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NEWSLETTER

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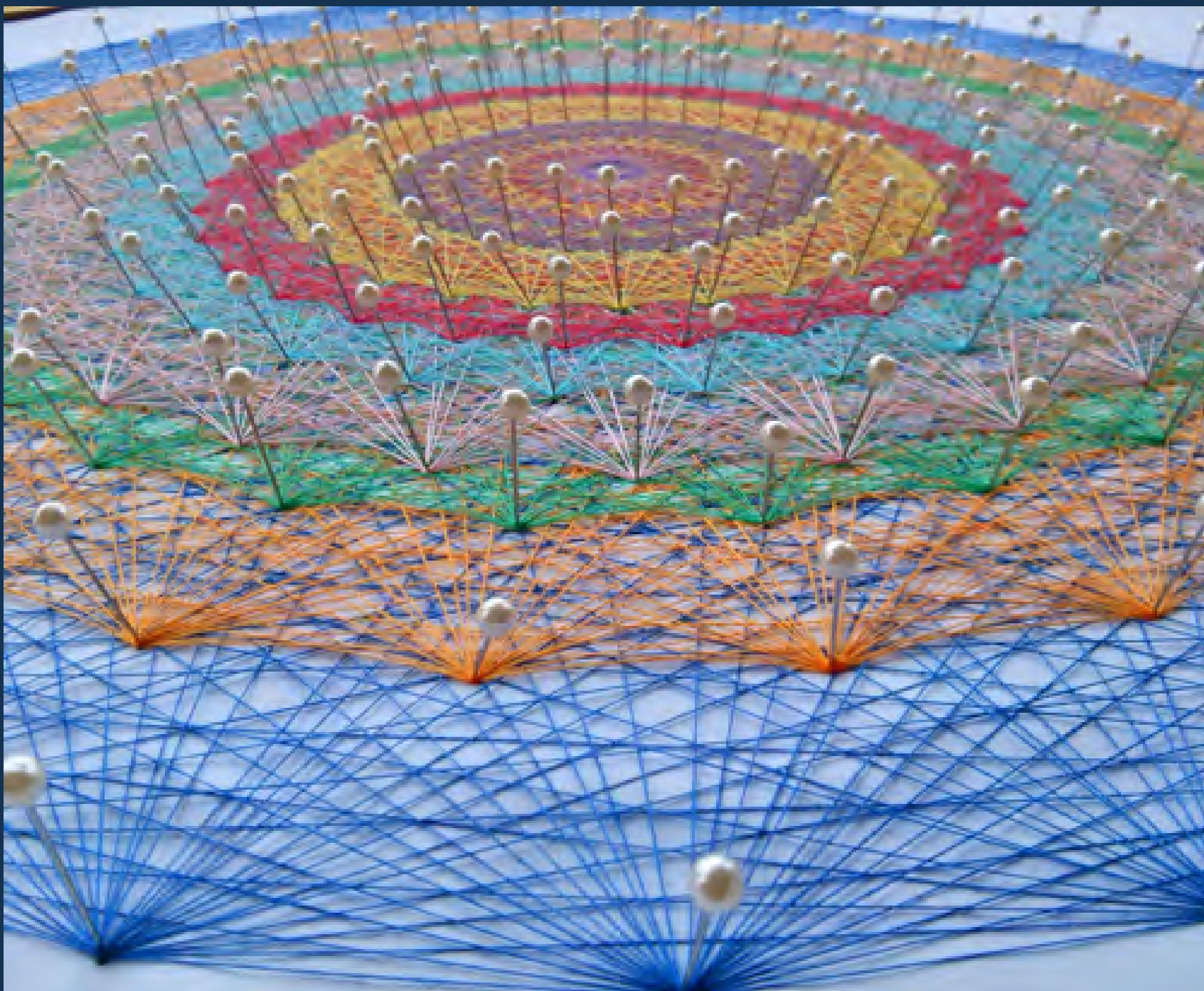

THE POETRY OF GEORGE BOOLE

FERMAT'S TWO SQUARES THEOREM

THE SHAPE OF A MODULE CATEGORY

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COVER IMAGE

The Coxeter projection of the E_8 root system, a string art created by Tamás Görbe (University of Leeds). For more details and images, see tamasgorbe.com/symmetry.

Do you have an image of mathematical interest that may be included on the front cover of a future issue? Email images@lms.ac.uk for details.

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MEMBERSHIP

Joining the LMS is a straightforward process. For membership details see lms.ac.uk/membership.

SUBMISSIONS

The Newsletter welcomes submissions of feature content, including mathematical articles, career related articles, and microtheses from members and non-members. Submission guidelines and LaTeX templates can be found at lms.ac.uk/publications/submit-to-the-lms-newsletter.

Feature content should be submitted to the editor-in-chief at newsletter.editor@lms.ac.uk.

News items should be sent to newsletter@lms.ac.uk.

Notices of events should be prepared using the template at lms.ac.uk/publications/lms-newsletter and sent to calendar@lms.ac.uk.

For advertising rates and guidelines see lms.ac.uk/publications/advertise-in-the-lms-newsletter.

CONTENTS

LMS President-Designate Appointed Next Director of INI

Professor Ulrike Tillmann FRS, LMS President designate, has been appointed the next Director of the Isaac Newton Institute (INI). Professor Tillmann will become the seventh Director since the INI opened in 1992, following in the footsteps of Sir Michael Atiyah, Professor Keith Moffatt, Sir John Kingman, Sir David Wallace, Professor John Toland and current Director Professor David Abrahams. Her five-year appointment begins on 1 October 2021.

Professor Tillmann is known for her leading contributions to algebraic topology and for her many contributions to the LMS and the broader mathematical community. Her research interests include Riemann surfaces and the homology of their moduli spaces. Her work on the moduli spaces of Riemann surfaces and manifolds of higher dimensions has been inspired by problems in quantum physics and string theory. More recently her work has broadened into areas of data science.

She has supported the wider mathematical community both nationally and internationally. This includes as a Fellow of the Alan Turing Institute and also serving on the scientific boards of several international insititutions, including the Oberwolfach Mathematical Research Institute (MFO). Professor Tillmann was also a member of Council of the Royal Society until the end of 2020 and served as an interim Vice-President in 2018.

Professor Tillmann has received several prestigious honours including the LMS Whitehead Prize in 2004. She was elected a Fellow of the American Mathematical Society in 2012 and a Member of the German National Academy of Sciences Leopoldina in 2017.

More information is available at tinyurl.com/2vskdjbo.

The 'Research Reboot' Scheme

In response to a proposal from its Covid Working Group, the Society's Council has approved a new 'Research Reboot' grant scheme. This scheme aims to help mathematicians restart their research activities following the intense disruption and upheaval of the pandemic.

Researchers may have found themselves with very little time for research due to illness, caring responsibilities, increased teaching or administrative loads, or other factors. This scheme offers funding for accommodation and caring costs for applicants so that they can leave their usual environment to focus entirely on research for a period from two days to a week. The initial period of the scheme will run from June to September 2021 and could, for example, cover the costs of a hotel or holiday apartment.

Applicants should be mathematicians based in the UK, the Isle of Man or the Channel Islands and may be at any career stage. We will prioritise applications from those who, as a result of the pandemic, faced increased caring responsibilities such as homeschooling. However, applications from those with additional professional demands such as teaching and administration are also welcome.

There will be an optional online support network for successful applicants. Applicants who are not able to commit to a period of time away from home can still apply to join the support network.

Research Reboot Scheme 2021
Application deadline: 15 May 2021

The value of the award will be a maximum of £100 per day for accommodation, subsistence, and other necessary expenses to enable the research project. An additional £100 per day may be applied for to cover Caring Costs for those who have dependents. For more information and the application form visit lms.ac.uk/grants/research-pairs-scheme-4. Prospective applicants are advised to consult the guidance available at tinyurl.com/63tmtc. Contact grants@lms.ac.uk if you have any queries.

News for US Taxpayers

We are delighted to announce that it has recently become possible for US taxpayers to make tax-deductible donations to the LMS. Until now this has not been possible and we are aware that it has been an issue for people wishing to make donations in the past.

The scheme works as follows. The LMS has recently been approved as an institutional member of the American non-profit organisation the British Schools and Universities Foundation (BSUF, bsuf.org). The sole purpose of BSUF is to allow tax-deductible donations from US donors to be made to British educational and cultural institutions. Anyone wishing to make a donation to the LMS through this means donates the money to BSUF, designating the LMS as their preferred institution. BSUF will then forward the full amount of the donation to the LMS. Donor advised funds may also be used for this purpose.

Donations can be made electronically via the 'Donate' button on the BSUF website; alternatively, cheques can be sent to the New York office of BSUF. For full details see the Donate section of the LMS website lms.ac.uk/content/donations. Professor Tim Riley of Cornell University has taken on the role of the LMS's US representative for this purpose and donations by cheque may also be sent to him; please check with him in advance. Queries can be addressed to development@lms.ac.uk or to Tim Riley at trr22@cornell.edu.

The LMS is always grateful for donations small or large. There are about 175 LMS members living in the United States and we are hopeful that many of them may take this new and exciting opportunity to donate to the LMS.

Forthcoming LMS Events

The following events will take place in forthcoming months:

LMS Spitalfields History of Mathematics Meeting: 14 May, online (tinyurl.com/y3kpv6ye)

Mary Cartwright Lecture 2021: 24 May, online

LMS Good Practice Scheme Workshop: 26 May, online (tinyurl.com/3ednhb82)

Midlands Regional Meeting and Workshop: 2–4 June, Lincoln (tinyurl.com/y5vtaytx)

LMS Meeting at the 8ECM: 22 June, online (tinyurl.com/pk4zy6nw)

Summer Graduate Student Meeting: 24 June, online (tinyurl.com/rt8xwn8)

Summer General Society Meeting: 2 July, London, online (tinyurl.com/rt8xwn8)

Northern Regional Meeting: 6–10 September, University of Manchester (tinyurl.com/yamy8uvq)

A full listing of upcoming LMS events can be found on page 50.

OTHER NEWS

International Mathematics Competition for University Students

Conditions permitting, the 28th IMC will be held on 2–8 August 2021 in Blagoevgrad, Bulgaria, organised by University College London and hosted by the American University in Bulgaria, Blagoevgrad. Conditions not permitting, the 28th IMC will be held online as was done last year. Universities are invited to send several students and one teacher as Team Leader; individual students without Team Leaders are welcome. The competition is planned for students just completing their first, second, third, or fourth years of university education and will consist of two sessions of five hours each. Problems will be from the fields of Algebra, Analysis (Real and Complex), Geometry and Combinatorics. The maximum age of participants is normally 23 years of age at the time of the IMC, although exceptions can be made. The working language will be English.

The IMC in Blagoevgrad is a residential competition, and all student participants are required to stay in the accommodation provided by the hosts. It aims to provide a friendly, comfortable and secure environment for university mathematics students to enjoy mathematics with their peers from all around the world, to broaden their world perspective and to be inspired to set mathematical goals for themselves that might not have been previously imaginable or thought possible. Past participants have gone on to distinguished careers in mathematics. Most notably, in 2018 Caucher Birkar received mathematics' most prestigious award, the Fields Medal. In 2000 he participated in the 7th IMC that was held at UCL.

Over the past 27 competitions the IMC has had participants from over 200 institutions from over 50 countries. For further information and online registration visit the website at www.imc-math.org.uk. Further details may be obtained from Professor John Jayne (j.jayne@ucl.ac.uk).

Atiyah Conference Update

Assuming there are no further untoward developments in regard to the pandemic, the LMS sponsored conference on the Unity of Mathematics in honour of Sir Michael Atiyah will take place as a hybrid meeting at the Isaac Newton Institute on 21–23 September 2021. Registration will open in late May or June and will be announced on the conference website, the LMS website and in LMS e-updates. For advance expressions of interest and notification when registration opens, please email Kathryn de Ridder at office@newton.ac.uk, using the subject line 'Sir Michael Atiyah Conference'. Those registering will be asked whether they plan to attend in person or online only. We shall endeavour to honour attendees' preferences as far we are able, subject to social distancing rules holding in September. For further details visit the conference website newton.ac.uk/atiyah.

The meeting is supported by the LMS, as well as by the Clay Mathematics Institute, the Heilbronn Institute for Mathematical Research, the National Science Foundation, the Isaac Newton Institute and the Oxford Mathematics Department. Thanks to the generosity of sponsors, some funding will be available for physical participants, with priority on early career researchers. NSF funding will be available to graduate students and postdoctoral researchers intending to travel to the UK from the USA provided the pandemic situation allows; for expressions of interest in regard to this fund please contact Laura Schaposnik at schapos@uic.edu.

