Mathematical Journeys - Supporting Students Through Key Transitions
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The importance of pre- and post-16 mathematics qualifications in supporting transition to mathematical and quantitative studies in HE

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Currently fulfilled (in England) through …

… acquisition and *measurement* of knowledge, understanding, skills, competencies in:

A levels in Mathematics & Further Mathematics (post 16/level 3)

Core Maths (post 16/level 3)

GCSE Mathematics (pre 16/level 2)

Questions

- Is there a mismatch between pre-requisite knowledge and skills required of students, current pre- and post-16 mathematics qualifications, and formal entry requirements?
- Did the 2017 reforms to post-16 mathematics qualifications meet the needs of students for transition and success on programmes?
- Do they currently meet these needs?
- Should the current pre- and post-16 curriculum and assessment in mathematics be reformed to better meet the future needs of students?
- If so, should the reforms be
    - minor as might be envisaged by the current independent review being undertaken for the Department for Education (in England), or
    - major as envisaged in the Royal Society's 'Mathematical and Data Education' report as part of their Mathematical Futures Programme?

Mathematical and Data Education

To be globally competitive in today's world, a strong foundation in mathematics is essential. Here are some areas of mathematics that are particularly important for global competitiveness:

1. *Numeracy Skills: Proficiency in basic arithmetic operations, including addition, subtraction, multiplication, and division, is crucial for everyday calculations, financial literacy, and problem-solving in various contexts.*
2. *Algebra and Calculus: A solid understanding of algebraic concepts, equations, inequalities, and functions is essential. Calculus, which includes differentiation and integration, is crucial for modelling and analysing complex systems and phenomena.*
3. *Statistics and Probability: Competence in statistical analysis, data interpretation, and probability theory is essential for making informed decisions, conducting research, and understanding trends and patterns in various fields.*
4. *Problem-Solving and Critical Thinking: Mathematical thinking involves logical reasoning, problem-solving strategies, and the ability to think analytically and critically. These skills are valuable for tackling real-world problems, making sound decisions, and developing innovative solutions.*
5. *Mathematical Modelling: The ability to construct mathematical models to represent and analyze real-world situations is increasingly important. This skill allows individuals to understand and predict complex systems, make data-driven decisions, and optimize processes.*
6. *Computational Skills: With the increasing role of technology, proficiency in computational mathematics, algorithmic thinking, and programming is highly beneficial. Understanding coding concepts and mathematical software tools can enhance problem-solving capabilities and open doors to various career paths.*
7. *Financial Mathematics: Knowledge of financial mathematics, including concepts such as interest rates, investments, risk management, and financial modeling, is crucial in today's global economy.*
8. *Geometry and Spatial Reasoning: Understanding geometric concepts and spatial relationships is valuable in fields such as engineering, architecture, computer graphics, and design.*
9. *Data Analysis and Machine Learning: In the era of big data and artificial intelligence, skills in data analysis, machine learning, and predictive modeling are increasingly in demand. These skills enable individuals to extract insights from large datasets and contribute to fields such as data science and artificial intelligence.*
10. *Mathematical Communication: Effective communication of mathematical ideas and results is essential. The ability to explain mathematical concepts clearly, write mathematical proofs, and present findings in a comprehensible manner is crucial for collaboration, research, and innovation.*

Developing proficiency in these areas will provide individuals with a strong mathematical foundation, enabling them to compete globally and excel in a wide range of academic, scientific, technological, and business fields.

2010 Schools White Paper: The Importance of Teaching
A level Mathematics

- A levels are a crucial way that universities select candidates for their courses, so it is important that these qualifications meet the needs of higher education institutions.

differences between the Intended, Enacted, and Assessed Curriculum

DfE←→Ofqual

- Primary purpose of A levels is to prepare students for degree-level study.
- All students should have access to qualifications that are highly respected and valued by leading universities.
- Current A levels do not always provide the solid foundation that students need to prepare them for degree-level study and for vocational education.
- Many leading universities are concerned about current A levels.
- There is support for much greater higher education involvement in A levels.
- There is clear dissatisfaction among leading university academics about the preparation of A level pupils for advanced studies.

AS/A level reform: Overarching Themes

The intentions of the 2017 reforms can be neatly summarised by the three Overarching Themes:

- OT1 Mathematical argument, language and proof
- OT2 Mathematical problem solving
- OT3 Mathematical modelling

and AS/A level specifications must require students to demonstrate these, and must be applied, along with associated mathematical thinking and understanding, across the whole of the detailed content.

There are two further themes:

- use of technology
- use of data in statistics

All five themes should have some impact on the way students have been taught, and an impact on their learning of mathematics.

As such, students who have been taught by teachers who have embraced the reforms and these intentions as set out in the Mathematics and Further Mathematics Subject Content documents, should be better prepared for studying mathematics in HE.

AS/A level Maths and Further Maths

Aims and objectives:

- understand mathematics and mathematical processes in a way that promotes confidence, fosters enjoyment and provides a strong foundation for progress to further study
- extend their range of mathematical skills and techniques
- understand coherence and progression in mathematics and how different areas of mathematics are connected
- apply mathematics in other fields of study and be aware of the relevance of mathematics to the world of work and to situations in society in general
- use their mathematical knowledge to make logical and reasoned decisions in solving problems both within pure mathematics and in a variety of contexts, and communicate the mathematical rationale for these decisions clearly
- reason logically and recognise incorrect reasoning
- generalise mathematically
- construct mathematical proofs
- use their mathematical skills and techniques to solve challenging problems which require them to decide on the solution strategy

AS/A level Mathematics

Aims and objectives:

- recognise when mathematics can be used to analyse and solve a problem in context
- represent situations mathematically and understand the relationship between problems in context and mathematical models that may be applied to solve them
- draw diagrams and sketch graphs to help explore mathematical situations and interpret solutions
- make deductions and inferences and draw conclusions by using mathematical reasoning
- interpret solutions and communicate their interpretation effectively in the context of the problem
- read and comprehend mathematical arguments, including justifications of methods and formulae, and communicate their understanding
- read and comprehend articles concerning applications of mathematics and communicate their understanding
- use technology such as calculators and computers effectively and recognise when such use may be inappropriate
- take increasing responsibility for their own learning and the evaluation of their own mathematical development

A level Mathematics

Statistics:

- primary focus not routine calculations of summary statistical measures
- use of real, large data sets should permeate the teaching, learning and assessment
- more emphasis upon understanding, interpretation of data and making inferences from data
- use of large pre-released data sets is suggested

Mechanics:

- two-way process between pure and mechanics
- emphasis upon understanding, interpretation and problem solving should pervade the teaching and assessment of mechanics
- natural linkages with pure these should be exploited
- emphasis on mathematical modelling

Use of technology:

- use of technology, in particular mathematical and statistical graphing tools and spreadsheets, must permeate the study of AS and A level Mathematics

A level reforms statistics

Use of data in statistics - require students to:

- become familiar with one or more specific large data set(s) in advance of the final assessment (these data must be real and sufficiently rich to enable the concepts and skills of data presentation and interpretation in the specification to be explored)
- use technology such as spreadsheets or specialist statistical packages to explore the data set(s)
- interpret real data presented in summary or graphical form
- use data to investigate questions arising in real contexts
- explore the data set(s), and associated contexts, during their course of study to enable them to perform tasks that assume familiarity with the contexts, the main features of the data and the ways in which technology can help explore the data
- demonstrate the ability to analyse a subset or features of the data using a calculator with standard statistical functions

The intention is that, rather than students focussing on performing routine calculations to determine summary statistics, they should use technology to do this and then focus on the understanding and interpretation of these statistics.

AS/A level reforms statistics

Large Data Sets (LDS) – the intention of the statements in the Content Document was that:

- the use of real, large data sets permeates the teaching, learning and assessment of statistics in AS and A level mathematics and further mathematics
- learners will have been provided with large sets of real data at the start of the course and some of the examination tasks will be based upon these data sets
- such examination tasks will explore how the application of suitable statistical techniques to the data can provide insight into the contexts from which they have been drawn
- well prepared candidates will have been working with the data sets for nearly two years and so should be familiar with the contexts
- most examination tasks based on the data sets will include elements of interpretation relevant to their contexts
- assessment should adequately reflect the intentions of the Content Document, in all respects, but in particular in respect of the 'use of large data sets'

HE

Outcome:

- variety of mathematical concepts, methods and techniques – pure mathematics and applied, with overlap and interplay between them
- better problem-solving skills: change in emphasis to problem solving, interpretation and testing understanding
- assessment with fewer structured questions that test understanding and help to develop strategies for solving problems either in a purely mathematical or in an applications context

A level content: OTs

OT1 Mathematical argument, language and proof

AS and A level mathematics specifications must use the mathematical notation set out in appendix A and must require students to recall the mathematical formulae and identities set out in appendix B.

