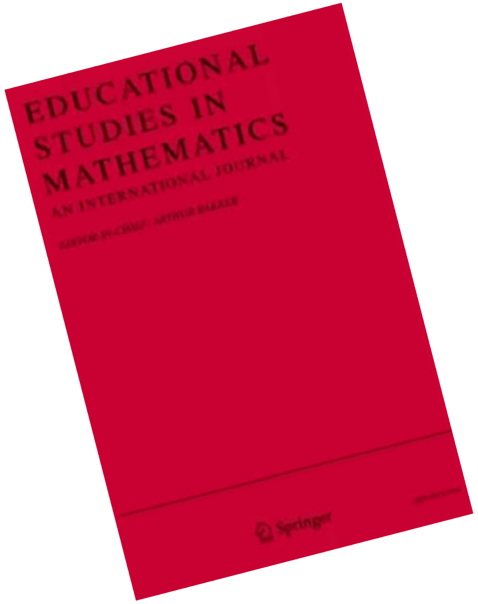
Theory of Didactic Situations

And so on…

Artigue, Gueudet et al., Nardi

three types of transition / Type I

- from school to university mathematics



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Transition from school into university mathematics: experiences across educational contexts
Editors:
Pietro Di Martino
Francesca Gregorio
Paola Iannone

Di Martino, Gregorio, Iannone

three types of transition / Type I, from school to university mathematics

an **exciting** and often **complex** experience

many facets: mathematical, curricular, academic and social

new discourse governed by new rules that may intrigue / alienate newcomers

obvious contextual differences but **key features** in common

academic and social differences in learning environment (changes in living arrangements, location, friendship groups, relationships with peers and teachers)

most emphatically: a **shift in thinking mathematically**

increased requirements for **rigour and precision**

moving from instrumental to more relational approaches

developing more **learner autonomy**

deficit discourses on mathematical under-preparedness

Gueudet et al, Solomon, Nardi et al, Bampili et al, Thomas et al

three types of transition / Type I, from school to university mathematics

foregrounding

mathematical content
students' approaches to study
institutional support

Gueudet et al, Solomon, Nardi et al, Bampili et al, Thomas et al

three types of transition / Type I, from school to university mathematics

foregrounding

mathematical content

- transition from **informal to formal** calculus
- what **means of justification** students bring to their mathematical work
- what forms of **scientific debate** may help students' arguments grow more robust
- what **guided intervention** processes may assist students' collective construction of a definition (convergence)
- classification of arriving students' **concept images** of decimals and irrationals
- **epistemological analyses** of the transition to a formal definition of real numbers
- increased complexity of **signs and reasoning** processes
- challenges in learning new **techniques** and diagrammatic reasoning
- problematising **compartmentalisation** of the curriculum (probability, continuous distributions; calculus, integrals)

Lecorre, Ghedamsi, Schüler-Meyer, Kidron, Bloch, Gibel, Derouet

three types of transition / Type I, from school to university mathematics

foregrounding

students' approaches to study

- how students' **independent work** changes as they enter university
- how they use **resources** (greater variety at university)
- what concerns, and sometimes **hesitation**, students have about how to deploy resources available to them (and deficits of strategy thereof)
- how university mathematics teachers expect students to use resources and how students actually use them (**epistemic versus pragmatic mediation**)
- **diminished role of the teacher** as the main resource
- associations between students' use of resources and **course organisation**
- poor alignment between how mathematical content is presented in **textbooks** and in lectures

Gueudet, Pepin, Kock, Quéré

foregrounding

institutional support

- transitions experienced differently by students with **different SES**
- part of concurrent, broader and deeper **identity** formation processes
- student **learning support** systems, personal tutoring and peer-support systems
- programmes that aim to raise **appreciation** of university mathematics
- identifying newly arriving **students' interest** in different aspects of mathematics such as real-world problems, calculations and proofs
- exploring who may be at **risk of dropping out** of university mathematics
- evaluating **tailored support** for students (bridging courses)