Please note that international travel to the conference will depend on the pandemic situation in September. The organisers can take no responsibility for difficulties caused by any travel or quarantine restrictions that may be in force at the time of the meeting. We therefore strongly advise participants not to book any transport into the UK without clear prior advice from the Newton Institute staff.

STEM for Britain 2021

(l to r) Scott Harper, Georgia Brennan and Gioia Boshci, winners of the gold, silver and bronze awards, respectively

Over 100 early career researchers took part in the annual STEM for Britain poster competition held online on 8 March 2021. The competition enables mathematicians, scientists and engineers to present their research to politicians and policy makers.

The competition is organised by the Parliamentary and Scientific Committee. The Committee Chair, Stephen Metcalfe MP, commented: "This annual competition is an important date in the parliamentary calendar because it gives MPs an opportunity to see the work of a wide range of the country's best young researchers. These early career engineers, mathematicians and scientists are the architects of our future". Ten finalists presented their posters in the Mathematical Sciences section with Gold, Silver and Bronze awards made to the three early career researchers whose posters were judged to best communicate their research to a non-specialist audience.

This year's gold award winner was Scott Harper, a Heilbronn Research Fellow at the University of Bristol, for his poster on *Classifying Isolated Symmetries*; the silver award went to Georgia Brennan, a DPhil student at the University of Oxford, for her poster on *Mathematically Modelling Clearance in Alzheimer's Disease*, and Gioia Boshci, a PhD student at King's College London, received the bronze award for her poster on *Opinion Dynamics and Collective Memory*. The gold and silver awards were supported by the Clay Mathematics Institute and the bronze award was supported by the Heilbronn Institute for Mathematical Research. The winning posters can be viewed at stemforbritain.org.uk/.

Young Researchers Quiz Ministers and Advisers

Voice of the Future returned online this year giving young researchers the unique opportunity to put their questions to politicians and policy makers. The event is organised by the Royal Society of Biology on behalf of a wide range of STEM organisations, including the Council for the Mathematical Sciences (CMS).

In the morning session Amanda Solloway MP, Minister for Science, Research and Innovation, and Chi Onwurah MP, Shadow Minister for Science, Research and Digital were in the hotseat to answer questions on topics such as the new R&D 'people and culture' strategy, the Advanced Research and Innovation Agency and the lower uptake of the covid-19 vaccine within the Black, Asian and minority ethnic communities. The CMS was represented by Dr Emma Bailey (University of Bristol) who put a question to Amanda Solloway on quantum computing and whether the UK is 'quantum ready'.

In the afternoon session Sir Patrick Vallance, Government Chief Scientific Adviser, discussed among other issues the lessons he would like to take forward following the pandemic, such as how to work collaboratively to develop medicines quicker. For the final part of the afternoon the House of Commons Science and Technology Select Committee took questions from researchers. The Committee members discussed a range of issues, from the new research agency, ARIA, to plastic pollution and the role of hydrogen in achieving net zero.

A recording of the event is available on the Royal Society of Biology YouTube Channel at tinyurl.com/da3xy2p9.

2021 Abel Prize Winners

László Lovász (left) and Avi Wigderson. Photo credit: Nora Green

The Norwegian Academy of Science and Letters has awarded the 2021 Abel Prize jointly to László Lovász (Eötvös Loránd University, Budapest, Hungary) and Avi Wigderson (Institute for Advanced Study, Princeton, USA) "for their foundational contributions to theoretical computer science and discrete mathematics, and their leading role in shaping them into central fields of modern mathematics."

Lovász has served his community as a writer of books, noted for their clarity and accessibility, as an inspirational lecturer, and as a leader, spending a term as President of the International Mathematical Union (2007–10). In addition to his work on the foundational underpinning of computer science, Lovász has also devised powerful algorithms with wide-ranging applications. One of these, the LLL algorithm, named after Lovász and the brothers Arjen and Hendrik Lenstra, represented a conceptual breakthrough in the understanding of lattices, and has had remarkable applications in areas including number theory, cryptography and mobile computing. Currently, the only known encryption systems that can withstand an attack by a quantum computer are based on the LLL algorithm.

Wigderson is known for his ability to see connections between apparently unrelated areas. He has conducted research into every major open problem in the field which has in many ways evolved around him in co-authoring papers with more than 100 people while illuminating fundamental connections between mathematics and computer science. Early in his career, Wigderson made key contributions to internet cryptography, including the zero-knowledge proof, now used in cryptocurrency technology.

LMS Members Elected FRSE

(l to r) Tara Brendle, Paul Glendinning and Bernd Schroers

Three LMS members (Professor Tara Brendle, Glasgow; Professor Paul Glendinning, Manchester; and Professor Bernd Schroers, Heriot-Watt) have been elected Fellows of the Royal Society of Edinburgh (FRSE). The elected Fellows are chosen for excellence, measured against three criteria: outstanding achievement, professional standing, and societal contribution.

Eighty-seven new Fellows were elected in 2021 from across the sciences, arts, education, business and public life and join the RSE's current roll of around 1,600 leading thinkers and practitioners from Scotland and beyond whose work has a significant impact on their own fields and more widely.

Irish Mathematical Society President

Dr Tom Carroll (Cork) has been elected as President of the Irish Mathematical Society (IMS). The IMS is one of the reciprocal societies of the LMS; see other reciprocal societies at tinyurl.com/27ycvvb7.

IMA Catherine Richards Prize

The best article in Mathematics Today (published by the Institute of Mathematics and Its Applications) is awarded the Catherine Richards Prize each year. Professor Adrian Rice (Randolph–Macon College) was awarded the prize for his article, Srinivasa Ramanujan (1887–1920): The Centenary of a Remarkable Mathematician which is available on the IMA website at tinyurl.com/MT-Ramanujan.

Summer Science Exhibition

The Summer Science Exhibition is the Royal Society's flagship public engagement event which takes place every July at the Royal Society's prestigious premises in central London. Showcasing cutting edge science, it reaches an audience of 13,000 people including 2,500 school children and attracts significant media attention. The Royal Society will shortly be accepting proposals for participation in the 2022 Summer Science Exhibition from UK-based researchers. If your team has research that you would like to share with the public and want to know more, sign up for the mailing list to be notified when the call for proposals opens: tinyurl.com/syr9bk2f.

Montucla Prize 2021

LMS member and Newsletter author Brigitte Stenhouse, who is a PhD student at the Open University, has been awarded the International Commission on the History of Mathematics 2021 Montucla Prize for her article 'Mary Somerville's early contributions to the circulation of differential calculus' published in Historia Mathematica 51 (2020), 1–25. In the citation Stenhouse's article is described as "a well-documented, convincing contribution to our knowledge of Mary Somerville's socialization in the male-dominated world of 19th-century mathematics" and shows "Somerville's active engagement in the circulation of the differential calculus twenty years earlier than previously appreciated." The Montucla Prize is awarded to the author of the best article by an early career scholar published in Historia Mathematica in the four years preceding the International Congress of History of Science and Technology.

MATHEMATICS POLICY DIGEST

EPSRC Publishes Detailed Ethnicity Data

Data published by EPSRC outline the underrepresentation of ethnic minority researchers in its funding portfolio. Addressing this underrepresentation is a key priority for EPSRC and UK Research and Innovation (UKRI) and these data will inform ongoing work that supports this goal. More information is available at tinyurl.com/3n27bwmv.

Research Agency Launch

The Advanced Research & Invention Agency (ARIA) will be led by scientists who will have the freedom to identify and fund transformational science and technology at speed. The new agency will be tasked with funding high-risk research that offers the chance of high rewards, supporting ground-breaking discoveries that could transform people's lives for the better. More information is available at tinyurl.com/34f8dcaf.

Digest prepared by Dr John Johnston
Society Communications Officer

Note: items included in the Mathematics Policy Digest are not necessarily endorsed by the Editorial Board or the LMS.

New EMS website

The European Mathematical Society is pleased to unveil a completely new and more secure internet platform, coinciding with a rebranding and a new logo for the EMS. Visit euromathsoc.org.

8ECM

The Executive Committee of the European Mathematical Society and the organisers of the 8th European Congress of Mathematics have now decided that all lectures, minisymposia and special sessions planned to take place in Portorož, Slovenia, 20–26 June 2021 will be moved online. For full details see 8ecm.si/news/82.

AMS–SMF–EMS Joint International Meeting

This meeting, originally scheduled for 2021, has been postponed to 18–22 July 2022 in Grenoble. The date was chosen to be in synergy with the 2022 International Congress of Mathematicians, which will meet 6–14 July in Saint Petersburg, Russia. If the pandemic situation remains problematic the meeting will be held fully online.

EMS Journals: Open Access

EMS Press is pleased to announce that all ten of its Subscribe To Open (S2O) journals will become open access for 2021, including its flagship publication Journal of the European Mathematical Society. The Press chose S2O as its preferred route to open access as it does not rely on author fees, while allowing for equitable and sustainable open access with a focus on high-quality research. For details of how S2O works see ems.press/subscribe-to-open. A reminder also that Zentralblatt Mathematics (zbmath) is also now open access: see zbmath.org.

Raising Public Awareness

The EMS Raising Public Awareness (RPA) Committee have launched a new web-portal Pop Math: popmath.eu. Pop Math is a calendar and map of mathematical outreach events in Europe and beyond. Anyone can submit an event via the website.

Laure Saint-Raymond appointed at IHES

Laure Saint-Raymond will join the Institut des Hautes Études Scientifiques (IHES) as a permanent professor of mathematics in September 2021. Laure Saint-Raymond's work focuses mainly on the asymptotic analysis of systems of partial differential equations, in particular those governing the dynamics of gases, plasmas or fluids. She has made fundamental contributions to Hilbert's Sixth Problem on the axiomatization of mechanics: with various collaborators, she has shown that there is a continuous transition between the models of non-equilibrium statistical physics and the equations of fluid mechanics, and more recently she has studied the validity of these statistical models based on Newtonian mechanics. She is working in parallel on models in fluid mechanics that describe ocean currents, including the effect of fluid rotation and stratification on wave propagation and boundary layer phenomena. She is a recipient of numerous awards, including the title of Chevalier of the French Legion of Honour, and in 2020 she was awarded the Bôcher Prize by the American Mathematical Society.

Institut Mittag-Leffler (IML) call for event proposals

The IML hosts week-long conferences and workshops in all areas of mathematics at the current research frontier, and has announced a call for proposals for conferences, workshops, and summer schools in June and July 2022. The deadline for proposals is 15 August 2021. For more details see mittag-leffler.se/conferences-workshops/call-conferences.

EMS Magazine

The latest edition of the EMS Magazine, and the first since it has been renamed from the Newsletter, is available at the EMS Press website ems.press/journals/mag .

EMS News prepared by David Chillingworth
LMS/EMS Correspondent

Note: items included in the European Mathematical Society News represent news from the EMS are not necessarily endorsed by the Editorial Board or the LMS.

OPPORTUNITIES

LMS Grant Schemes

The next closing date for research grant applications (Schemes 1,2,4,5,6 and AMMSI) is 15 May 2021. Applications are invited for the following grants to be considered by the Research Grants Committee at its June 2021 meeting. Applicants for LMS Grants should be mathematicians based in the UK, the Isle of Man or the Channel Islands. For grants to support conferences/workshops, the event must be held in the UK, the Isle of Man or the Channel Islands:

Conferences (Scheme 1)

Grants of up to £7,000 are available to provide partial support for conferences. This includes support towards actual travel, accommodation and subsistence expenses for Principal speakers, UK-based Research students and Participants from Scheme 5 countries. Applicants unsure if the proposed country is eligible under a Scheme 5 grant should contact the Grants team. In addition, the Society allows the use of the grant award to cover Caring Costs for those attendees who have dependents.

Visits to the UK (Scheme 2)

Grants of up to £1,500 are available to provide partial support for a visitor who will give lectures in at least three separate institutions. Awards are made to the host towards the travel, accommodation and subsistence costs of the visitor. Potential applicants should note that it is expected the host institutions will contribute to the costs of the visitor. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents.