	Knowledge/Skill
OT1.1	**[Construct and present mathematical arguments through appropriate use of diagrams; sketching graphs; logical deduction; precise statements involving correct use of symbols and connecting language, including: constant, coefficient, expression, equation, function, identity, index, term, variable]**
OT1.2	**[Understand and use mathematical language and syntax as set out in the content]**
OT1.3	**[Understand and use language and symbols associated with set theory, as set out in the content]** **[Apply to solutions of inequalities]** and probability
OT1.4	Understand and use the definition of a function; domain and range of functions
OT1.5	**[Comprehend and critique mathematical arguments, proofs and justifications of methods and formulae, including those relating to applications of mathematics]**

OT2 Mathematical problem solving

	Knowledge/Skill
OT2.1	**[Recognise the underlying mathematical structure in a situation and simplify and abstract appropriately to enable problems to be solved]**
OT2.2	**[Construct extended arguments to solve problems presented in an unstructured form, including problems in context]**
OT2.3	**[Interpret and communicate solutions in the context of the original problem]**
OT2.4	Understand that many mathematical problems cannot be solved analytically, but numerical methods permit solution to a required level of accuracy
OT2.5	**[Evaluate, including by making reasoned estimates, the accuracy or limitations of solutions]**, including those obtained using numerical methods
OT2.6	**[Understand the concept of a mathematical problem solving cycle, including specifying the problem, collecting information, processing and representing information and interpreting results, which may identify the need to repeat the cycle]**
OT2.7	**[Understand, interpret and extract information from diagrams and construct mathematical diagrams to solve problems, including in mechanics]**

OT3 Mathematical modelling

	Knowledge/Skill
OT3.1	**[Translate a situation in context into a mathematical model, making simplifying assumptions]**
OT3.2	**[Use a mathematical model with suitable inputs to engage with and explore situations (for a given model or a model constructed or selected by the student)]**
OT3.3	**[Interpret the outputs of a mathematical model in the context of the original situation (for a given model or a model constructed or selected by the student)]**
OT3.4	**[Understand that a mathematical model can be refined by considering its outputs and simplifying assumptions; evaluate whether the model is appropriate]**
OT3.5	**[Understand and use modelling assumptions]**

A level content: technology & data in statistics

Use of technology

8. The use of technology, in particular mathematical and statistical graphing tools and spreadsheets, must permeate the study of AS and A level mathematics. Calculators used must include the following features:

- an iterative function
- the ability to compute summary statistics and access probabilities from standard statistical distributions

Use of data in statistics

9. AS and A level mathematics specifications must require students to:

- become familiar with one or more specific large data set(s) in advance of the final assessment (these data must be real and sufficiently rich to enable the concepts and skills of data presentation and interpretation in the specification to be explored)
- use technology such as spreadsheets or specialist statistical packages to explore the data set(s)
- interpret real data presented in summary or graphical form
- use data to investigate questions arising in real contexts

10. Specifications should require students to explore the data set(s), and associated contexts, during their course of study to enable them to perform tasks that assume familiarity with the contexts, the main features of the data and the ways in which technology can help explore the data. Specifications should also require students to demonstrate the ability to analyse a subset or features of the data using a calculator with standard statistical functions, as detailed in paragraph 8.

A level content

A	Proof
B	Algebra and functions
C	Coordinate geometry in the (x,y) plane
D	Sequences and series
E	Trigonometry
F	Exponentials and logarithms
G	Differentiation
H	Integration
I	Numerical methods
J	Vectors
K	Statistical sampling
L	Data presentation and interpretation
M	Probability
N	Statistical distributions
O	Statistical hypothesis testing
P	Quantities and units in mechanics
Q	Kinematics
R	Forces and Newton's laws
S	Moments

AS/A level reform: benefits to HE

In particular, students should be better at:

- understanding what proof is and what it means to prove (or disprove) a statement
- using mathematical language
- developing and articulating mathematical arguments
- providing extended responses
- drawing together different areas of knowledge, skills and/or understanding from across the subject content
- tackling ‘synoptic’ problems that can now be set in the linear framework where any content from across the qualification could appear in an assessment
- ‘problem solving’
- determining summary statistics using technology, and interpreting real data presented in summary or graphical form (as opposed to focussing on calculation of summary statistics by hand)
- understanding modelling (in application settings), and the limitations of models.

AS/A Levels in Mathematics – Ofqual accreditation

Conditions, Requirements & Guidance, including assessment objectives.
Ofqual Report on Mathematical Problem Solving, Modelling and the Use of Large Data Sets in Statistics.

AO1 – 50%

Use and apply standard techniques

Learners should be able to:

- select and correctly carry out routine procedures; and
- accurately recall facts, terminology and definitions

AO2 – 25%

Reason, interpret and communicate mathematically

Learners should be able to:

- construct rigorous mathematical arguments (including proofs);
- make deductions and inferences;
- assess the validity of mathematical arguments;
- explain their reasoning; and
- use mathematical language and notation correctly.

AO3 – 25%

Solve problems within mathematics and in other contexts

Learners should be able to:

- translate problems in mathematical and non-mathematical contexts into mathematical processes;
- interpret solutions to problems in their original context, and, where appropriate, evaluate their accuracy and limitations;
- translate situations in context into mathematical models;
- use mathematical models; and
- evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them.

AS/A level Mathematics – mathematical problem solving

- Tasks have little or no scaffolding: there is little guidance given to the candidate beyond a start point and a finish point. Questions do not explicitly state the mathematical process(es) required for the solution.
- Tasks provide for multiple representations, such as the use of a sketch or a diagram as well as calculations.
- The information is not given in mathematical form or in mathematical language; or there is a need for the results to be interpreted or methods evaluated, for example, in a real-world context.
- Tasks have a variety of techniques that could be used.
- The solution requires understanding of the processes involved rather than just application of the techniques.
- The task requires two or more mathematical processes or may require different parts of mathematics to be brought together to reach a solution.

Not all attributes would be required within a single task to establish it as problem solving.
Neither does the presence of one or more attributes within a task automatically imply problem solving is taking place.

A level Mathematics assessment - 2019 (legacy) – spec 1

It is given that

$$y = 15x + 108x^{\frac{1}{2}} + 4x^{\frac{5}{2}} \qquad x > 0$$

Find, in simplest form,

(a) $\dfrac{dy}{dx}$

(b) $\dfrac{d^2y}{dx^2}$

(c) Find the value of $\dfrac{d^2y}{dx^2}$ when $x = 9$

Given that $y = 4$ when $x = 1$ and that

$$\frac{dy}{dx} = 12x^2 + \frac{4x+2}{3x^4} \qquad x \neq 0$$

find y in terms of x, giving each term in a simplified form.

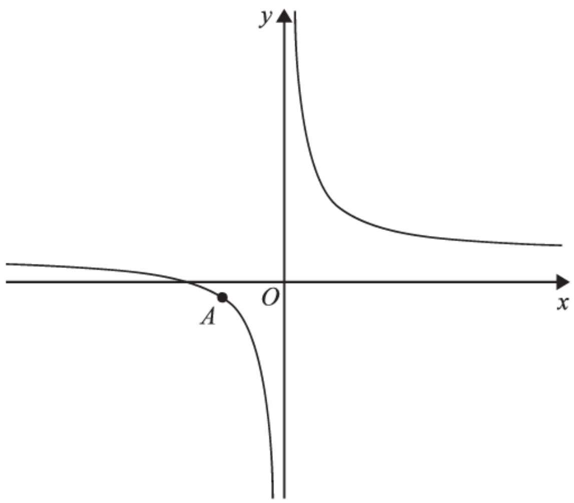


Figure 1

Figure 1 shows a sketch of part of the curve H with equation

$$y = \frac{12}{x} + 5 \quad x \neq 0$$

(a) Find an equation for the normal to H at the point A (–2, –1), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

The points B and C also lie on the curve H.
The normal to H at the point B and the normal to H at the point C are each parallel to the straight line with equation $4y = 3x + 5$

(b) Find the coordinates of the points B and C, given that the x coordinate of B is positive.

(5)

A level Mathematics assessment - 2019 (legacy) – spec 1

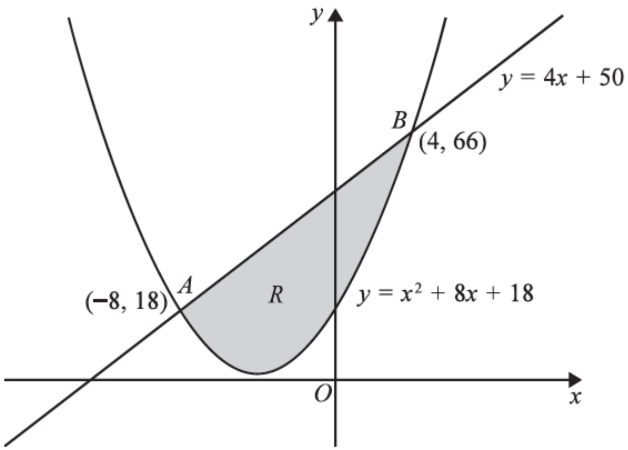


Figure 2

Figure 2 shows the line with equation $y = 4x + 50$ and the curve with equation $y = x^2 + 8x + 18$. The line cuts the curve at the points A $(-8, 18)$ and B $(4, 66)$.