Biehler, Eichler, Kuklinski, Landgärds, Törner, Rach, Bracke

three types of transition / Type I, from school to university mathematics

foregrounding

institutional support

- analysis of **affective variables** such as interest, mathematical self-concept, basic needs and self-efficacy and goal-fulfilment
- **tracing decline in interest** and shifts in student beliefs e.g. about the "toolbox" nature of mathematics
- the **impact of anxiety and personality** factors

  Mathematics Anxiety Rating Scale (MARS)
  "Big Five" inventory (extraversion, agreeableness, conscientiousness, neuroticism and openness)

[as transition from school to university mathematics ease… Type II…]

Biehler, Eichler, Kuklinski, Landgärds, Törner, Rach, Bracke

three types of transition / Type II, within university courses

transitions between mathematics courses that occur in succession

(i.e., one is the prerequisite of the other, mainly in the Analysis path / single variable Calculus, multivariable Calculus, Real Analysis, Metric Spaces, …)

transitions from foundational mathematics courses to specialist courses for disciplines that use mathematics

(e.g. engineering)

Hardy, Broley, Winsløw, Bergé, Hamza, O'Shea, González-Martín, Hernandes-Gomes

three types of transition / Type II, within university courses

transitions between mathematics courses that occur in succession

Example / Bergé: from one course to the next, activities given to students (first find suprema of concrete sets, then use the supremum as a tool in proof tasks). Students first saw the supremum either as not being useful, or as an ordinary upper bound; then could name situations where the supremum functions as a tool.

transitions from foundational mathematics courses to specialist courses for disciplines that use mathematics

Example / González-Martín (engineering): comparative analysis of how integrals are presented in Calculus textbooks and in Strength and Materials textbooks. Norm is that engineers learn about them. However, techniques for calculating integrals are not needed to solve Strength and Materials tasks. Engineering students question need to learn integrals...

Hardy, Broley, Winsløw, Bergé, Hamza, O'Shea, González-Martín, Hernandes-Gomes

three types of transition / Type III, from university to the workplace

to primary / secondary school teaching

to post-secondary teaching

…transitions to different workplaces increasingly being studied …

Cooper, Pinto, Biza, Nardi, Hanke, Schäfer, Stender, Corriveau, Florensa, Broley

three types of transition / Type III, from university to the workplace

to primary / secondary school teaching

MathTASK

(student) teachers engage with fictive, albeit realistic classroom situations

data analysed per four characteristics

consistency (between intended and actual practice)

specificity (to the situation)

reification of pedagogical discourses / reification of mathematical discourses

we analyse / facilitate transition process

Student teachers do not naturally **gain the "higher standpoint"** on school mathematics that is intended by curriculum developers; we need to help them

There are **natural connections between university and school mathematics**; let's find and focus on these (e.g. understand the problem, change representation, consider special cases, break the problem into sub-problems…)

Cooper, Pinto, Biza, Nardi, Hanke, Schäfer, Stender, Corriveau, Florensa, Broley

three types of transition / Type III, from university to the workplace

to post-secondary teaching

- **challenge of transferring results** about schoolteachers to the post-secondary level
- transition as a **discursive shift** in one’s narratives about mathematics and its pedagogy
- **designing and evaluating** attempts at easing transition
- lecturer / TA courses and the pitfalls of “contentless”, **“pedagogical generalism”**
- exploring the **interplay** between mathematical research and pedagogical practice (gaps, opportunities, authenticity, workplaces)

Cooper, Pinto, Biza, Nardi, Hanke, Schäfer, Stender, Corriveau, Florensa, Broley

three types of transition / Type III, from university to the workplace

...transitions to different workplaces increasingly being studied ...

MatRIC Centre for Research, Innovation and Coordination of Mathematics Teaching

UNIVERSITY OF BERGEN

The Learning and Teaching of Calculus Across Disciplines

5-9 Jun 2023 Bergen (Norway)

Biology, Chemistry, Economics, Engineering, Physics

Special Issue: coming soon!