Research in Pairs (Scheme 4)

For those mathematicians inviting a collaborator, grants of up to £1,200 are available to support a visit for collaborative research either by the grant holder to another institution abroad, or by a named mathematician from abroad to the home base of the grant holder. For those mathematicians collaborating with another UK-based mathematician, grants of up to £600 are available to support a visit for collaborative research either by the grant holder to another institution or by a named mathematician to the home base of the grant holder. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents.

Collaborations with Developing Countries (Scheme 5)

For those mathematicians inviting a collaborator to the UK, grants of up to £3,000 are available to support a visit for collaborative research, by a named mathematician from a country in which mathematics could be considered to be in a disadvantaged position, to the home base of the grant holder. For those mathematicians going to their collaborator's institution, grants of up to £2,000 are available to support a visit for collaborative research by the grant holder to a country in which mathematics could be considered to be in a disadvantaged position. Applicants will be expected to explain in their application why the proposed country fits the circumstances considered eligible for Scheme 5 funding. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents. Contact the Grants team if you are unsure whether the proposed country is eligible, or check the IMU's Commission for Developing Countries definition of developing countries (tinyurl.com/y9dw364o).

Research Workshop Grants (Scheme 6)

Grants of up to £10,000 are available to provide support for Research Workshops. Research Workshops should be an opportunity for a small group of active researchers to work together for a concentrated period on a specialised topic. Applications for Research Workshop Grants can be made at any time but should normally be submitted at least six months before the proposed workshop.

African Mathematics Millennium Science Initiative (AMMSI)

Grants of up to £2,000 are available to support the attendance of postgraduate students at conferences in Africa organised or supported by AMMSI. Application forms for LMS-AMMSI grants are available at ammsi.africa.

For 2021 only, there will be a third round of applications for ECR Travel Grants with the deadline on 15 May 2021. Applications will be considered by the Early Career Research Committee at its June 2021 meeting:

ECR Travel Grants

Grants of up £500 are available to provide partial travel and/or accommodation support for UK-based Early Career Researchers to attend conferences or undertake research visits either in the UK or overseas.

For full details of these grant schemes, and to find information on how to submit application forms, visit the LMS website: lms.ac.uk/content/research-grants. Queries regarding applications can be addressed to the Grants Administrator Lucy Covington (020 7927 0807, grants@lms.ac.uk), who will be pleased to discuss proposals informally with potential applicants and give advice on the submission of an application.

Applicants seeking support for travel are advised to consider any travel restrictions in place due to covid-19 when planning their itinerary and whether they may need to make alternative arrangements.

Introducing Royal Society Read & Publish

If you are from an institution signed up to a Royal Society Read & Publish agreement, all fees are automatically covered by your library when you choose to publish an open access paper in a Royal Society journal. Check to see whether you are eligible at tinyurl.com/6ss6urx4.

ICIAM Prizes 2023: Call for Nominations

The International Council for Industrial and Applied Mathematics (ICIAM) Prize Committee for 2023 calls for nominations for the six ICIAM Prizes to be awarded in 2023: Collatz Prize, Lagrange Prize, Maxwell Prize, Pioneer Prize, Su Buchin Prize and Industry Prize. Each ICIAM Prize has its own special character, but each one is international in character. Nominations are welcomed from all over the world. A nomination should take into account the specifications for a particular prize (see iciam.org/iciam-prizes). Nominations should be made electronically through the website iciamprizes.org. The deadline for nominations is 1 September 2021. Contact president@iciam.org if you have any question regarding the nomination procedure.

Ya-xiang Yuan
ICIAM President

Royal Society Grants

The Royal Society provides grants and fellowships for outstanding researchers in the UK and internationally. The Society funds researchers at the postdoctoral level and above (undergraduate, Masters and PhD students cannot apply for funding). Grants can be found on the website at tinyurl.com/8v4s3j2v and the grants schedule at tinyurl.com/2drv77ab.

Mastermind Contestants

Contestants are currently being cast for the next series of the BBC Two quiz show Mastermind. Do you have what it takes to sit in the famous black chair? Know enough about your specialist subject to be forensically tested? Many start, but only one will finish... will it be you?

The organisers are looking to cast a diverse range of people throughout the whole of the UK. Anyone can apply as long they are aged 18 or over and are a resident of UK. You do need some general knowledge, but real interest is in your three specialist subjects — the wider the range the better.

To apply email mastermind.hth@hattrick.com for a link to the online application form. After you have submitted your application, one of the casting team may get in touch to organise a Skype audition. There will be a short general knowledge quiz to learn more about why you've chosen your specialist subjects. Applications close on Monday 24 May 2021 at midnight.

William Benter Prize in Applied Mathematics 2022

Call for NOMINATIONS

The Liu Bie Ju Centre for Mathematical Sciences of City University of Hong Kong is inviting nominations of candidates for the William Benter Prize in Applied Mathematics, an international award.

The Prize

The Prize recognizes outstanding mathematical contributions that have had a direct and fundamental impact on scientific, business, financial, and engineering applications.

It will be awarded to a single person for a single contribution or for a body of related contributions of his/her research or for his/her lifetime achievement.

The Prize is presented every two years and the amount of the award is US$100,000.

Nominations

Nomination is open to everyone. Nominations should not be disclosed to the nominees and self-nominations will not be accepted.

A nomination should include a covering letter with justifications, the CV of the nominee, and two supporting letters. Nominations should be submitted to:

Selection Committee
c/o Liu Bie Ju Centre for Mathematical Sciences
City University of Hong Kong
Tat Chee Avenue, Kowloon, Hong Kong

Or **by email to: lbj@cityu.edu.hk**

Deadline for nominations: 30 September 2021

Winner of the Prize 2020

The 2020 Prize went to Michael S. Waterman (University Professor Emeritus at the University of Southern California, Distinguished Research Professor at Biocomplexity Institute, University of Virginia). Due to the pandemic of Covid-19, the award ceremony will be held in summer 2022 at the **International Conference on Applied Mathematics**.

Presentation of the Prizes 2020 and 2022

The recipient of the Prize (2022) will be announced at the **International Conference on Applied Mathematics** to be held in summer 2022. The Prize Laureates (2020 and 2022) are expected to attend the award ceremony and present a lecture at the conference.

The Prize was set up in 2008 in honor of Mr William Benter for his dedication and generous support to the enhancement of the University's strength in mathematics. The inaugural winner in 2010 was George C Papanicolaou (Robert Grimmett Professor of Mathematics at Stanford University), and the 2012 Prize went to James D Murray (Senior Scholar, Princeton University; Professor Emeritus of Mathematical Biology, University of Oxford; and Professor Emeritus of Applied Mathematics, University of Washington), the winner in 2014 was Vladimir Rokhlin (Professor of Mathematics and Arthur K. Watson Professor of Computer Science at Yale University). The winner in 2016 was Stanley Osher, Professor of Mathematics, Computer Science, Electrical Engineering, Chemical and Biomolecular Engineering at University of California (Los Angeles), and the 2018 Prize went to Ingrid Daubechies (James B. Duke Professor of Mathematics and Electrical and Computer Engineering at Duke University).

The Liu Bie Ju Centre for Mathematical Sciences was established in 1995 with the aim of supporting world-class research in applied mathematics and in computational mathematics. As a leading research centre in the Asia-Pacific region, its basic objective is to strive for excellence in applied mathematical sciences. For more information about the Prize and the Centre, please visit ***https://www.cityu.edu.hk/lbj/***

Liu Bie Ju Centre for Mathematical Sciences
香港城市大學
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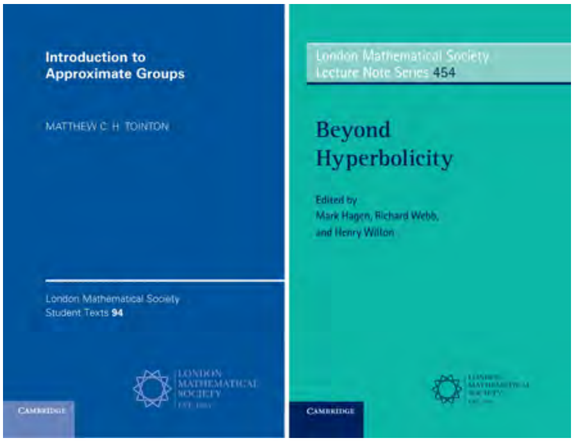


In this *Newsletter* issue we will be looking at the book series published by the Society and discounts that are available to all members.

The LMS has published world-class mathematical texts since its founding in 1865. There are 12 peer-reviewed journals, two book series and some individual book titles that are produced in collaboration with different partners.

All publications have high quality well-written articles that appeal to a broader mathematical audience. The Society's members can benefit from free online access to the *Bulletin, Journal* and *Proceedings of the LMS*, and receive discounts on books purchased directly from publishing partners.

The LMS Lecture Notes and LMS Student Texts are two book series produced in collaboration with Cambridge University Press. All members can save 25% on printed volumes in these series.

The LMS Lecture Notes Series was founded in 1968 and has become an established and valuable source of information for mathematicians and research professionals. Most of the volumes are short monographs written in an informal way that present an overview of a current development and provide a path to understanding recent advancements across a wide range of mathematical topics. The series also covers conference proceedings and similar collections that meet its general objectives.

The LMS Student Texts Series was introduced in 1983 to complement the LMS Lecture Notes and is designed for undergraduates or beginning graduate students. The textbooks cover the whole range of pure mathematics, as well as topics in applied mathematics and mathematical physics that involve a substantial use of modern mathematical methods. The series allows non-specialists and students with some background and knowledge in specific topics to get to grips with the subject. Materials of current interest of a non-standard nature are also covered in this series.

All publications from these book series are available to individual members at a discounted price when ordered directly from the Cambridge University Press website; see bit.ly/LMSmember for more information.

Independently, if your institution has purchased electronic editions of titles in the series, you can get free online access to them. This will be indicated by the word 'Access' (in green) and a tick mark next to these titles on the Cambridge Core platform.

The discount codes and other exciting news about Society's activities and events are sent to all members in our monthly eUpdates. See lms.ac.uk/publications for more information about the Society's publications or contact membership@lms.ac.uk if you have any other queries.

Valeriya Kolesnykova
Accounts and Membership Assistant

LMS Council Diary — A Personal View

Council met via video conference on Friday 5 February 2021. The meeting began with the President welcoming all members to the meeting, in particular the new members of Council, and also the President Designate, Professor Ulrike Tillmann FRS, who was attending the meeting as an observer. The President's business included a report on recent successful fundraising by the Development Committee, an update on the Michal Atiyah Memorial Projects, and an outline of the ongoing work of the Covid-19 Working Group by Vice-President Gordon. A draft Corporate Environmental Policy statement was also discussed and approved, and it was agreed that the Society should make a public statement related to the case of Azat Miftakhov, which has subsequently been posted on the Society's website.

Under communications business, it was agreed that most of the LMS blogs, with the exception of those related to MARM and the LMS Elections, could be discontinued as they have become used less and less.

The Publications Secretary then provided an update on recent contract negotiations. There followed a discussion on financial matters, in which the Treasurer reported a reduction in the projected deficit due to donations and an underestimate of Publications income, while a loss of income on De Morgan House facilities as a result of the pandemic has been offset by savings on operations. It was agreed to increase the 2021 budget of the Committee for Women and Diversity in Mathematics in order to fund a number of excellent proposals for Women in Mathematics Days and Diversity in Mathematics Days.

The LMS Elections Scrutineer reported on the 2020 Council and Nominating Committee Elections.

We also discussed committee membership, the 2021 Membership Census, and various annual committee reports, including the annual report of the Research Grants Committee, where it was noted that there had been a collapse in grant applications due to uncertainty over when it will be possible to host conferences again.

The meeting concluded with Council agreeing that, if possible, an LMS delegation should attend in person the 8ECM in Portorož, Slovenia, although this should be a smaller delegation than had originally been planned. (Sadly, since the Council meeting, the organisers have decided to make 8ECM an entirely online event, but there will be a strong LMS presence amongst the online attendees.) There was also an extended discussion on the proposed re-structuring plans for the mathematics department at the University of Leicester, about which a public statement has subsequently been posted on the Society's website.

Elaine Crooks
Member-at-Large

Highlighting the Impact of LMS Education Grants

The LMS has a major commitment to mathematics education, and provides a number of funding opportunities for teachers and other educators. The Society's Small Grants for Education scheme is intended to fund activities that stimulate interest in mathematics for Key Stage 1 students and beyond, while the Grants for Teacher CPD scheme is broadly intended to facilitate CPD for mathematics teachers by, for example, supporting teacher attendance at events held by professional mathematics organisations, or by assisting providers of professional development to run conferences or courses. The scheme also provides partial funding for workshops which disseminate good practice in teaching undergraduate mathematics.