The shaded region R is bounded by the line and the curve, as shown in Figure 2.

Using calculus, find the area of R.

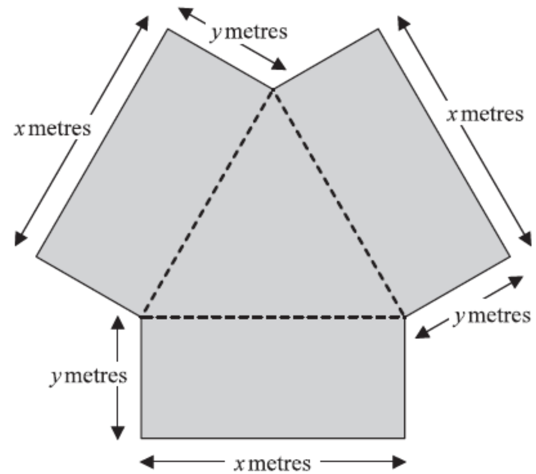


Figure 4

Figure 4 shows a plan view for a flower bed. Its shape is an equilateral triangle of side x metres with three congruent rectangles attached to the triangle along its sides. Each rectangle has length x metres and width y metres, as shown in Figure 4.

Given that the total area of the flower bed is 3 m^2 and that $0 < x < 2.632$ (3d.p.),

(a) show that the perimeter P metres, around the outside of the flower bed, is given by the equation

$$P = 3x + \frac{6}{x} - \frac{\sqrt{3}}{2}x$$

(6)

(b) Use calculus to find the minimum value of P, giving your answer to 3 significant figures.

(5)

(c) Justify, using calculus, that the value you have found in part (b) is a minimum value.

(2)

A level Mathematics assessment - 2019 (legacy) – spec 1

(i)

$$y = \frac{(2x-1)^3}{(3x-2)} \qquad x \neq \frac{2}{3}$$

(a) Find $\frac{dy}{dx}$ writing your answer as a single fraction in simplest form.

(b) Hence find the set of values of x for which $\frac{dy}{dx} \geqslant 0$

(ii) Given

$$y = \ln(1 + \cos 2x) \qquad x \neq (2n+1)\frac{\pi}{2} \quad n \in \mathbb{Z}$$

show that $\frac{dy}{dx} = C\tan x$, where C is a constant to be determined.

(You may assume the double angle formulae.)

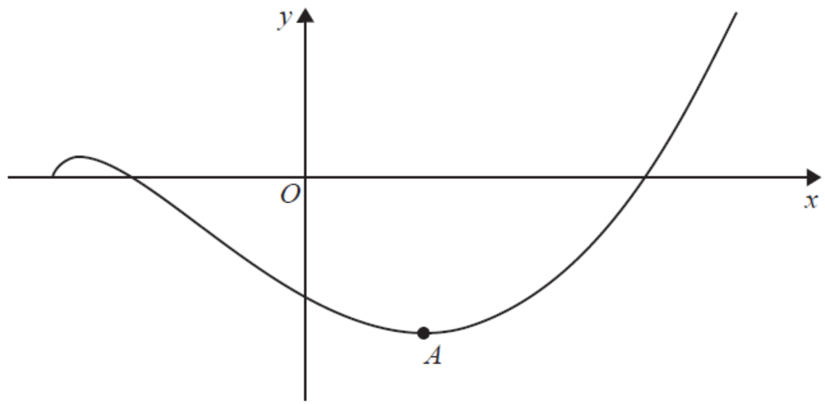


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (x^2 - x - 12)\ln(x+3), \qquad x \in \mathbb{R}, x > -3$$

(a) Find $f'(x)$.

A level Mathematics assessment - 2019 (legacy) – spec 1

A study is being carried out on two colonies of ants.

The number of ants N_A in colony A, t years after the start of the study, is modelled by the equation

$$N_A = 3000 + 600e^{0.12t} \qquad t \in \mathbb{R}, t \geqslant 0$$

Using the model,

(a) find the time taken, from the start of the study, for the number of ants in colony A to double. Give your answer, in years, to 2 decimal places.

(5)

(b) Show that $\dfrac{dN_A}{dt} = pN_A + q$, where p and q are constants to be determined.

(3)

The number of ants N_B in colony B, t years after the start of the study, is modelled by the equation

$$N_B = 2900 + Ce^{kt} \qquad t \in \mathbb{R}, t \geqslant 0$$

where C and k are positive constants.

According to this model, there will be 3100 ants in colony B one year after the start of the study and 3400 ants in colony B two years after the start of the study.

(c) (i) Show that $k = \ln\left(\dfrac{5}{2}\right)$

(ii) Find the value of C.

(4)

The curve C has equation

$$x = \frac{1}{1 + \cot y} \qquad 0 < y < \frac{3\pi}{4}$$

(a) Show that

$$\frac{dx}{dy} = 2x^2 - 2x + 1$$

The point A with y coordinate $\arctan\left(\dfrac{1}{3}\right)$ lies on C.

(b) Find the x coordinate of A.

(c) Find the value of $\dfrac{dy}{dx}$ at A.

A level Mathematics assessment - 2019 (legacy) – spec 1

The curve C has equation

$$x^2 - y^3 - x - x\sin(\pi y) = -2$$

(a) Find $\dfrac{dy}{dx}$ in terms of x and y.

The point P with coordinates $(3, 2)$ lies on C.

The tangent to C at P meets the y-axis at the point Q.

(b) Find the y coordinate of Q, giving your answer in the form $\dfrac{a\pi + b}{\pi + c}$ where a, b and c are integers to be found.

(i) Find, in its simplest form,

$$\int \frac{5}{6e^{3x}}\,dx$$

(2)

(ii) (a) Express $\dfrac{4y^2 + 3y - 4}{y(2y-1)}$ in partial fractions.

(4)

(b) Hence find

$$\int \frac{4y^2 + 3y - 4}{y(2y-1)}\,dy \qquad y > \frac{1}{2}$$

(3)

(iii) Use integration by parts to find

$$\int_1^4 \frac{1}{\sqrt{x}}\ln(2x)\,dx$$

giving your answer in the form $a + b\ln 2$, where a and b are constants to be found.

(5)

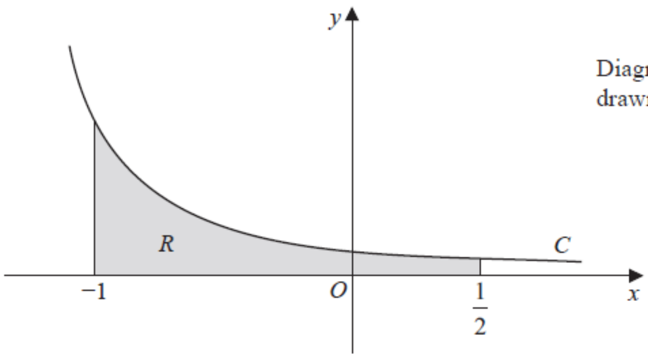


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{8}{5(2x+3)^2} \qquad x > -\frac{3}{2}$$

The finite region R, shown shaded in Figure 2, is bounded by the line with equation $x = -1$, the curve C, the line with equation $x = \frac{1}{2}$ and the x-axis. The region R is rotated through 360° about the x-axis to form a solid of revolution with volume V.

Use calculus to find the exact value of V, giving your answer in its simplest form.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

A level Mathematics assessment - 2019 (legacy) – spec 1

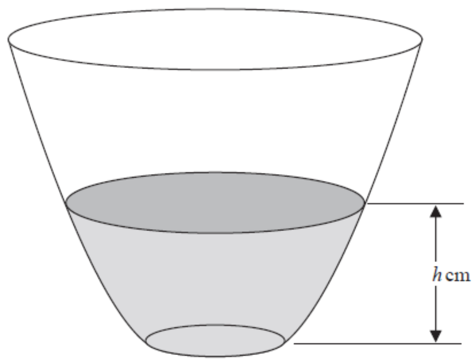


Figure 3

A container with a circular cross-section is shown in Figure 3.

Initially the container is empty. At time t seconds after water begins to flow into the container, the height of water in the container is h cm.

The height of water in the container satisfies the differential equation

$$\frac{dh}{dt} = \frac{k}{h+4} \qquad 0 \leqslant h \leqslant 35$$

where k is a constant.

When $h = 16$, the height of water in the container is increasing at a rate of $0.6\,\text{cm s}^{-1}$

(a) Find the value of k.

