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Whitehead Prize: citation for Richard Montgomery

Short citation:

Dr Richard Montgomery of the University of Warwick is awarded a Whitehead Prize for his outstanding work on the absorption method, and on sublinear expanders, in extremal and probabilistic combinatorics.

Long citation:

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In his PhD thesis, Montgomery found the threshold for an n-vertex binomial random graph to contain (simultaneously) all bounded degree trees on n vertices. Later, with Pokrovskiy and Sudakov, he proved Ringel's Conjecture, showing that one can decompose the edges of a complete graph on 2n-1 vertices into copies of any n-vertex tree. Very recently, he proved half of the Ryser–Brualdi–Stein conjecture, finding an optimal transversal of any Latin square of even order. All these problems were old conjectures and until recently considered unapproachable.

In each of these papers, the theme is that one can solve an approximate version of the problem by relatively standard probabilistic methods, constructing close to n-vertex trees or nearly-optimal transversals, but it is not possible to then complete these approximate solutions to exact ones because the random methods will have made a few 'mistakes'. The absorption method is to start the construction with a carefully chosen small and very flexible part, then continue to an approximate solution, then use the flexibility of the first part to correct any mistakes and obtain the desired exact solution.

This is easier said than done: Montgomery has developed a number of new approaches to absorption, in particular template absorption, which have been picked up by the community and which are what enable the solution to these old problems.

In another strand of work, together with Liu, Montgomery revived the old technique of applications of sublinear expanders in graphs. Here the theme is that one wants to construct a large object in a graph, and this is relatively easy to do if it is known that the graph has a simple structure, or if the graph is a strong expander. Unfortunately, in many cases of interest there is a gap: a graph can fail to have a simple structure but also not contain any strong expanders. But it will contain a weaker 'sublinear' expander. Liu and Montgomery developed a new and powerful method to find desired objects in sublinear expanders, and used it to prove a conjecture of Mader on clique subdivisions in C4-free graphs, of Erdős and Hajnal on the odd cycles in graphs of high chromatic number, and of Erdős on cycles with length a power of 2 in graphs of high average degree. This method too has been taken up by the community and powers several recent papers: a particularly nice example is Montgomery's attack, with Bucić, on the Erdős-Gallai cycle decomposition problem.

The work of Montgomery and his coauthors has brought a new energy to extremal and probabilistic combinatorics, solving central problems and creating new techniques which allow the community to attack conjectures which two decades ago seemed hopelessly out of reach.