The LMS Education Committee, which awards these grants, is particularly keen to receive applications for activities that have a wide impact and that enhance and enrich mathematical study beyond the curriculum, and/or that stimulate interactions across the mathematical community. Some examples of recently funded activities are below.

Small Grants for Education

1. Resources for 'House of Maths' workshop

This full-day workshop for Key Stage 1 pupils included activities and games designed to engage pupils in broad mathematical concepts. Following the workshop, maths packs were created, which included worksheets, dice, counters and a 100-square work board. These were sent to families to help them teach their children during the covid-19 lockdown. The school also used a small portion of the grant to create prizes for mathematics competitions while learning from home, intended to engage families with maths during the lockdown.

2. Primary school outdoor mathematical activity

The grant was used for materials needed to create an outdoor 'wigloo' (igloo constructed as a living willow structure). The construction, which was carried out by a group of Year 6 children, was intended for use by the whole school but especially targeted towards Key Stage 1 children. The children were given a floor plan of the structure and followed detailed instructions to build it, employing mathematical techniques along the way to work out the correct measurements. Building the structure enabled the children to gain a deeper understanding of load and balance, symmetry, measuring and counting.

Grants for Teacher CPD

1. Numicon training for teaching staff

The LMS grant was used to pay for training for teachers working with pupils with severe learning difficulties (SLD). Four members of the SLD team took part in a full day of training, which built on their knowledge of concrete and operational activities that can be used to support pupils in acquiring and recalling early mathematical concepts. Following this, the training was shared with the wider staff. This included sharing of virtual activities and access to the Numicon resources currently held in the school. The impact of the training has been significant in providing further tools for staff to support pupils with special educational needs.

2. Online workshops on assessment in undergraduate mathematics

Funding was provided to support a student intern at the University of Edinburgh who would administer three workshops on different aspects of online assessment using the STACK system during June/July 2020. The workshops were originally intended to take place in person, but were moved online owing to the lockdown.

Assessment of proofs in STACK
This seminar-style event featured three speakers: Juan Pablo Mejía-Ramos (Rutgers), Chris Sangwin (Edinburgh), and Siri Chongchitnan (Warwick). There were 40 participants from a number of different universities. Responses to the evaluation were uniformly positive about the “insight into pedagogical theory” and “seeing real examples”.

Addressing common student errors
This was a repeat of a workshop that had run successfully online in July, and previously as an in-person event at Loughborough University in 2019. It was led by Ian Jones (Loughborough) with input from Chris Sangwin (Edinburgh). There were 11 participants from a number of universities, and again feedback was very positive.

Effective use of Maxima
This workshop featured a demonstration by Chris Sangwin (Edinburgh) of some of the advanced features of STACK based on the Maxima computer algebra system, followed by time for participants to try out these features in small groups. There were 23 participants from a number of universities, and those who completed the evaluation form rated it as either very useful or extremely useful.

Further information on the LMS Education Grants can be found at lms.ac.uk/grants/education-grants. You can read more about the work of the Education Committee at tinyurl.com/44yh3d4r.

Katherine Wright
Society & Research Officer

LEVELLING UP

Maths Launches at Leicester and Durham

Both the Leicester and Durham branches of Levelling Up: Maths Scheme were launched in the week beginning 15 March 2021. Leicester’s programme has five student tutors and 25 A-Level students and Durham has six student tutors and 30 A-Level students.

At the time of writing this article Leicester students had two back-to-back sessions. The first session introduced them to working together on mathematics problems in a group, supported by their tutor, and the second session was on algebra. After this, the online tutor sessions will normally be 90 minutes every other week during school terms and the students will have sessions on coordinate geometry, algebra, differentiation, integration and trigonometry.

To prepare them for the sessions, the students were given some pre-reading to familiarise them with each topic and remind them of relevant material from their school teaching.

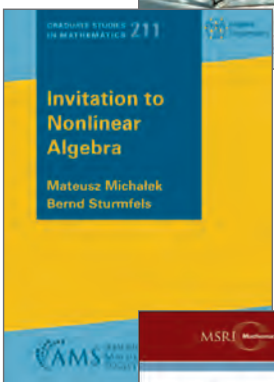
During the sessions, the students work on problems together, and there are also activities that help them to think about how they interact with mathematics, how they behave while problem solving, and other topics. The tutors are guided by tutor notes and have access to the problems and their solutions before each session. The students see the problems fresh and then have access to the solutions after the session.

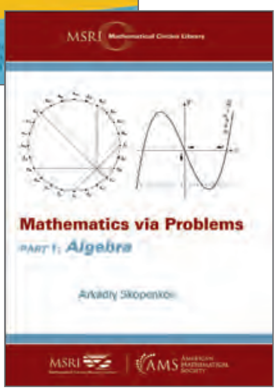
It is important in mathematics to write out ideas, diagrams, calculations and proofs etc. and in face-to-face sessions students would normally work together on a whiteboard. Thanks to the graphics tablets that the students have received as part of the Scheme they are able to work collaboratively using an online whiteboard, with the benefit of being able to interact in a group and exchange ideas. Providing graphics tablets also ensures that no students are disadvantaged by varying access to technology.

More information about the Levelling Up Scheme is available at levellingupscheme.co.uk.

John Johnston
Society Communications Officer

AMERICAN MATHEMATICAL SOCIETY





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Second Edition

Ethan D. Bolker, University of Massachusetts Boston & Maura B. Mast, Fordham University

Using this text, students work regularly with real data in moderately complex everyday contexts, using mathematics as a tool and common sense as a guide. The focus is on problems suggested by the news of the day and topics that matter to students, like inflation, credit card debt, and loans.

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Thomas Q. Sibley, St. John's University

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Mateusz Michałek, Max Planck Institute for Mathematics in the Sciences and University of Konstanz & Bernd Sturmfels, Max Planck Institute for Mathematics in the Sciences

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Graduate Studies in Mathematics, Vol. 211

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Arkadiy Skopenkov, Moscow Institute of Physics and Technology and Independent University of Moscow

A translation from Russian of Part I of the book *Mathematics Through Problems: From Olympiads and Math Circles to Profession*. The main goal of this book is to develop important parts of mathematics through problems.

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Cover Image: Exceptionally Beautiful Symmetries

TAMÁS GÖRBE

The classification of (semi)simple Lie algebras over the field of complex numbers is regarded by many as a gem of 19th century mathematics. It was first described by German mathematician Wilhelm Killing in a series of papers published between 1888–1890. A more rigorous proof (and the case of real Lie algebras) was presented by Élie Cartan in his 1894 PhD thesis. In 1947 the 22-year-old Eugene Dynkin formulated a modern, streamlined proof of the classification theorem which states that every semisimple complex Lie algebra is a "sum of building blocks", most of which belong to one of four infinite families. These are denoted by A_n, B_n, C_n, D_n with n being an arbitrary positive integer. Surprisingly, there exist five exceptional "building blocks" that don't fit into the above families. They are named E_6, E_7, E_8, F_4, G_2.

This result is reminiscent of, and in many ways serves as a precursor to, a much larger classification project: the classification of finite simple groups. In that case there are 18 infinite families and 26 exceptional pieces, known as the "sporadic groups". Another remarkable feature the two constructions have in common is their humble origins, as both stem from concepts that are defined via only a handful of axioms: *groups* and *Lie algebras*.

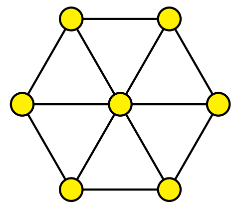
Now we introduce the main character of this story: a *Lie algebra* is a vector space $\mathfrak{g}$ over a field $\mathbb{F}$ equipped with a binary operation $[\cdot,\cdot] : \mathfrak{g} \times \mathfrak{g} \to \mathfrak{g}$ (called the *Lie bracket*) that satisfies the following:

- alternativity: $[x,x] = 0$, for all $x \in \mathfrak{g}$
- bilinearity: $[ax + by, z] = a[x,z] + b[y,z]$ and $[x, by + cz] = b[x,y] + c[x,z]$ for all $a,b,c \in \mathbb{F}$ and $x,y,z \in \mathfrak{g}$
- Jacobi identity: $[x,[y,z]]+[y,[z,x]]+[z,[x,y]] = 0$ for all $x,y,z \in \mathfrak{g}$

The simplest examples of Lie algebras are the space of 3-dimensional vectors with the cross product as Lie bracket and the space of $n \times n$ matrices with the commutator $[A,B] = AB - BA$ as Lie bracket.

The classification theorem of simple Lie algebras hinges on objects called *root systems*, which are symmetric configurations of vectors (usually) sitting in higher dimensional space. The dimension of this space is indicated by the subscripts, e.g. E_8 lives in an 8-dimensional Euclidean space.

Coxeter projection of root systems



The frame of a cube can cast shadows of different shapes, but only a few orientations lead to the most symmetric shadows, namely the projections through one of the four body diagonals. See the figure above. The method of finding the "most symmetric shadows" can be generalised to higher dimensions, and this is how the string art of the *exceptional root system* E_8, depicted on this Newsletter's cover page, was made. The pins are where the endpoints of root vectors land after projection, whilst the connections are obtained by connecting each vector to its nearest neighbours before projection. A fun fact is that due to the left-right symmetry of the connections, we have an even number of threads meeting at each pin resulting in *Eulerian Circuits*. This means that all connections of a given colour can be drawn using a single (really long) piece of string. For reference, when the outermost pins lie on a circle of radius 1 foot, the corresponding (blue) string has a length of 800 feet!

Unlike this article, the story of Lie algebras does not end here. It continues as an astonishing success story in the physics of the 20th century and beyond...

FURTHER READING

[1] T. Görbe, Exceptionally Beautiful Symmetries, string art, tamasgorbe.com/symmetry.
[2] J.E. Humphreys, Introduction to Lie Algebras and Representation Theory, GTM9, Springer, 1972.

Tamás Görbe

Tamás is an Academic Development Fellow in the School of Mathematics at the University of Leeds. His research interests include integrable systems, symplectic geometry, Lie theory and special functions.

Notes of a Numerical Analyst

Multivariate polynomials

NICK TREFETHEN FRS

My past twenty years have been spent working with polynomials. Polynomials are the starting point of numerical algorithms for integration, differentiation, root-finding, optimisation, and approximation, and in the software system Chebfun, every function is converted to this universal currency before you do anything with it. A typical f defined on $[-1,1]$ might be approximated to 16 digits by $p(x) = \sum_{k=0}^{n} a_k T_k(x)$ with $n = 500$, say, where T_k is the degree k Chebyshev polynomial.

But all this is univariate. What does one do in three dimensions, the base case of science and engineering? And how about dimensions $n > 3$, with applications from the many-particle systems of quantum physics to the high-dimensional search spaces of data science?

We all have our lacunae, and for years, one of mine was multivariate polynomials. You couldn't ask for a more respectable citizen of pure mathematics, as attested by ten Fields medals related to algebraic geometry, and I knew that one day, I would have to get serious and learn something about this subject. An excuse to put my house in order came recently in teaching a course at NYU. I decided to show the students case-by-case how, for each numerical problem, the basic 1D method you already know starts from univariate polynomials, and then in nD, there's a powerful analogue based on multivariate polynomials.

But as I tried to prepare my lecture I discovered, it wasn't so! When it comes to numerical computation, *multivariate polynomials are not used much.* Tensor products of univariate polynomials are used all the time (a special case), but not the multivariate version as normally understood, where we start from P_k, the set of polynomials of total degree $\leq k$.

In numerical integration in a square or a cube, for example, there's an elegant cubature idea introduced by James Clerk Maxwell: interpolate function samples by an element of P_k, then integrate the interpolant. But implementation is difficult (challenges of unisolvency), and although there's plenty of theory, these formulae are rarely employed. Or in numerical PDE, the dominant method is finite elements, which in principle can be based on multivariate polynomials of arbitrary degree. But in practice, most applications stick to degrees 1–4. Or in approximation of functions, you *could* use multivariate polynomials, but few do.

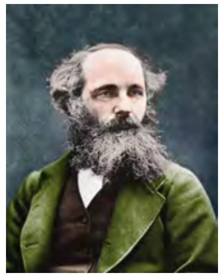


Figure 1. Maxwell proposed integrating a function via a multivariate polynomial interpolant. Why is this method so rarely used?