(1)

(b) Find the time taken to fill the container with water from empty to a height of 30 cm.

(4)

Given that the water flows into the container at a constant rate of $96\pi\,\text{cm}^3\,\text{s}^{-1}$

(c) find the volume of water in the container when $h = 30$
Give your answer in cm^3 to 3 significant figures.

(2)

The curve C has parametric equations

$$x = -3 + 6\sin\theta \qquad y = 4\sqrt{3}\cos 2\theta \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

where θ is a parameter.

(a) Find an expression for $\frac{dy}{dx}$ in terms of θ.

(2)

The curve C cuts the y-axis at the point A.

The line l is the normal to C at the point A.

(b) Show that an equation for l is

$$\sqrt{3}x - 4y + 8\sqrt{3} = 0$$

(6)

The line l intersects the curve C again at the point B.

(c) Find the coordinates of B. Give your answer in the form $(p, q\sqrt{3})$, where p and q are rational constants.
(Solutions based entirely on graphical or numerical methods are not acceptable.)

(6)

A level Mathematics assessment 2019 - (reform) – spec 1

Find

$$\int \frac{x^{\frac{1}{2}}(2x-5)}{3}\,\mathrm{d}x$$

writing each term in simplest form.

The curve C has equation $y = \mathrm{f}(x)$ where $x \in \mathbb{R}$

Given that

- $\mathrm{f}'(x) = 2x + \frac{1}{2}\cos x$
- the curve has a stationary point with x coordinate α
- α is small

(a) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

The point $P(0, 3)$ lies on C

(b) Find the equation of the tangent to the curve at P, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

$$y = \sin x$$

where x is measured in radians.

Use differentiation from first principles to show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \cos x$$

You may

- use without proof the formula for $\sin(A \pm B)$
- assume that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

A level Mathematics assessment 2019 - (reform) – spec 1

$$f(x) = x^3 + 2x^2 - 8x + 5$$

(a) Find $f''(x)$

(b) (i) Solve $f''(x) = 0$

(ii) Hence find the range of values of x for which $f(x)$ is concave.

The curve C has equation $y = f(x)$

The curve

- passes through the point $P(3, -10)$
- has a turning point at P

Given that

$$\frac{dy}{dx} = 2x^3 - 9x^2 + 5x + k$$

where k is a constant,

(a) show that $k = 12$

(b) Hence find the coordinates of the point where C crosses the y-axis.

A curve has equation $y = f(x)$, where

$$f(x) = \frac{7xe^x}{\sqrt{e^{3x} - 2}} \qquad x > \ln\sqrt[3]{2}$$

(a) Show that

$$f'(x) = \frac{7e^x(e^{3x}(2 - x) + Ax + B)}{2(e^{3x} - 2)^{\frac{3}{2}}}$$

where A and B are constants to be found.

(b) Hence show that the x coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4}$$

The equation $x = \frac{2e^{3x} - 4}{e^{3x} + 4}$ has two positive roots α and β where $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

in an attempt to find approximations for α and β

Diagram 1 shows a plot of part of the curve with equation $y = \frac{2e^{3x} - 4}{e^{3x} + 4}$ and part of the line with equation $y = x$

Using Diagram 1 on page 42

(c) draw a staircase diagram to show that the iteration formula starting with $x_1 = 1$ can be used to find an approximation for β

Use the iteration formula with $x_1 = 1$, to find, to 3 decimal places,

(d) (i) the value of x_2

(ii) the value of β

Using a suitable interval and a suitable function that should be stated

(e) show that $\alpha = 0.432$ to 3 decimal places.

A level Mathematics assessment 2019 - (reform) – spec 1

A curve has equation

$$x^3 + 2xy + 3y^2 = 47$$

(a) Find $\frac{dy}{dx}$ in terms of x and y

The point $P(-2, 5)$ lies on the curve.

(b) Find the equation of the normal to the curve at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

The curve C has parametric equations

$$x = t^2 + 6t - 16 \qquad y = 6\ln(t+3) \qquad t > -3$$

(a) Show that a Cartesian equation for C is

$$y = A\ln(x+B) \qquad x > -B$$

where A and B are integers to be found.

The curve C cuts the y-axis at the point P

(b) Show that the equation of the tangent to C at P can be written in the form

$$ax + by = c\ln 5$$

where a, b and c are integers to be found.

$$f(x) = \frac{3kx - 18}{(x+4)(x-2)} \qquad \text{where } k \text{ is a positive constant}$$

(a) Express $f(x)$ in partial fractions in terms of k.

(b) Hence find the exact value of k for which

$$\int_{-3}^{1} f(x)\, dx = 21$$

Coffee is poured into a cup.

The temperature of the coffee, H °C, t minutes after being poured into the cup is modelled by the equation

$$H = Ae^{-Bt} + 30$$

where A and B are constants.

Initially, the temperature of the coffee was 85 °C.

(a) State the value of A.

Initially, the coffee was cooling at a rate of 7.5 °C per minute.

(b) Find a complete equation linking H and t, giving the value of B to 3 decimal places.

A level Mathematics assessment 2019 - (reform) – spec 1

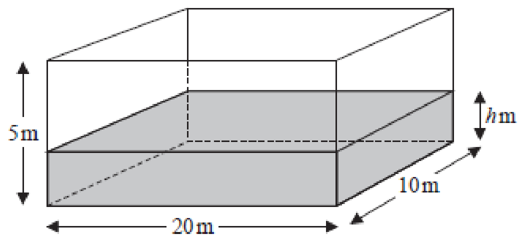


Figure 1

A tank in the shape of a cuboid is being filled with water.

The base of the tank measures 20 m by 10 m and the height of the tank is 5 m, as shown in Figure 1.

At time t minutes after water started flowing into the tank the height of the water was h m and the volume of water in the tank was $V\text{m}^3$

In a model of this situation

- the sides of the tank have negligible thickness
- the rate of change of V is inversely proportional to the square root of h

(a) Show that

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\lambda}{\sqrt{h}}$$

where λ is a constant.

Given that

- initially the height of the water in the tank was 1.44 m
- exactly 8 minutes after water started flowing into the tank the height of the water was 3.24 m

(b) use the model to find an equation linking h with t, giving your answer in the form

$$h^{\frac{3}{2}} = At + B$$

where A and B are constants to be found.

(c) Hence find the time taken, from when water started flowing into the tank, for the tank to be completely full.

(a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$(3 + x)^{-2}$$

writing each term in simplest form.

(b) Using the answer to part (a) and using algebraic integration, estimate the value of

$$\int_{0.2}^{0.4} \frac{6x}{(3 + x)^2}\,\mathrm{d}x$$

giving your answer to 4 significant figures.

(c) Find, using algebraic integration, the exact value of

$$\int_{0.2}^{0.4} \frac{6x}{(3 + x)^2}\,\mathrm{d}x$$

giving your answer in the form $a \ln b + c$, where a, b and c are constants to be found.

A level Mathematics assessment 2019 - (reform) – spec 2

Given that $y = 2x^3$ find $\dfrac{dy}{dx}$

Circle your answer.

$\dfrac{dy}{dx} = 5x^2$ $\qquad$ $\dfrac{dy}{dx} = 6x^2$ $\qquad$ $\dfrac{dy}{dx} = \dfrac{x^4}{2}$ $\qquad$ $\dfrac{dy}{dx} = 6x^3$

A curve with equation $y = f(x)$ passes through the point $(3, 7)$

Given that $f'(3) = 0$ find the equation of the normal to the curve at $(3, 7)$

Circle your answer.

$y = \dfrac{7}{3}x$ $\qquad$ $y = 0$ $\qquad$ $x = 3$ $\qquad$ $x = 7$

A curve has equation $y = 3e^{2x}$

Find the gradient of the curve at the point where $y = 10$

Use the substitution $u = x^5 + 2$ to show that

$$\int_0^1 \frac{x^9}{(x^5+2)^3}\,dx = \frac{1}{180}$$

It is given that

$$\int_0^6 f(x)\,dx = 20 \quad \text{and} \quad \int_3^6 f(x)\,dx = -10$$

Find the value of $\displaystyle\int_0^3 f(x)\,dx$

Circle your answer.

-30 $\qquad$ -10 $\qquad$ 10 $\qquad$ 30

A curve has equation

$$y = \frac{x^2}{8} + 4\sqrt{x}$$

(a) Find an expression for $\dfrac{dy}{dx}$

(b) The point P with coordinates $(4, 10)$ lies on the curve.

Find an equation of the tangent to the curve at the point P

(c) Show that the curve has no stationary points.