Cooper, Pinto, Biza, Nardi, Hanke, Schäfer, Stender, Corriveau, Florensa, Broley

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three types of transition / Part I, concluding remarks

a **vibrant and broadening** research area

internationality / importance of **contexts**

Type II / Type III: rather **new**

beyond Year 1 or 2: even less!
more Calculus / Analysis than Algebra or Statistics

less **within-discipline uses** of mathematics,
other than engineering

need to **broaden** mathematical **skills studied**
(numeracy, computational thinking, logic, statistical thinking)

three types of transition / Part I, concluding remarks

From a commognitive (ToC) perspective, transitions are currently studied mostly in terms of ***changes in meta-discursive rules****, changes in word use and emerging* ***commognitive conflicts*** *in the interactions between students and teachers. There is ample space for data collection and* ***analyses of a different grain size****, for example for studies of how changes in the* ***curriculum*** *and in* ***institutional factors*** *play out in the way students' discursive activity changes from one educational level to another, from one course to another, from the world of study to the world of work, etc.*

three types of transition / Part I, concluding remarks

need for research that evaluates **interventions** or explores **sustainability** issues

challenge tendency for **transition measures as "add-ons"** to existing systems

need for **deeper systemic changes**

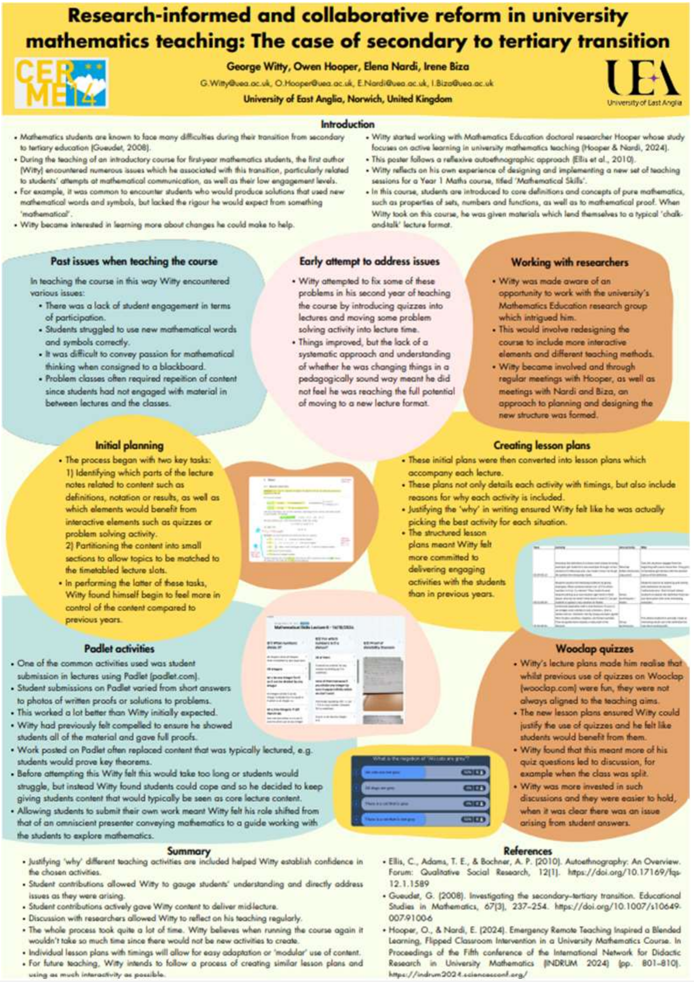
need for **cross-community collaboration** …

LONDON MATHEMATICAL SOCIETY EST. 1865

Transitions to, across and beyond university mathematics *Elena Nardi, LMS Education Day,* 14 May 2025

University of East Anglia

Part II Example of cross-community collaboration on a transition initiative



https://tinyurl.com/CERME14-Poster-Witty-et-al

math TASK

Thank you

Q&A after Paul's keynote!

INDRUM2026: Sixth conference of the International Network for Didactic Research in University Mathematics

15-19 Jun 2026 Dubrovnik (Croatia)

Submit a paper or a poster proposal!

Join us!