This got me thinking about another lacuna. Complex variables are my best-loved tool—why had I never mastered the multivariate case, several complex variables (SCV)? Given how much we gain from convergent series of polynomials, surely there's all the more to be gained from convergent series of multivariate polynomials? Well, with apologies to the experts, I now believe it isn't so. SCV is a fascinating field, full of challenges, but when it comes to developing numerical algorithms, it is the one-variable case we leverage.

Maybe the bedrock example is the Laplacian operator, the starting point of mathematical physics. In 3D we write it like this:

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}.$$

The Laplacian is the very archetype of an isotropic process—rotation-invariant—yet to work with it, we break it into univariate pieces.

Nick Trefethen
Trefethen is Professor of Numerical Analysis and head of the Numerical Analysis Group at the University of Oxford.

Mathematics News Flash

Jonathan Fraser reports on some recent breakthroughs in mathematics.

Breaking the logarithmic barrier in Roth's theorem on arithmetic progressions

AUTHORS: Thomas F. Bloom and Olof Sisask
ACCESS: https://arxiv.org/abs/2007.03528

A famous conjecture of Erdős asserts that a set of positive integers whose reciprocals form a divergent series should contain arbitrarily long arithmetic progressions. The celebrated Green-Tao Theorem which states that the *primes* contain arbitrarily long arithmetic progressions is a notable special case of this conjecture.

This much talked about paper proves the first non-trivial case of the conjecture by proving that such sets of integers must contain arithmetic progressions of length 3. This follows from a quantitative statement bounding the size of a subset of $\{1, \ldots, N\}$ which avoids 3-term arithmetic progressions.

It is a straightforward but instructive exercise (for non-experts) to consider if the converse of the Erdős conjecture holds, that is, is it true that if a set of positive integers contains arbitrarily long arithmetic progressions, then the reciprocals necessarily form a divergent series?

Bourgain's slicing problem

AUTHORS: Yuansi Chen
ACCESS: https://arxiv.org/abs/2011.13661

'Bourgain's slicing problem' is a famous open problem in convex geometry. It was formulated by Bourgain in the 1980s and is both fiendishly difficult and ripe with applications in other fields, perhaps most notably in statistics via sampling theory and random walks. It is also easy to state:

Conjecture: There exists a constant $\varepsilon > 0$ such that, for all integers n and all bounded convex sets in $\mathbb{R}^n$ with volume 1, there must exist a hyperplane section with volume at least ε.

The key here is that the constant ε must not depend on the ambient dimension n. Indeed it is easy to prove the conjecture if we allow $\varepsilon = \varepsilon_n$ to depend on n. In fact it is not too hard to show that ε_n can be taken to be $c/\sqrt{n}$ for some absolute constant c. Bourgain himself proved that one can take ε_n to be $c/(\sqrt[4]{n}\log n)$.

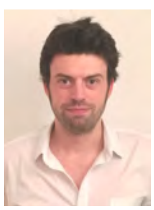
In November 2020 an enormous step forward towards resolving Bourgain's slicing problem was made by Yuansi Chen, a postdoc in Statistics in the Swiss Federal Institute of Technology, Zurich. Chen proved that ε_n can be taken to be $cn^{-\delta_n}$ with $\delta_n \to 0$.

A surprising observation in the quarter-plane diffraction problem

AUTHORS: Raphael C. Assier, I. David Abrahams
ACCESS: https://arxiv.org/abs/1905.03863

This paper was published in 2021 in *SIAM Journal on Applied Mathematics*. The authors consider the long-standing 3D canonical problem of wave diffraction by a quarter-plane (an infinitely thin angular sector with a right inner angle) using a two-complex-variable Wiener-Hopf approach. This approach was first considered by J. Radlow in the 1960s, resulting in a closed-form solution—an ansatz—obtained in a non-constructive way that has long been known to be erroneous (on physical grounds).

The authors show constructively how Radlow's ansatz arises and evidence the presence of a 'correction term', thus providing another reason for the invalidity of the ansatz. The diffraction coefficient resulting from the erroneous ansatz is then compared with the exact one obtained by the recently developed *modified Smyshlyaev formulae*. A surprising observation is that the two are almost indistinguishable, leading to numerous research questions to be addressed in future work.



Jonathan Fraser is a Reader of Mathematics at the University of St Andrews. His research interests centre on fractal geometry, geometric measure theory and analysis.

The Poetry of George Boole

DESMOND MACHALE

George Boole FRS (1815–1864) is well-known for his work on mathematics and logic, but he also wrote translations of classical verse, original sonnets and poetry. We reflect on some of the themes of his poetic work, the influences on his poetry and the relationships in his creative life.

Introduction

George Boole (1815–1864) has variously been described as the inventor of symbolic logic, the founder of pure mathematics, and one of the fathers of computer science [1]. He was almost entirely self-educated, first in Latin, Greek, French, German and Italian, and later in Optics, Astronomy, and Mechanics, and finally in Mathematics. His extensive knowledge of languages meant that he was able to read and follow in the originals the works of the great continental mathematicians such as Lagrange, Laplace, Legendre and Jacobi, and adopt their notation for the Calculus, at a time while many British mathematicians were still using the more cumbersome dot notation of Isaac Newton, often for patriotic reasons.

Bust of George Boole at University College Cork

However, Boole had many other strings to his bow. He was involved voluntarily in several social schemes and activities such as a Savings Bank, an early Trade Union, a Building Society, a Home for Unmarried Mothers and Reformed Prostitutes, and the Lincoln Mechanics' Institute [2]. From his mother, Mary Ann (nee Joyce) he inherited a great love of music, literature and poetry, just as he had inherited a love of science and mathematics from his father John Boole. Throughout his life, George Boole liked to relax by writing and translating poetry and left behind some seventy poems which have recently been collected with commentary and analysis [3].

Lincoln Cathedral

The early years

When he was only fourteen, Boole caused quite a stir in his native city of Lincoln by publishing translations into English from the classical Greek poets in a local newspaper. Here are the first eight lines of 'To The Evening Star':

> Hail! brightest wanderer of the west,
> Light of the golden realms of love:
> Hail! glory of the starry vest
> That evening spreads above;
> Night's sacred queen alone can boast,
> The splendour of thy silver ray;
> Though fairest of the heavenly host
> Star of departing day!

Controversy followed because a local classics master wrote to the editor claiming that the young Boole was a plagiarist and that no boy of fourteen could have produced such a translation unaided. These allegations were refuted in a series of letters to the editor, but this was perhaps the first indication that the people of Lincoln had that they had a young man of exceptional talent living among them.

W. R. Hamilton

Boole was not the only mathematician-to-be who was producing poetry around this time. William Rowan Hamilton (1805–1865) also took writing verse very seriously and often described himself as a poet rather than a mathematician. In 1821, he had written an original poem in English, that had precisely the same title as Boole's. It is interesting to compare the two poems and the poets' styles — Hamilton was sixteen and Boole fourteen when they were written. We include the first eight lines of Hamilton's poem here:

How fondly do I hail thee, Star of Eve!
In all thy beauty shining in the west;
And, as if loath our firmament to leave,
Slow and majestic sinking to thy rest!
Ere Night ascends her throne, while tinged the sky,
And yet all glowing with expiring day,
Floats thy fair orb upon the ravished eye,
Beaming a pure celestial living ray.

Hamilton, by the way, was one of the few mathematicians to write poetry about mathematics. He wrote a sonnet, 'Recollections of Collingwood', after he enjoyed the hospitality of Sir John and Lady Herschel at Collingwood in Kent. It was subtitled 'The Tetractys', in connection with the mystical symbol of Pythagoras, consisting of ten circles, arranged in an equilateral triangle, to represent the fourth triangular number $10 = 1 + 2 + 3 + 4$. Hamilton's sonnet is a celebration of his invention of quaternions, and contains the lines:

And how the One of Time, of Space the Three,
Might in the chain of symbol girdled be;
And when my eager and reverted ear
Caught some faint echoes of an ancient strain,
Some shadowy outline of old thoughts sublime,
Gently he smiled to mark revive again,
In later age, and occidental clime,
A dimly traced Pythagorean lore;
A westward floating, mystic dream of FOUR.

Here Hamilton is very tentatively suggesting an imagined lineage between his quaternions and the tetractys of Pythagoras, but on the surface all they would seem to have in common is the number four. Sadly, the hundred and twenty four poems Hamilton wrote during his lifetime have not appeared in book form (a nice retirement project for some enthusiastic follower?) but there is an excellent website (tinyurl.com/sxtc69ra) containing Hamilton's poems and commentary on them.

Further translations

After George Boole had translated much classical Greek verse into English he then moved on to translating Latin poetry, especially the Odes of Horace, some of the most beautiful verse ever written in any language, despite the fact that it was constrained by strict metrical rules. One of his best efforts was a translation into English of Horace's 'Non Omnis Moriar' ('I Shall Not Wholly Die'), whose theme is that part of the poet will continue to live even after his physical death, for as long as his poems are read and quoted. The same is true of course of mathematicians—what educated person on the planet has not heard of the Pythagorean Theorem?

Horace dedicated his poem to his patron Maecenas, whose name thereby lives on too. It is a very beautiful poem indeed, which the reader is encouraged to read in the original, where even with a lack of knowledge of Latin, the beauty of the language shines through. The translation by the young George Boole is sensitive, as one may see from the first six lines:

On pinions strong aloft I'll fly,
My airy course through clouds and sky,
And here no longer wait:
In this dull world I'll not delay
Nor in its splendid cities stay
Above pale envy's hate.

Boole soon moved to "loose translations or rather imitations of Horace" [3] where his own original contributions were now becoming more significant. At the same time he produced verse translations of French poetry ('Eliza Trizel' from the works of Chateaubriand), translations and extensions of Italian poetry (Petrarch's sonnets), and a translation of 'To a Great Oak', from the German of Fulleborn.

Original verse: sonnets

When Boole moved to write his own original verse in the English language, it was perhaps inevitable that he should choose a known and structured poetic form in which to express his thoughts and feelings. He had long admired the sonnets of Wordsworth, Milton, Shakespeare, Spenser, Dante and Petrarch. Boole wrote many sonnets in the period 1832–1855 and twenty-three of these survive. Their themes include nature, religion, friendship, English places, music, and Ireland. Of particular interest is his 'Sonnet to the Number Three', written in May 1846:

When the great Maker, on Creation bent,
Thee from thy brethren chose, and fram'd by thee
The world to Sense reveal'd, yet left it free,
To those whose intellectual gaze intent
Behind the veil phenomenal is sent
Space diverse, systems manifold to see,
Reveal'd by thought alone; was it that we,
In whose mysterious spirits thus are blent
Finite of Sense and Infinite of thought,
Should feel how vast, how little is thy store;
As yon excelling arch with orbs deep fraught
To the light wave that dies along the shore,
Till from our weakness and our strength may rise
One worship unto Him the only wise?

This poem has unexpected depths, which need to be explained. Though Boole professed to being a Christian, he was hardly an orthodox one, and had a particular difficulty with the doctrine of the Trinity. He saw God as a unity, one supreme being, in an almost Judaic fashion, and he had leanings towards Unitarianism, though he never formally became a member of that church. These beliefs had important consequences when he came to formulate his Algebra of Classes, nowadays called Boolean Algebra, where he used the symbol 1 for the universal class. In the poem just quoted, he appears to be suggesting that belief in the Trinity is connected with the fact that we interpret space in three dimensions. Interestingly, this was written around the time that Hamilton was suggesting that Time was a fourth dimension, and Boole himself wrote a paper on quaternions in 1848.

Boole also wrote sonnets with more personal themes. Here is 'Sonnet 12—Carisbrooke' which he wrote in 1849 in memory of his father John, who had died the previous year.

I saw the moonlight stream o'er Carisbrooke
I heard the owl sing there her doleful lay
Deep fell the shadows o'er my lonely way
As the dark ivy in the night wind shook.
Then standing there my parable I took
And said, 'Thou wreck of storm and bloody fray
Selected home of things that hate the day,
Why art thou thus, O ruined Carisbrooke?'
Because another day on men has broken
Because the reign of wrong has passed forever
Because a higher law to us has spoken
And war is not the height of man's endeavour,
Because in thy decay we see the token
Of that which banished once returneth never.

Carisbrooke Castle is located near Newport in the Isle of Wight, a favourite haunt of George Boole, often to take the water cure. King Charles the First had been imprisoned at Carisbrooke prior to his execution and it had a long history of war and conflict dating back to Roman times. Boole emerges as a confirmed pacifist and optimistically believes that mankind has seen the light and that there will be no more wars; one wonders what he would have made of two World Wars in the twentieth century, with his beloved England on one side and Germany, whose political and social system and scientific progress he admired so much, on the other.