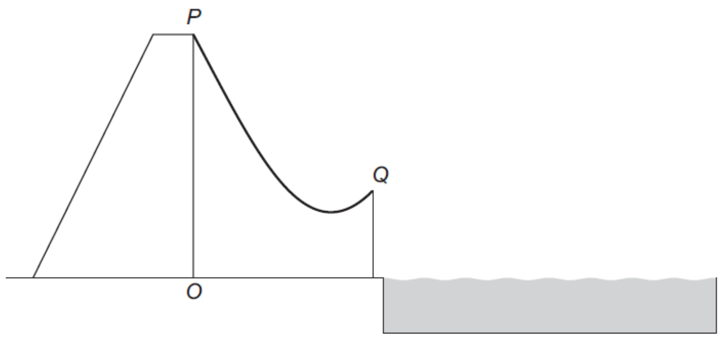
Show that

$$\int_0^{\frac{\pi}{2}} (x \sin 4x)\,dx = -\frac{\pi}{8}$$

A level Mathematics assessment 2019 - (reform) – spec 2

A water slide is the shape of a curve PQ as shown in **Figure 1** below.

Figure 1



The curve can be modelled by the parametric equations

$$x = t - \frac{1}{t} + 4.8$$

$$y = t + \frac{2}{t}$$

where $0.2 \leq t \leq 3$

The horizontal distance from O is x metres.

The vertical distance above the point O at ground level is y metres.

P is the point where $t = 0.2$ and Q is the point where $t = 3$

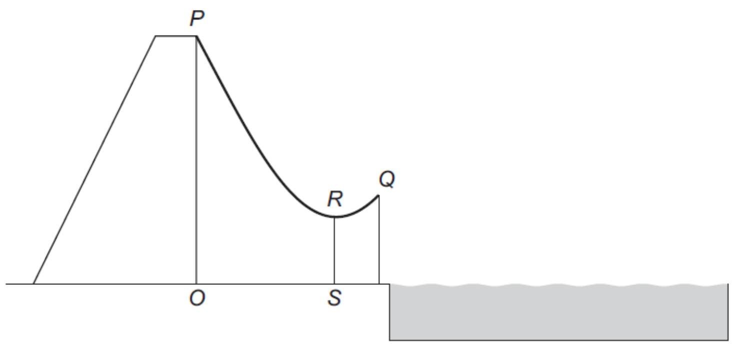
(a) To make sure speeds are safe at Q, the difference in height between P and Q must be less than 7 metres.

Show that the slide meets this safety requirement.

(b) (i) Find an expression for $\frac{dy}{dx}$ in terms of t

(b) (ii) A vertical support, RS, is to be added between the ground and the lowest point on the slide as shown in **Figure 2** below.

Figure 2



Find the length of RS

(b) (iii) Find the acute angle the slide makes with the horizontal at Q

Give your answer to the nearest degree.

A level Mathematics assessment 2019 - (reform) – spec 3

(a) Given that $f(x) = x^2 + 2x$, use differentiation from first principles to show that $f'(x) = 2x + 2$. **[4]**

(b) The gradient of a curve is given by $\frac{dy}{dx} = 2x + 2$ and the curve passes through the point $(-1, 5)$.

Find the equation of the curve. **[3]**

A curve has equation $y = e^{x^2+3x}$.

(a) Determine the x-coordinates of any stationary points on the curve.

(b) Show that the curve is convex for all values of x.

Conservationists are studying how the number of bees in a wildflower meadow varies according to the number of wildflower plants. The study takes place over a series of weeks in the summer. A model is suggested for the number of bees, B, and the number of wildflower plants, F, at time t weeks after the start of the study.

In the model $B = 20 + 2t + \cos 3t$ and $F = 50e^{0.1t}$.

The model assumes that B and F can be treated as continuous variables.

(a) State the meaning of $\frac{dB}{dF}$. **[1]**

(b) Determine $\frac{dB}{dF}$ when $t = 4$. **[4]**

(c) Suggest a reason why this model may not be valid for values of t greater than 12. **[1]**

(a) Use the substitution $u = e^x - 2$ to show that

$$\int \frac{7e^x - 8}{(e^x - 2)^2}\,dx = \int \frac{7u + 6}{u^2(u + 2)}\,du.$$

(b) Hence show that

$$\int_{\ln 4}^{\ln 6} \frac{7e^x - 8}{(e^x - 2)^2}\,dx = a + \ln b$$

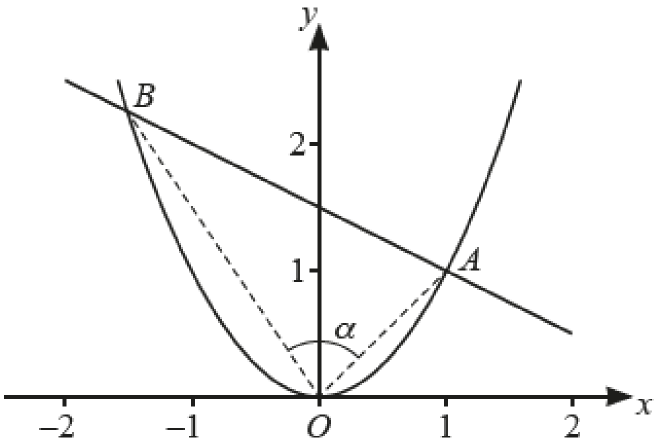
where a and b are rational numbers to be determined.

A level Mathematics assessment 2019 - (reform) – spec 3

In this question you must show detailed reasoning.

Find the exact area of the region enclosed by the curve $y = \dfrac{1}{x+2}$, the two axes and the line $x = 2.5$.

The diagram shows part of the graph of $y = x^2$. The normal to the curve at the point $A(1, 1)$ meets the curve again at B. Angle AOB is denoted by α.



(a) Determine the coordinates of B. **[6]**

(b) Hence determine the exact value of $\tan\alpha$. **[3]**

A level Mathematics assessment 2019 - (reform) – spec 3

In this question you must show detailed reasoning.

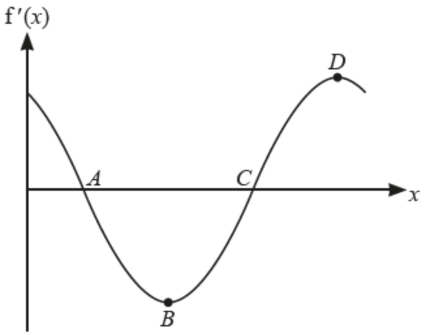
The function f is defined by $f(x) = \cos x + \sqrt{3}\sin x$ with domain $0 \leqslant x \leqslant 2\pi$.

(a) Solve the following equations.

(i) $f'(x) = 0$ **[4]**

(ii) $f''(x) = 0$ **[3]**

The diagram shows the graph of the gradient function $y = f'(x)$ for the domain $0 \leqslant x \leqslant 2\pi$.

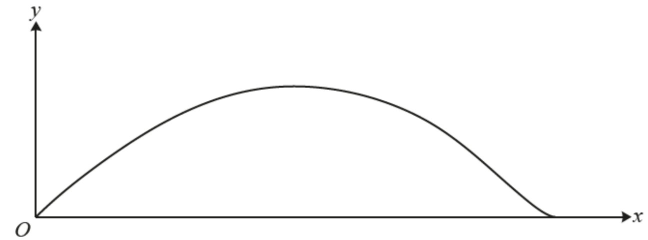


(b) Use your answers to parts **(a)(i)** and **(a)(ii)** to find the coordinates of points A, B, C and D. **[2]**

(c) **(i)** Explain how to use the graph of the gradient function to find the values of x for which $f(x)$ is increasing. **[1]**

(ii) Using set notation, write down the set of values of x for which $f(x)$ is increasing in the domain $0 \leqslant x \leqslant 2\pi$. **[2]**

A mathematics department is designing a new emblem to place on the walls outside its classrooms. The design for the emblem is shown in the diagram below.



The emblem is modelled by the region between the x-axis and the curve with parametric equations

$x = 1 + 0.2t - \cos t, \qquad y = k\sin^2 t,$

where k is a positive constant and $0 \leqslant t \leqslant \pi$.

Lengths are in metres and the area of the emblem must be $1\ \text{m}^2$.

(a) Show that $k\int_0^{\pi}(0.2 + \sin t - 0.2\cos^2 t - \sin t\cos^2 t)\,dt = 1$. **[3]**

(b) Determine the exact value of k. **[6]**

A circle C has equation $x^2 + y^2 - 6x + 10y + k = 0$.

(a) Find the set of possible values of k. **[2]**

(b) It is given that $k = -46$.

Determine the coordinates of the **two** points on C at which the gradient of the tangent is $\frac{1}{2}$. **[5]**

A level Mathematics assessment 2019 - (reform) – spec 3

A car C is moving horizontally in a straight line with velocity $v\,\text{m s}^{-1}$ at time t seconds, where $v > 0$ and $t \geqslant 0$. The acceleration, $a\,\text{m s}^{-2}$, of C is modelled by the equation

$$a = v\left(\frac{8t}{7+4t^2} - \frac{1}{2}\right).$$

(a) **In this question you must show detailed reasoning.**

Find the times when the acceleration of C is zero. **[3]**

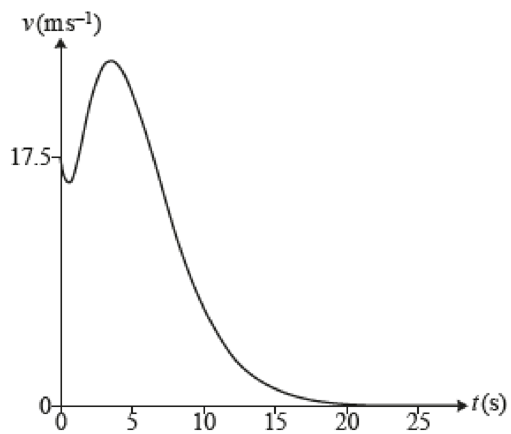
At $t = 0$ the velocity of C is $17.5\,\text{m s}^{-1}$ and at $t = T$ the velocity of C is $5\,\text{m s}^{-1}$.

(b) By setting up and solving a differential equation, show that T satisfies the equation

$$T = 2\ln\left(\frac{7+4T^2}{2}\right).$$ **[6]**

(c) Use an iterative formula, based on the equation in part **(b)**, to find the value of T, giving your answer correct to 4 significant figures. Use an initial value of 11.25 and show the result of each step of the iteration process. **[2]**

(d) The diagram below shows the velocity-time graph for the motion of C.



Find the time taken for C to decelerate from travelling at its maximum speed until it is travelling at $5\,\text{m s}^{-1}$. **[1]**

A level Mathematics assessment

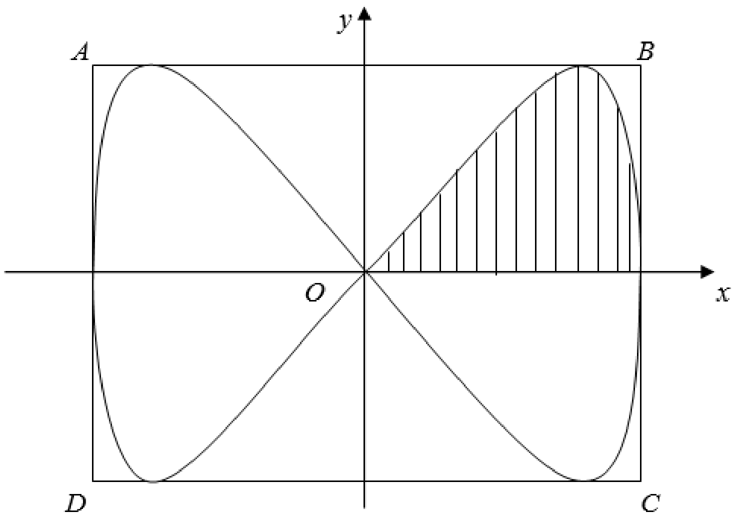


Figure 1

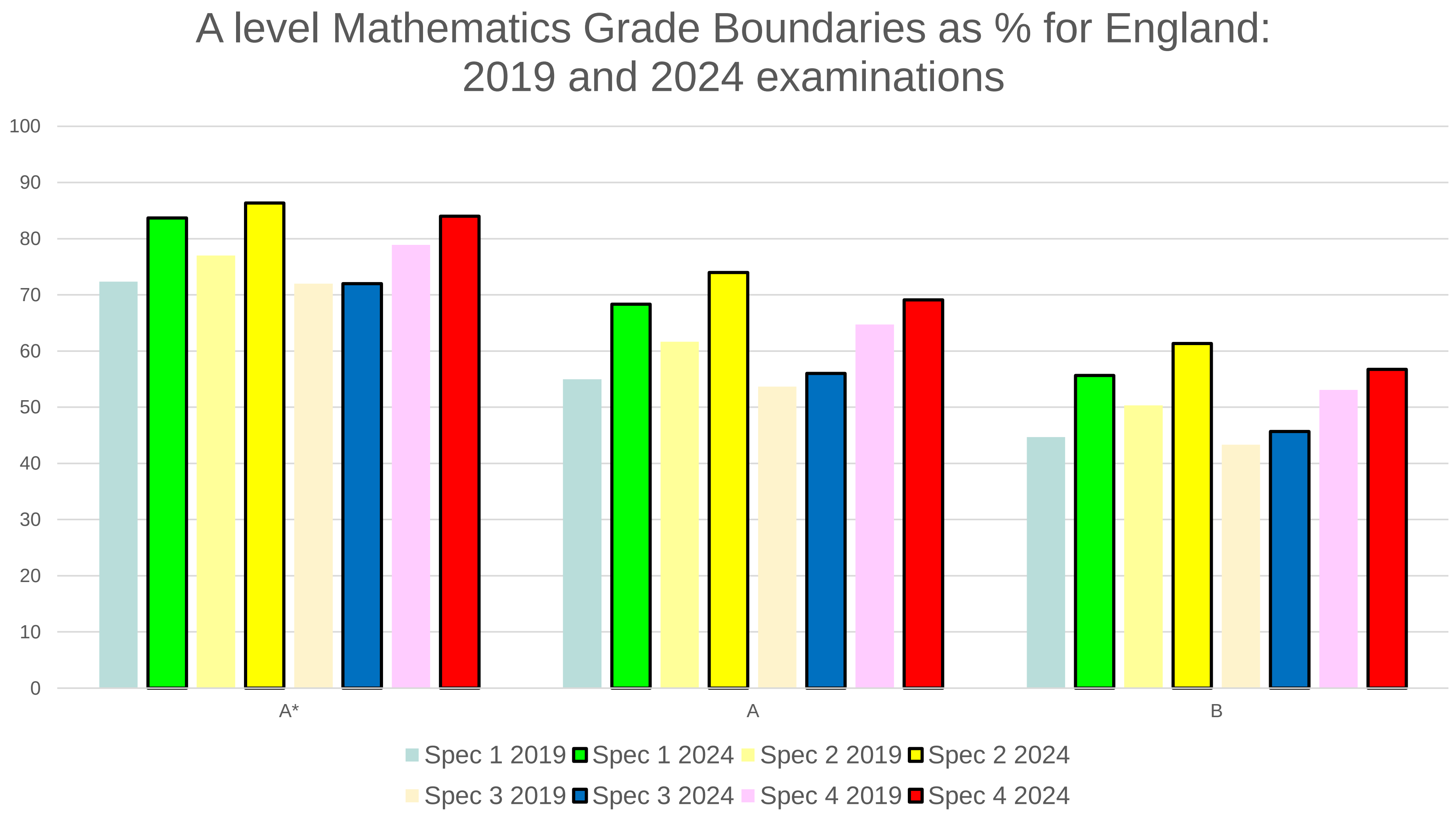
Part of the design of a stained-glass window is shown in Figure 1. The two loops enclose an area of blue glass. The remaining area within the rectangle *ABCD* is red glass.

The loops are described by the curve with parametric equations

$$x = 3\cos t, \quad y = 9\sin 2t, \quad 0 \le t < 2\pi.$$

The sides of the rectangle *ABCD*, in Figure 1, are the tangents to the curve that are parallel to the coordinate axes. Given that 1 unit on each axis represents 1 cm, find the total area of the red glass.

A level 2019 -



A level Mathematics Grade Boundaries as % for England:
2019 and 2024 examinations
100
90
80
70
60
50
40
30
20
10
0
A*
A
B
Spec 1 2019
Spec 1 2024
Spec 2 2019
Spec 2 2024
Spec 3 2019
Spec 3 2024
Spec 4 2019
Spec 4 2024

Views

1. Acquisition of knowledge and understanding to develop ***basic****
mathematical and quantitative skills.

2. Acquisition of knowledge and understanding to develop ***advanced****
mathematical and quantitative skills.

3. Application of 1 and 2 in meaningful and relevant contexts (to the individual).

4. What do we mean by ***mathematical and quantitative*** skills?

GCSE 2017 -

General quantitative literacy (GQL)

The second, which we call general quantitative literacy (GQL) , addresses the growing and currently unmet need for all students to confidently apply their mathematical and data skills to the common, real-world, quantitative problems they are likely to face, and in a range of educational, employment and everyday contexts. These skills need to be developed beyond GCSE study.

The ability to use and apply mathematical concepts and use digital tools to address real-world quantitative problems.

With the exception of those with the highest grades, GCSE performance currently provides little information about what learners can and cannot do. An essential element of a mathematical and data education will be qualifications and assessment methods that reliably describe the competences of learners.