Carisbrooke Castle

Life in Ireland

Boole arrived in Ireland in 1849 to take up the first Professorship of Mathematics at the new university, Queen's College Cork, now University College Cork. He wrote two sonnets to Ireland, but sadly misunderstood its complex political and social situation. Here is one of those sonnets:

Why sing of Ireland's wrongs? The Past is dead,
The shades of years departed are its pall,
And they who love their land should least recall
The dark career of barbarous ages fled,
Before her see a glorious Future spread,
With peace, with knowledge rich - the virtues all,
With Art's rich stores, and Nature's festival
In liberal harvests o'er her valleys shed.
In noble breasts past injuries are tame;
Courage distains them, Patience makes them light,
The wise forget them, and the good forgive.
O ye who seek the Patriot's holy aim!
Teach Ireland this—the self-sustaining might
Of Duty teach—instruct her how to live.

It is difficult to see how someone of Boole's undoubted intelligence could have misjudged the Irish situation so profoundly, just as he did in another sonnet written a few weeks previously. He makes no mention of the fact that nearly a million people had died of hunger in a horrendous famine over the previous five years, and that many others had been forced to emigrate, or of the underlying oppression and economic neglect that had brought the country to its knees. The past was not dead either then or now, and there lay ahead over a century of bitter conflict before even a partial solution was found. Boole's simplistic solution was that the Irish should forget their bitter thoughts and brooding about the past, and look forward to a bright future, but he had no suggestions on how to bring this about in reality, except for a vague plea for instruction on how to live and above all to do their duty.

Family, religion and other themes

Many of Boole's poems were dedicated to his family and friends. In 1836 he wrote a poem To My Very Dear Sister, celebrating the first ball she attended, and later he wrote her a poem entitled Life in Earnest. In 1845 Boole wrote a letter poem to his friend William Brooke from the Isle of Wight. He refers to his visit to Cambridge for his first annual meeting of the British Association for the Advancement of Science and some of the scientists and mathematicians he met there:

'Twas something on the banks of Cam to see
Men known to Science, known to History;
Airy, who trod the subtlest paths of light,
And Herschel, worn by many a watchful night,
Whewell and Brewster, Challis, large of brow,
And Hamilton, the first of those to know.

The grave of George Boole at Saint Michael's Church, Blackrock, Cork

Boole wrote a great deal of hymns and religious poetry too, with titles such as 'Consider the Lilies of the Field', 'Ode to Truth', 'The Council of Constance', 'Virtue', and 'Paraphrase of the 137th Psalm'. In these he showed a deep and humble faith in God. This is the first verse one of his most beautiful religious poems 'The Communion of Saints', which was sung as a hymn at a service to mark the 150th anniversary of his death on 8 December 2014, in Saint Michael's Church, Blackrock, Cork, in whose churchyard he is buried.

When the day of light declineth,
And the fields in shadow lie,
And the dewy Hesper shineth
Fairest in the western sky,
Visions in the twilight rise,
Night unseals the spirit's eyes.

Boole wrote on many other themes, especially his beloved English countryside—Hornsea, the Isle of Wight, Lincoln and its magnificent cathedral, but also on his feelings of sadness and loneliness in exile in Ireland. He despaired of ever marrying and settling down. It is believed he wrote a good deal of poetry, maybe even love-poetry, to his wife-to-be Mary Everest during their courtship in the years 1851–1855, but none of this seems to have survived; perhaps as she was wont to do, she destroyed it as being too personal. However, in 1845 Boole wrote a sorrowful poem called 'Love—Three Aspects of Nature', full of despair about his seemingly eternal bachelorhood, but in 1855, the year of his marriage to Mary Everest, he had changed his tune, as we can see from this excerpt of 'Ten Years Later':

> The woods and fields did once alone suffice
> To fill the heart with joy. I was not nice
> To question Heaven's good gifts, but took the blessing
> Just as it came, without or care or guessing
> But with departed youth the splendour fell,
> The meads were daisies and not asphodel.
> But now the common earth again is bright,
> Sweet peace is on the grass and on the flower;
> The rainbow spans the fields and gilds the shower,
> And sunset glows once more with golden light.

Boole also wrote many poems with great historical themes, with titles such as 'The Roman Triumph', 'The Departing of the Crusaders', and 'The Ruined City', a tribute to Babylon, where much of early mathematics had originated.

Married life

In 1855, soon after his marriage, his poetic activities came to an abrupt end. Mary Everest Boole explained:

> "Soon after our marriage, I found a sheet of paper covered with blank verse on some classical subject. 'What's this?' I asked. 'Some poetry of mine,' he replied. I read half through the passage. I had been told he was overworked. I had to preserve his brain from needless exertion, and save it for science, at least so I thought; and I supposed that writing poetry was hard work. I walked over to the fire and dropped the paper into the flames, asking him never to write verses, but, if he had any poetry in his composition, to let it out in talk to me. He promised compliance, but with a curious smile which puzzled me at the time. These were, I believe, the only verses of his I saw during his lifetime. The treasures which I found in an old box after his death - hymns, metrical versions of psalms, sonnets, and verses about myself (but all written before my prohibition) were a revelation to me of powers in him which I had not suspected."

As a devoted new husband, George Boole took the ruling of 'she who must be obeyed' quite cheerfully, and wrote to his friend William Brooke:

> "She says I may write anything but poetry. This she forbids, having a theory that the poetry that is in a man ought to be for home consumption and ought not to evaporate in words. Hence it is she says that professed poets are dull and prosaic in common life. I am disposed to think that she is right."

Thus Boole's poetic life came to a premature end, as did his physical life less than ten years later. Perhaps it did not occur to his wife that he wrote poetry largely for relaxation and pleasure, and that her prohibition may have had the opposite effect to what she intended. And if the poems he wrote about her still survive somewhere, it would be very interesting to read them.

Mary Everest Boole (1832–1916)

In honour of Boole

Many poems too have been written about Boole, such as 'Elegy' by John Fitzgerald, 'Leaving Lichfield' by Daw Harding, as well as acrostics, clerihews and limericks, one of the cleverest of which is:

> Though it's many a year since Boole died
> His work is well-known far and wide
> AND believe it OR NOT
> It's still used quite a lot
> Wherever computers reside.

George Boole was not an outstanding poet, but he was an able versifier who sometimes wrote lines of real poetic merit. A detailed critique of his poetry in the English language by the late Professor Sean Lucy, poet and critic, as well as an evaluation of his classical translations by Dr Patrick Cronin, are to be found in 'The Poetry of George Boole' [3]. But we are fortunate that Boole did write poetry because this enabled him to express some thoughts and feeling on topics such as religion, mathematics, loneliness, love and patriotism, that he would have been reluctant to express in any other medium.

Boole was an exceedingly modest individual who never referred to his own work or discoveries in his poems as Hamilton had done. We finish with a modest tribute to Boole by the present author entitled 'Boole's Equation $x^2 = x$'.

> Amalgamate
> Two flocks of sheep
> And what do you get?
> Just a single
> Flock of sheep.
> A banal and trivial
> Observation
> The unlikely inspiration
> For electronic revolution.
> All classes are idempotent
> X squared is just
> Another name for X.
> George Boole you showed
> There is beauty
> And utility
> In utter simplicity.

Acknowledgement

I would like to thank my former student Yvonne Cohen for supplying the illustrations and for her help in the layout of this article.

FURTHER READING

[1] D. MacHale, The Life and Work of George Boole; a Prelude to the Digital Age, University Press, Cork, 2014.
[2] D. MacHale and Y. Cohen, New Light on George Boole, Atrium Books, Cork, 2018.
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Desmond MacHale

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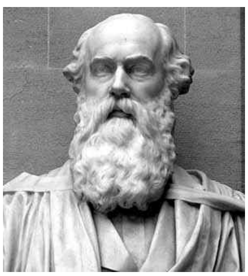
Fermat's Two Squares Theorem for a Prime $p \equiv 1 \pmod 4$

PETER SHIU

The proof, by H. J. S. Smith, of Fermat's theorem on the representation of a prime $p \equiv 1 \pmod 4$ as a sum of two squares shows why J. Brillhart's remarkable algorithm delivers the representation.

Introduction

The first recorded proof that a prime $p \equiv 1 \pmod 4$ is representable as a sum of two squares was given by Euler in 1749, but the theorem is usually attributed to Fermat, who stated in 1659 that he possessed an irrefutable proof by his method of infinite descent—indeed he even laid stress on the fact that the representation is unique, calling it 'the fundamental theorem on right angled triangles'. G. H. Hardy, in *A Mathematician's Apology* [4], considered the theorem to be one of the finest of arithmetic, but that the proofs could only be understood by "a fairly expert mathematician". There is now a proof which is within the grasp of an intelligent person willing to devote some effort to digest the short and simple argument set out at the end of the article.



H. J. S. Smith

There are many proofs of the theorem, and most of them require the initial stage of showing that the congruence $u^2 + 1 \equiv 0 \pmod p$ is soluble, the proof of which was given by Euler after repeated failures over several years. Such proofs only deal with the existence of the representation of p, paying scant regard for the delivery of the actual representation. The exception is the beautifully written book *The Higher Arithmetic* [3] by H. Davenport, who wrote "Once we know that any prime of the form $4k + 1$ is representable uniquely as $x^2 + y^2$, it is natural that mathematicians should have tried to find constructions for the numbers x and y in terms of p. A construction often gives greater mental satisfaction than a mere proof of existence. . . ." Davenport then mentioned four constructions, due to Legendre (1808), Gauss (1825), Serret (1848), and Jacobsthal (1906), adding that Serret's construction was given again in a slightly different form by H. J. S. Smith in 1855. Only brief sketches for the constructions, together with a couple of relevant sentences on Smith's argument, are given.

The ubiquity of computing machines nowadays means that readers will want to know about algorithms for the delivery of solutions to problems, especially those in number theory, even from abstract arguments. We consider the simplest, and also the best, algorithm for the delivery of the two squares corresponding to the given prime. The algorithm was given by J. Brillhart [1] in 1972: the 'how' part is simple, but the 'why' part would require the full attention of "a fairly expert mathematician".

Brillhart wrote that his algorithm is an improvement of one by Hermite, which in turn is an improvement on the one by Serret mentioned by Davenport. Here we explain the why part of Brillhart's algorithm from the results in Smith [10], which was written in Latin, but it has been given a new rendition in [2], together with a short account of the life of Smith.

The Legendre symbol (n/p)

The Legendre symbol $\left(\frac{n}{p}\right)$ takes the value $+1$, or -1, depending on whether n is a quadratic residue, or a non-residue, that is whether the quadratic congruence $x^2 \equiv n \pmod p$ is soluble, or not. The value of the symbol can be computed efficiently by applying the law of quadratic reciprocity (proved by Gauss), which states that, for distinct odd primes p, q,

$$\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{(p-1)(q-1)/4}.$$

Brillhart's algorithm

For the decomposition of a prime $p \equiv 1 \pmod 4$ as $p = a^2 + b^2$, we apply the following procedure:

(1) Find the solution to $u^2 + 1 \equiv 0 \pmod p$, with $1 < u < p/2$; if $p = u^2 + 1$ then return $(a, b) = (u, 1)$.

(2) Apply the Euclidean algorithm to p and u, keeping three successive remainders r, r', r'' in each step; if $r > \sqrt{p} > r'$ then return $(a, b) = (r', r'')$.

There is a catch: if it took Euler years just to show that u exists, how are we expected to find it easily when p is no longer small? Well, in the days of our ancestors, the first item in the recipe for fish soup might be to go and catch a fish, and if none was caught there would be no soup. Nowadays, we just go to the supermarket for the fish, which is our number u here.

After showing that $u^2 + 1 \equiv 0$ (mod p) is soluble, Euler established his eponymous criterion. Note that, by Fermat's little theorem $a^{p-1} \equiv 1$ (mod p), so that $a^{(p-1)/2} \equiv \pm 1$ (mod p). Moreover, if a is a quadratic residue then there exists x such that $x^2 \equiv a$ (mod p), and hence $a^{(p-1)/2} \equiv (x^2)^{(p-1)/2} \equiv x^{p-1} \equiv 1$ (mod p). Euler proved that $a^{(p-1)/2} \equiv -1$ (mod p) when a is a non-residue.

Euler's criterion (1755)

$$a^{(p-1)/2} \equiv \left(\frac{a}{p}\right) \pmod p.$$

A theorem of Lagrange's states that the number of solutions of a polynomial congruence with a prime modulus cannot exceed the degree of the polynomial. If a is a non-residue, we consider the polynomial congruence $x^{(p-1)/2} \equiv 1$ (mod p), which has at most $(p-1)/2$ solutions. Since the $(p-1)/2$ quadratic residues are solutions, it follows that a cannot be a solution, so that $a^{(p-1)/2} \equiv -1$ (mod p).

It follows from Euler's criterion that, for any non-residue n, the numbers $u \equiv \pm n^{(p-1)/4}$ (mod p) satisfy $u^2 \equiv n^{(p-1)/2} \equiv -1$ (mod p). We may take successive prime values for n until the least non-residue is delivered—thanks to the law of quadratic reciprocity, the computation of the value of the Legendre symbol (n/p) is efficient. In particular, if $p \equiv 5$ (mod 8) then $n = 2$, and if $p \equiv 17$ (mod 24) then $n = 3$. Alternatively, we may choose n randomly and evaluate (n/p) until -1 is delivered. Once n is found, the value of $u \equiv n^{(p-1)/4}$ (mod p) can be evaluated by a repeated squaring and reduction (mod p) process; we may assume that $1 < u < p/2$, since otherwise we can replace u by $p - u$.

For an integer arithmetic algorithm we also require the integer part of $\sqrt{p}$, but this can be delivered efficiently by Newton's method, for example.

Examples

A Python program is written for experiments on a desk-top machine. The following are the results for $p \equiv 1 \pmod 4$, $p \le 617$. Thus $p = a^2 + b^2$, n is the least non-residue, and $p | u^2 + 1$, with $u < p/2$.

p	(n,u)	(a,b)	p	(n,u)	(a,b)
5	(2,2)	(2,1)	281	(3,53)	(16,5)
13	(2,5)	(3,2)	293	(2,138)	(17,2)
17	(3,4)	(4,1)	313	(5,25)	(13,12)
29	(2,12)	(5,2)	317	(2,114)	(14,11)
37	(2,6)	(6,1)	337	(5,148)	(16,9)
41	(3,9)	(5,4)	349	(2,136)	(18,5)
53	(2,23)	(7,2)	353	(3,42)	(17,8)
61	(2,11)	(6,5)	373	(2,104)	(18,7)
73	(5,27)	(8,3)	389	(2,115)	(17,10)
89	(3,34)	(8,5)	397	(2,63)	(19,6)
97	(5,22)	(9,4)	401	(3,20)	(20,1)
101	(2,10)	(10,1)	409	(7,143)	(20,3)
109	(2,33)	(10,3)	421	(2,29)	(15,14)
113	(3,15)	(8,7)	433	(5,179)	(17,12)
137	(3,37)	(11,4)	449	(3,67)	(20,7)
149	(2,44)	(10,7)	457	(5,109)	(21,4)
157	(2,28)	(11,6)	461	(2,48)	(19,10)
173	(2,80)	(13,2)	509	(2,208)	(22,5)
181	(2,19)	(10,9)	521	(3,235)	(20,11)
193	(5,81)	(12,7)	541	(2,52)	(21,10)
197	(2,14)	(14,1)	557	(2,118)	(19,14)
229	(2,107)	(15,2)	569	(3,86)	(20,13)
233	(3,89)	(13,8)	577	(5,24)	(24,1)
241	(7,64)	(15,4)	593	(3,77)	(23,8)
257	(3,16)	(16,1)	601	(7,125)	(24,5)
269	(2,82)	(13,10)	613	(2,35)	(18,17)
277	(2,60)	(14,9)	617	(3,194)	(19,16)

For the larger prime $p = 9999999999999937$, we find that $(5/p) = -1$, so that $u^2 \equiv -1 \pmod p$ has the solution $u \equiv 5^{(p-1)/4} \pmod p$. Repeated squaring and reduction (mod p) then delivers $u \equiv 539227494331899 \pmod p$, which satisfies $1 < u < p/2$. Clearly $\lfloor\sqrt{p}\rfloor = 99999999$ without needing Newton's method; the output of the Euclidean algorithm for p and u is:

$$\begin{aligned}
9999999999999937 &= 18\times 539227494331899+\\
&\qquad + 293905102025755\\
539227494331899 &= 1\times 293905102025755\\
&\qquad + 245322392306144\\
293905102025755 &= 1\times 245322392306144\\
&\qquad + 48582709719611\\
245322392306144 &= 5\times 48582709719611\\
&\qquad + 2408843708089\\
48582709719611 &= 20\times 2408843708089\\
&\qquad + 405835557831\\
2408843708089 &= 5\times 405835557831\\
&\qquad + 379665918934\\
405835557831 &= 1\times 379665918934\\
&\qquad + 26169638897\\
379665918934 &= 14\times 26169638897\\
&\qquad + 13290974376\\
26169638897 &= 1\times 13290974376 + 12878664521\\
13290974376 &= 1\times 12878664521 + 412309855\\
12878664521 &= 31\times 412309855 + 97059016\\
412309855 &= 4\times 97059016 + 24073791.
\end{aligned}$$

We stop here because

$$(r, r', r'') = (412309855, 97059016, 24073791)$$

and $r' < \lfloor\sqrt{p}\rfloor < r$, giving

$$p = 9999999999999937 = 97059016^2 + 24073791^2.$$

The total time taken by the machine is under one millisecond; in fact, the number of steps involved is small enough for the task to be carried out with pencil, paper, a rudimentary calculator, and some perseverance.

For $p = 21\times 2^{899} + 1 \approx 9\times 10^{271}$, the least non-residue is $n = 11$, and we find that

$$\begin{aligned}
u &= 29881990457\ldots 25205268,\\
a &= 73428205432\ldots 17318785,\\
b &= 59022425993\ldots 18181168;
\end{aligned}$$

the time taken is less than half a second. We conclude that Brillhart's algorithm does deliver the solution efficiently.

From Euler, Serret and Hermite to Smith

In 1773 Lagrange pointed out that, by Wilson's theorem, $u^2 + 1 \equiv 0 \pmod p$ has the explicit solution $u \equiv \pm(2k)! \pmod p$ if $p = 4k+1$. Indeed, Gauss gave the explicit solution for (a, b):

$$a \equiv \frac{(2k)!}{2(k!)^2} \pmod p, \qquad b \equiv (2k)!a \pmod p.$$

There may be some theoretical interest in such formulae but they are useless for the purpose of evaluating the solutions when p is no longer small. The same can be said of Jacobsthal's construction, which is given in terms of sums involving quadratic residues and non-residues.


Joseph Alfred Serret

Whereas Legendre's method for the representation makes use of the continued fraction expansion of $\sqrt{p}$, Serret's much simpler method [8] involves the expansion for the fraction u/p instead, but it does not contain a precise termination condition. Hermite's single page article, which appeared on the next page after Serret's, gives the condition that the denominators of two successive convergents should straddle $\sqrt{p}$.

Brillhart's contribution is that the computation of the convergents in the continued fraction expansion can be dispensed with completely by applying the Euclidean algorithm to the numbers p and u instead. The proof that the algorithm delivers the representation will therefore involve properties of continued fractions, to which Euler was the first person to make substantial contributions.

Meanwhile, Smith's neglected paper [10] is remarkable in that he succeeded in giving a proof of the two squares theorem without using the fact that $u^2 + 1 \equiv 0 \pmod p$ is soluble. More precisely, he showed the existence of a special number u in the set $\{2, 3, \ldots, (p-1)/2\}$ by considering an involution on the set—an involution is a mapping f such that the composition $f \circ f$ is the identity map. Yes, the special number is the same u as before, but the fact that $p | u^2 + 1$ is a consequence of the argument. Indeed, we find it expedient to use Smith's method to prove that Brillhart's algorithm works.

Continued fractions and continuants

A fraction $p/q > 1$ has the continued fraction expansion

$$\frac{p}{q} = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \ddots + \cfrac{1}{a_{n-1} + \cfrac{1}{a_n}}}},$$

where a_i are positive integer—we denote the right-hand side by $[a_0, a_1, \ldots, a_n]$. If $a_n = 1$ we can replace n by $n-1$ and increase the value of a_{n-1} by 1; with such an adjustment the representation for p/q by $[a_0, a_1, \ldots, a_n]$ is unique. The term a_k is called the k-th partial quotient, and $[a_0, a_1, \ldots, a_k] = p_k/q_k$ the k-th convergent; in particular $p/q = p_n/q_n$ is the last convergent. The well known recurrence formulae

$$p_{k+1} = a_{k+1}p_k + p_{k-1}, \quad q_{k+1} = a_{k+1}q_k + q_{k-1}, \qquad (1)$$

can be established by induction, starting from the appropriate initial values.

We also require the notion of a *continuant* for $a_0, a_1, \ldots, a_n$, which was introduced by Euler in his investigation of continued fractions; Smith defined it by the tridiagonal determinant:

$$K(a_0, a_1, \ldots, a_n) = \begin{vmatrix} a_0 & 1 & 0 & \cdots & 0 & 0 \\ -1 & a_1 & 1 & \cdots & 0 & 0 \\ 0 & -1 & a_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & a_{n-1} & 1 \\ 0 & 0 & 0 & \cdots & -1 & a_n \end{vmatrix}.$$

Thus $K(a_0) = a_0$, $K(a_0, a_1) = a_0a_1 + 1$, and we let $K() = 1$. Properties of continuants discovered by Euler can also be derived from the usual properties of determinants. For example, expansion by the first row of the determinant shows that $K(a_0, \ldots, a_n) = a_0K(a_1, \ldots, a_n) + K(a_2, \ldots, a_n)$, which can be shown to be equivalent to the more useful $K(a_0, \ldots, a_n) = a_nK(a_0, \ldots, a_{n-1}) + K(a_0, \ldots, a_{n-2})$; more generally, we have

$$\begin{aligned} K(a_0, \ldots, a_n) &= K(a_0, \ldots, a_\ell)K(a_{\ell+1}, \ldots, a_n) \qquad (2) \\ &\quad + K(a_0, \ldots, a_{\ell-1})K(a_{\ell+2}, \ldots, a_n). \end{aligned}$$

A most useful property of the continuant is

$$K(a_0, a_1, \ldots, a_n) = K(a_n, a_{n-1}, \ldots, a_0), \qquad (3)$$

from which it follows that the convergents can be written as

$$[a_0, a_1, \ldots, a_k] = \frac{p_k}{q_k} = \frac{K(a_0, a_1, \ldots, a_k)}{K(a_1, a_2, \ldots, a_k)},$$

and hence

$$\frac{p_n}{p_{n-1}} = [a_n, a_{n-1}, \ldots, a_0], \quad \frac{q_n}{q_{n-1}} = [a_n, a_{n-1}, \ldots, a_1]. \qquad (4)$$

Smith's involution

Let $p \equiv 1 \pmod 4$ be a prime, and consider the $(p-3)/2$ continued fractions

$$\frac{p}{q} = [a_0, a_1, \ldots, a_n], \quad q \in \{2, 3, \ldots, \frac{p-1}{2}\},$$

where $a_0, a_n \geq 2$. Corresponding to each q there is a unique $r \in \{2, 3, \ldots, (p-1)/2\}$ such that the ordering of the partial quotients for p/r is reversed:

$$\frac{p}{r} = [a_n, a_{n-1}, \ldots, a_0].$$

The mapping $q \leftrightarrow r$ is then an involution on the set $\{2, 3, \ldots, (p-1)/2\}$, so that members of the set form pairs (q, r). Since the cardinal of the set is the odd number $(p-3)/2$, it follows that there is at least one 'fixed point' u, that is a 'self-paired' (u, u). For such a fixed point, the continued fraction for p/u is, by (3), palindromic:

$$\frac{p}{u} = [a_0, a_1, \ldots, a_1, a_0] = \frac{K(a_0, a_1, \ldots, a_1, a_0)}{K(a_1, \ldots, a_1, a_0)}. \qquad (5)$$

Smith then showed that there is no central term in the palindromic continuant K for p, because the existence of such a term a_ℓ, together with (2), led to p having the proper divisor $K(a_0, a_1, \ldots, a_{\ell-1})$, which is impossible for a prime. Therefore

$$p = K(a_0, a_1, \ldots, a_\ell, a_\ell, \ldots, a_1, a_0), \qquad (6)$$

and a further application of (2) delivers the representation

$$p = K(a_0, \ldots, a_{\ell-1})^2 + K(a_0, \ldots, a_\ell)^2. \qquad (7)$$

This is Smith's proof of Fermat's theorem, in which he also mentioned that $p | u^2 + 1$ at the end of the short paper. The proof of uniqueness of the representation by his method is given in [2].