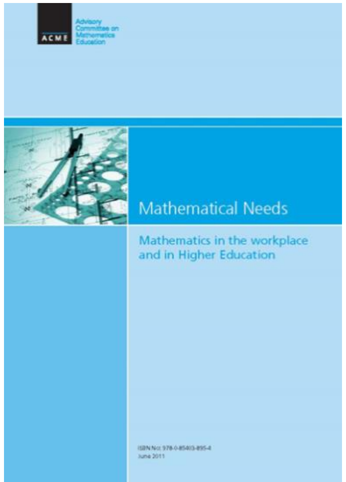
GCSE 2017 -

GCSE 2024 Higher Tier						
Pass Grade	4	5	6	7	8	9
Grade boundary (%)	14	25	37	48	65	82

RS ACME report – ‘Mathematical Needs: Mathematics in the workplace and in higher education’, 2011

The report’s key findings included the following:

- **We estimate that of those entering higher education in any year, some 330,000 would benefit from recent experience of studying some mathematics (including statistics) at a level beyond GCSE, but fewer than 125,000 have done so.**
- **Data on higher education acceptances suggest that some 180,000 of those accepted will encounter a significant amount of mathematics on their courses.**
- **An additional 150,000 students in the social sciences will also encounter some mathematics on their courses.**
- **The total demand figure for mathematically competent students is thus over 330,000 per year, but the supply is only 125,000.**



University of Reading

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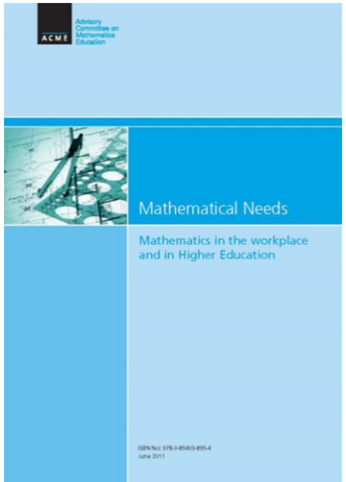


University of Reading

Core Maths Qualifications

1. **Deepen competence in the selection and use of mathematical methods and techniques.**

2. **Develop confidence in representing and analysing authentic situations mathematically and in applying mathematics to address related questions and issues.**

3. **Build skills in mathematical thinking, reasoning and communication.**

‘Content’

- Modelling
- Statistics
- Finance
- Working with exponentials
- Working with graphs and gradients
- Geometry and measures
- Risk
- Statistical problem solving
- Use of technology
- Estimation
- Problem solving
- Communicating solutions
- Numerical problem solving
- Mathematical processes
- Probability

Core Maths Qualifications

Core Maths qualifications:

- consolidate and build on students' mathematical understanding and develop further mathematical understanding and skills in the application of maths to authentic problems;
- provide a sound basis for the mathematical demands that students will face at university and within employment across a broad range of academic, professional and technical fields;
- prepare students for the varied contexts they are likely to encounter in vocational and academic study and in future employment and life, for example, financial modelling and analysis of data trends;
- foster the ability to think mathematically and to apply mathematical techniques to variety of unfamiliar situations, questions and issues with confidence;
- are likely to be particularly valuable for students progressing to higher education courses with a distinct mathematical or statistical element such as psychology, geography, business and management

Core Maths Premium

- 2024: £900 **additional** funding for **<u>EVERY</u>** student studying a Core Maths qualification

Core Maths assessment

TASK 3: HEALTH CENTRE

10 The table shows some information from a survey of 500 people about the waiting time, in days, between making an appointment to see a doctor at a health centre and the date of the appointment.

Waiting time (t days)	Frequency
$0 \leqslant t < 1$	124
$1 \leqslant t < 7$	192
$7 \leqslant t < 14$	112
$14 \leqslant t < 28$	56
$28 \leqslant t$	16

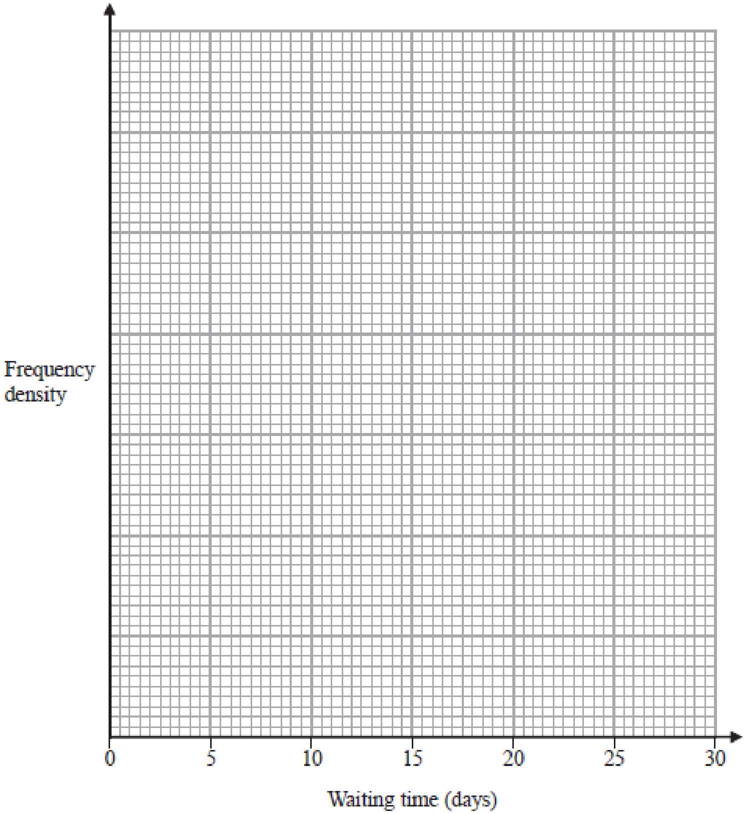
An additional 80 people make an appointment to see a doctor at the health centre.

(a) Using the information in the table, work out an estimate for the number of the additional 80 people who will have a waiting time of less than one day.

16 of the 500 people who completed the survey waited at least 28 days between making an appointment and the date of the appointment.

(b) Explain why the interval $28 \leqslant t$ used to represent these people **cannot** be shown on the histogram.

(c) On the grid opposite, draw a histogram to represent the information for all the people who have a waiting time of less than 28 days.



A newspaper reports that the average waiting time in the UK between making an appointment to see a doctor and the date of the appointment has recently exceeded 2 weeks for the first time.

(d) By finding an estimate for the median, compare the waiting time at the health centre with the average waiting time in the UK.

The health centre aims to provide people with an appointment with a waiting time of less than 10 days.

(e) Work out an estimate for the number of people, from the 500 people surveyed, who had a waiting time of less than 10 days.

Core Maths assessment

TASK 1: VACCINES AND BLOOD GROUPS

4 Kevin teaches at a college.

He carries out a survey of 100 of the students at the college.

He finds out that:

87 students have had the DTP vaccine

77 students have had the Meningitis C (Men C) vaccine

86 students have had the MMR vaccine

75 students have had all three vaccines

2 students have not had any vaccines

2 students have had just the Men C and the MMR vaccine

No students have had just the DTP and MMR vaccines

(a) Work out the probability that a student, chosen at random, has had only the DTP vaccine.

(b) Two students are chosen at random.

Work out the probability that both students have received fewer than two of the vaccines.

Core Maths assessment

5 The MMR vaccine protects against measles, mumps and rubella.

In Kevin's survey of 100 students, 86 of them have had the MMR vaccine.

If a student has received the vaccine then the probability that they will get measles is 0.01
If a student has **not** received the vaccine then the probability that they will get measles is 0.9

One of the 100 students in Kevin's sample is chosen at random.
Work out the probability that this student will get measles.

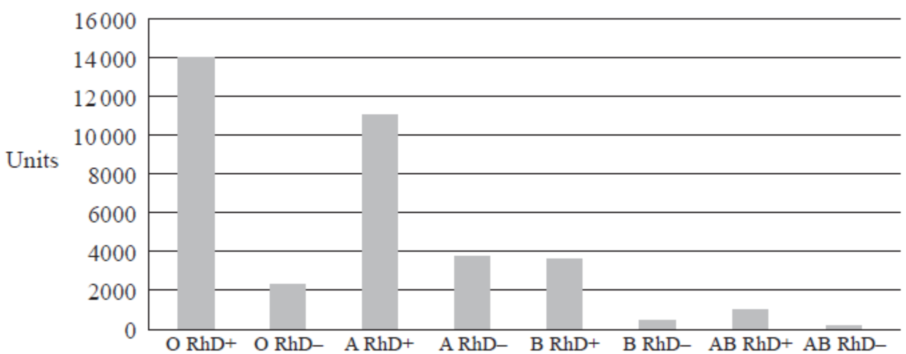
Core Maths assessment

6 Kevin teaches his biology class about blood groups.

Red blood cells sometimes contain a protein known as the RhD antigen. If this is present then the blood group is RhD positive (RhD+). If this is absent then the blood group is RhD negative (RhD–).

UK population		
Blood group	**RhD status**	**Percentage**
O	RhD+	37%
	RhD–	7%
A	RhD+	35%
	RhD–	7%
B	RhD+	8%
	RhD–	2%
AB	RhD+	3%
	RhD–	1%

Blood Stocks



(Source: adapted from NHS Blood.co.uk)

Blood is normally stored for a maximum of 30 days.
8 000 units of blood are needed each day by hospitals in the UK.

(a) Is there enough of stock B RhD+ blood available for the next 8 days?

All the students in Kevin's biology class test their blood to find out their blood group.

The table shows their results.

Kevin's class	
Blood group	**Number of students**
O	13
A	7
B	4
AB	0

A student from Kevin's biology class is picked at random.

(b) Work out an estimate for the probability that this student is RhD–.

A person is selected at random from the UK population.

(c) Given that this person is RhD+, work out the probability that they have blood group A.

Core Maths assessment

Drug decay

Drugs can be considered to decay exponentially in the body, and therefore can be considered to have a half-life.

In such cases the amount of drug, *A*, in your body at time, *t*, can be modelled by the exponential function

$$A = A_0 e^{-\left(\frac{0{\cdot}693}{T_{1/2}}\right)t}$$

where A_0 is the amount of drug in your body at time $t = 0$ and $T_{1/2}$ is the half-life of the drug.

When prescribed a drug by a doctor, or if taking a pain killer such as paracetamol, for example, it is usual to take some of the drug at regular intervals throughout the day.

For example:

- if you take a single 250 mg dose of a painkiller with a half-life of two hours there will be a level of 62.5 mg of the drug in your body after 4 hours;
- if you take 250 mg of the painkiller every 4 hours, after 8 hours, having just taken the third dose of the painkiller, you will have a level of 328.125 mg of the drug in your body.

Task:

Model the level of painkiller for a patient who is told to take up to a maximum of 4000 milligrams per day with doses at regular intervals. For example, they could take 1000 milligrams at 8 am, 12 noon, 4pm and 8pm every day.

Consider the effect of the person taking different doses at different regular intervals during the day.

The painkiller has a half-life of between 1 and 4 hours.

Use what you find out from your models to write a brief note to explain to patients how the concentration of a drug in general varies with time.

Show clearly all your working and highlight your final briefing note.

Bond Review 2018

"Skilled mathematicians of a high calibre are needed and they are in short supply. Demand for mathematical expertise across a wide range of subjects is booming: in addition to perennial demand for first-rate mathematical talent from financial markets, developing fields such as AI and machine learning, genomics, autonomous vehicle development, robotics, data science, the digital economy and many others are creating highly paid jobs for appropriately skilled people."

"The world requires 21st century mathematics to create 21st century technologies, and from smart cities to personalised medicine, new mathematics will lie at the heart of every major innovation."

"Innovation across the entire economy is a fundamental driver of living standards for the UK and mathematics is arguably the single most pervasive and powerful of all drivers of innovation in the world today."

"Financial services, security, defence, health, manufacturing, transport, film-making, and many other sectors all make use of many fields within the mathematical sciences. Developments in genomics, data science, economics, physics, quantum computing, biology, advanced engineering, epidemiology, zoology, sociology, geography, ecology, climate science, cybersecurity, social media analytics and numerous other fields all require the use not only of existing mathematical methods, but also the development of new, more powerful mathematical tools to continually spur advances and innovation."

"Mathematical departments in universities produce people who go out into the world and change the world, both in industry and also by crossing into and transforming other disciplines."

"We live in the era of mathematics. Its influence permeates economic and social activity and its influence and impact are profound."

The era of mathematics: the Bond review, 2018

Quantitative/Data science skills 2019 -

- The Royal Society's 2019 Dynamics of Data Science Skills report identified as a key recommendation that:
    - **ensuring our education system provides all young people with data science knowledge and skills will require curriculum change within ten years**

Mathematical and Data Education 2022

"In our daily lives, we are continually faced with an avalanche of data, figures, numbers and statistics across both legacy and social media.

In modern Britain, too many people have not been taught the skills to process all this digital data. And while we need many more scientists and mathematicians who can create and manage complex algorithms, we also need to upskill the many millions who lack these essential 21st century skills."

*"The introduction of **Core Maths** as an alternative to A level has been a popular and positive change. The base of those who continue to study maths post-16 has been broadened, and this is extremely encouraging news. We should try and use this change as a springboard for wider changes, as has happened elsewhere in the world."*

*As an early step in a transition to a broader education, Smith advocates for all young people to receive mathematics training post-16 through the **Core Maths** qualification which equips students with the mathematical, statistical and data skills essential for future study, employment, and life."*

Professor Sir Adrian Smith
President of the Royal Society
June 2022

Maths to 18
2011 -

“Maths, data, statistics and numeracy are essential skills for a modern world, whether for the workplace or for playing an active role in society. If we want our economy to thrive and young people to be prepared for well paid jobs, we need a radical overhaul of our education system that will include all young people doing some level of maths to 18 years of age. The PM understands this and today’s announcement is welcome.

*“While we have some elements in place to increase maths and data skills, we need to **upgrade the post-16 approach as part of wider reform at secondary and post-16**. It is time for a **baccalaureate style system** that will give a broader education than the exceptionally narrow A-levels.”*

Professor Sir Adrian Smith
President of the Royal Society
January 2023

RS Mathematical and Data Education 2024

- Mathematical and data sciences are everywhere.
- They increasingly support thinking and decisions in government, in industry, finance and business, and in academic disciplines.
- They influence the day-to-day lives of individuals as employees, citizens and consumers of information.
- The massive increase in the use and availability of data through digital technologies means that this influence can only grow.
- Scope and application of mathematics have undergone a remarkable expansion, partly driven by an unprecedented surge in data availability, computing capabilities, and statistical methodologies.
- Data now plays a pivotal role in both employment and everyday life.

RS Mathematical and Data Education 2024

- There has been a steady movement away from manual and low-skill jobs towards those requiring higher levels of expertise and problem-solving skills.
- There will be an increasing need in the future for mathematically and data educated people, and mathematical and data literacy has become increasingly necessary for daily life and as an engaged citizen.
- Too few of our citizens are trained to the high levels of mathematical and data competence that will be needed in the future.
- The rise of big data, machine learning and AI demands a shift towards statistics, data science and computing.
- The scope of mathematical education needs to change from ‘mathematics’ to a combination of mathematics, statistics, data science and computing.

RS Mathematical and Data Education 2024

- Need to equip future citizens with the capabilities, skills, adaptability and resilience they need to lead fulfilled lives in a fast-changing, data-rich world where mathematics and data play increasingly important roles in everyone's lives.

- Provide a better mathematics education for everyone, from the everyday needs of citizens to the brilliant mathematicians of the future.

- A new approach will result in many more people wanting to continue with the study of mathematics and data science and will lead to a more mathematically skilled labour force.

- The improvement in the skills of future generations would make a transformative contribution to the UK's economy and to our preparedness for the future.

- By contrast, continuing with our present arrangements condemns the UK to life in the slow lane.

RECOMMENDATION

Area for action 2: curriculum

Design and implement a curriculum that integrates appropriate data, statistics, and computational tools coherently with mathematics.

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Foundational and advanced mathematics

An evolution of the maths currently taught in school, with greater emphasis on data, technology and computing. It will continue to reflect the subject as a canon of knowledge that can be studied to the highest level.

General quantitative literacy

Addressing the need for all students to confidently apply their mathematical and data skills to common, real-world, quantitative problems in a range of educational, employment and everyday contexts.

Domain-specific competences

Recognition that mathematical and data skills are increasingly used within the classroom and beyond, in job or domain specific contexts

All citizens need foundational mathematics skills and general quantitative literacy for their daily lives. Individuals in vocational and technical roles often need domain-specific competences particular to those occupations, while roles traditionally seen as non-quantitative now require increased mathematical and data skills. At the same time, the demand for employees with advanced levels of mathematical and data skills is already high and is certain to increase substantially.

DfE (independent) Curriculum & Assessment Review 2024-5

Review of the existing national curriculum and statutory assessment system in England, to ensure they are fit for purpose and meeting the needs of children and young people.

The review will ensure that the curriculum appropriately balances ambition, excellence, relevance, flexibility and inclusivity for all children and young people.

DfE (independent) CAR – Interim Report 2024-5

Subject specific knowledge remains the best investment we have to secure the education young people need in a world of rapid technological and social change. Being secure in foundational subjects such as maths and science will remain pivotal, now and in the future.

We must ensure that young people are equipped to shape an increasingly AI-powered world. They need to be able to navigate misinformation and other challenges, and they also need to be able to take the opportunities that will be available to those who can become the most skilful shapers and operators of AI. This requires a strong focus on maths, but also the development of sophisticated analytical skills, and higher order domain-specific problem-solving ability, rooted in secure knowledge.

Thank you

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