Corollary. *Brillhart's algorithm delivers the representation of $p \equiv 1 \pmod 4$ as a sum of two squares.*

Proof. It follows from (5), (6), and (1) that p_ℓ, $p_{\ell-1}$ are successive remainders in the Euclidean algorithm for p and u. Also, from (7), we have $p = p_\ell^2 + p_{\ell-1}^2$, and $p_{\ell+1}^2 \geq (p_\ell + p_{\ell-1})^2 > p$, so that $p_{\ell+1} > \sqrt{p} > p_\ell$. Therefore Brillhart's algorithm does deliver the required presentation. □

The partial quotients a_k for p/u in the example with $p = 9999999999999937$ form only the first half of the palindromic sequence:

$$\begin{aligned}\frac{p}{u} = [&18,1,1,5,20,5,1,14,1,1,31,4,\\ &4,31,1,1,14,1,5,20,5,1,1,18].\end{aligned}$$

Heath-Brown's involutions

The basic tenet of Smith's proof is that there must be a fixed point for an involution on a set with an odd cardinal—indeed the number of fixed points and the cardinal of the set must have the same parity. In more recent times, D. R. Heath-Brown [5], while studying some papers by Liouville, found another proof of Fermat's theorem which is also based on the same tenet.

Let $p \equiv 1 \pmod 4$ be a prime, x,y,z be positive integers, and

$$S = \{(x,y,z) : x^2 + 4yz = p\}.$$

There is a natural involution on the finite set S, namely $b : S \to S$ given by $b(x,y,z) = (x,z,y)$; the fixed points of the involution are of the form (x,y,y) which correspond to the decomposition $p = x^2 + (2y)^2$. It therefore remains to show that the cardinal of S is odd to ensure that such a fixed point exists. For this purpose, Heath-Brown considered the mapping $h : S \to S$ where

$$h(x,y,z) = \begin{cases} (x+2z, & z & , y-x-z) \\ (2y-x, & y & , x-y+z) \\ (x-2y, & x-y+z & , y \quad) \end{cases}$$

with $y - z < x < y - 2x$, $y - z < x < 2y$, $2y < x$, for the corresponding cases. Let H_1, H_2, H_3 denote the matrices

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & -1 \end{pmatrix}, \quad \begin{pmatrix} -1 & 2 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & -2 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

That h is an involution on S amounts to verifying that $H_2^2 = H_1H_3 = I$, the identity matrix.

The fixed points of h now take the form (x,x,z), for which p then has the purported factorisation $p = x(x+4z)$. That p is a prime implies $x = 1$, so that there is one, and only one, fixed point of h, namely $(1,1,(p-1)/4)$. Therefore the cardinal of S is odd, so that there must be a fixed point for the involution b. This is Heath-Brown's proof of Fermat's theorem. The argument has since been dubbed a 'one-sentence proof' of the theorem by D. Zagier [12]; indeed, there is a visual version of the proof on YouTube [11].

Summary and remark

We summarise the two proofs of Fermat's theorem for a prime $p \equiv 1 \pmod 4$. Smith's argument involves a single involution on the set $\{2,3,\ldots,(p-1)/2\}$ with an odd cardinal in order to establish the existence of u such that the continued fraction for p/u is palindromic, and the representation is delivered using properties of continuants. Heath-Brown's argument involves two involutions b and h on the set S. The fixed point of b delivers the representation, and h shows the cardinal of S is odd if p is a prime.

That the congruence $u^2 + 1 \equiv 0 \pmod p$ is soluble is not involved at all in Heath-Brown's proof, and it appears that the existence proof may not lead to a construction of the fixed point of b, never mind a proof of uniqueness by the method. Nevertheless it is shown in [9] that the use of the composite map $f = b \circ h$ possesses enhanced power which leads to a construction of the fixed point of b. Indeed, for an input composite $n \equiv 1 \pmod 4$ instead of a prime p, the fixed points of h provide factorisations of n. However, when compared to better known methods of factorisation of n, or Brillhart's algorithm for the representation of p, the method is woefully inadequate.

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Computing a square-root, and a quadratic non-residue, modulo p

The efficient evaluation of the square-root of a general quadratic residue a (mod p), and the finding of a quadratic non-residue (mod p), are difficult problems in computational number theory. Roughly speaking, the 'efficiency' of the two tasks are 'equivalent' in the sense that if a quadratic non-residue (mod p) is found then we may determine the square-root of a (mod p) by Euler's criterion, and the efficient scheme of squaring and reduction (mod p); conversely if there is an efficient method of extracting the square-root of any quadratic residue a (mod p), then we can evaluate $\sqrt{-1}, \sqrt[4]{-1}, \sqrt[8]{-1}, \ldots$ successively until a non-residue emerges. However, there is no known deterministic polynomial-time algorithm for either task—polynomial-time is the technical jargon for being efficient.

Nevertheless, on the assumption of the Generalised Riemann Hypothesis (the Riemann Hypothesis for Dirichlet L-functions), it is known that the least quadratic non-residue is bounded by $\log^2 p$ (see [6]), so that a systematic search for that non-residue is efficient under the hypothesis. Furthermore, for any fixed quadratic residue a (mod p), there is an efficient algorithm by R. Schoof [7] for $\sqrt{a}$ (mod p); in particular, the evaluation of $u = \sqrt{-1}$ (mod p) is efficient, but the method involves very sophisticated notions. In practice, we opt for the random choice procedure to find a non-residue n, because the probability of having $(n/p) = -1$ is about $\frac{1}{2}$, so that success should come before long. Such an efficient scheme is called a probabilistic polynomial-time algorithm.

The Shape of a Module Category

ROSANNA LAKING

We take a leisurely tour of Auslander-Reiten theory and explain how the “shape" of the finite-dimensional module category over a finite-dimensional algebra is described by its Auslander-Reiten quiver.

In the representation theory of associative algebras we study categories of modules. This means that we are not just interested in the structure of our chosen type of mathematical object (modules over a fixed algebra), but we are also interested in describing the collection of all such objects, together with the structure-preserving functions between them.

Of course, this is often, if not always, an immense amount of data. Nevertheless, after some experimentation, combined with sufficient knowledge of the general theory, one can develop a sense, some intuition, of the shape of the category.

What is an algebra?

An *(associative) algebra* over a field K is a K-vector space A with a multiplication

$$A \times A \to A$$

that is associative, distributive over addition and compatible with scalar multiplication. We also assume that A has a multiplicative unit. Some examples include:

- For a finite group G, we define the *group algebra* $K[G]$ by taking a K-vector space with basis $\{v_g \mid g \in G\}$ and the multiplication determined by $v_g v_h = v_{gh}$ for any $g, h \in G$.
- The set of polynomials over K is a vector space with basis $\{X^n \mid n \geq 0\}$. The usual multiplication of polynomials gives us the *polynomial algebra* $K[X]$.
- Let Q be a finite directed graph and $\mathscr{P}$ be the set of paths in Q. The **path algebra** KQ is a K-vector space with basis $\{v_p \mid p \in \mathscr{P}\}$ with multiplication determined by $v_p v_q = v_{pq}$ when $p, q \in \mathscr{P}$ are composable and zero otherwise.

The smallest possible example, the one-dimensional algebra, is just the field K itself. Let's consider the case where K is the real numbers $\mathbb{R}$. By definition, the category of modules over $\mathbb{R}$ is nothing more than the collection of $\mathbb{R}$-vector spaces together with the $\mathbb{R}$-linear maps between them. With a quick application of Zorn's lemma, we discover that every vector space has a basis, albeit a possibly infinite one. This is enough for us to get a feel for the whole category: every $\mathbb{R}$-vector space is a direct sum of copies of $\mathbb{R}$. We conclude that the category is built from a unique building block: $\mathbb{R}$ itself.

Once our intuition for the shape of the category sinks in, we are faced with the next challenge: how can we communicate what we have understood to other mathematicians? This problem of transferring a mathematical idea from one brain into another is widely-shared. In his 1922 address to the British Association for the Advancement of Science, G.H. Hardy compared this process to explorers drawing maps of vast newly-discovered landscapes.

It may seem foolish to hope that we can do something as straightforward as “drawing a map" of a module category, but if we narrow our parameters a bit we can do just that. Both algebras and modules have underlying vector spaces, so we can consider the category of finite-dimensional modules over a finite-dimensional algebra A. We can then draw a directed graph $\Gamma(A)$, called its *Auslander-Reiten quiver*, that maps out the indecomposable modules (the building blocks) of this category and indicates how they relate to each other. As one might reasonably surmise from the discussion above, the Auslander-Reiten quiver of the category of finite-dimensional $\mathbb{R}$-modules is just the vertex $\mathbb{R}$. Later we will see that the Auslander-Reiten quiver can have a much more sophisticated structure than this.

The discovery of this mathematical object in the mid-1970s gave birth to a beautiful branch of representation theory, leading to interactions with other areas of mathematics such as commutative algebra, algebraic geometry, combinatorics and even, more recently, topological data analysis.

How do finite-dimensional algebras look?

Let A be a finite-dimensional algebra that is basic (i.e. A is as "small as possible" up to some sensible notion of equivalence) and connected (i.e. A cannot be "broken down" into smaller algebras). Then there exists a finite directed graph (in this context, called a *quiver*) such that $A \cong KQ/I$ where I is an ideal in KQ generated by a finite set ρ of relations.
For example, consider the quiver Q

$$\begin{tikzcd} 1 \arrow[loop left, "a"] \arrow[r, "b"] & 2 \arrow[loop right, "c"] \end{tikzcd}$$

If I is generated by the relations

$$\rho = \{a^3, \lambda ab - bc, c^2\}$$

for some $\lambda \in K$, then one can show that KQ/I is a 7-dimensional algebra.

How do finite-dimensional modules look?

The category of finite-dimensional A-modules is equivalent to the finite-dimensional representations of Q satisfying the relations ρ.
For example, a representation of the example above is given by

$$\begin{tikzcd} V_1 \arrow[loop left, "V_a"] \arrow[r, "V_b"] & V_2 \arrow[loop right, "V_c"] \end{tikzcd}$$

where V_1, V_2 and V_3 are finite-dimensional K-vector spaces and V_a, V_b and V_c are K-linear maps satisfying

$$V_a^3 = 0 \text{ and } \lambda V_b V_a - V_c V_b = 0 \text{ and } V_c^2 = 0.$$

The vertices of the Auslander-Reiten quiver

To begin understanding the Auslander-Reiten quiver of a finite-dimensional algebra, we start with its vertices: the indecomposable finite-dimensional modules. These are the building blocks of the category, but what qualifies them for such a title?

Firstly, a building block should be as small as possible and so we must spell out what it means for a module to break down into smaller pieces. Turning this on its head, let's start with what it means to build up. Given two non-trivial finite-dimensional modules, we can construct a new larger one by sticking them together to form their direct sum: first we take their direct sum as vector spaces and then we allow the algebra to act componentwise.

Accordingly, we think of the newly constructed module as breaking down into two smaller pieces. In fact, any module that is isomorphic to such a direct sum is considered to be decomposable. Our building blocks are, of course, the modules that cannot be decomposed; by definition, these are the *indecomposable modules*.

So far we have seen that these modules are, indeed, as small as possible but, to truly be building blocks, everything else must be constructed from them. This is the essence of the Krull-Remak-Schmidt Theorem.

The Krull-Remak-Schmidt Theorem. *Any non-trivial finite-dimensional module is isomorphic to a finite direct sum of indecomposable finite-dimensional modules. The decomposition is unique up to permutation and isomorphism of the summands.*

At this point we must address a crucial point: representation theorists do everything up to isomorphism. To illustrate this, consider modules over the real numbers $\mathbb{R}$ (a.k.a. $\mathbb{R}$-vector spaces). The field $\mathbb{R}$ is the only indecomposable finite-dimensional module, so the Krull-Remak-Schmidt Theorem tells us that every module decomposes into a direct sum of copies of $\mathbb{R}$ up to isomorphism. For example, although the two-dimensional $\mathbb{R}$-module given by the complex numbers $\mathbb{C}$ is not literally a direct sum of modules, we consider it to be essentially the same as the direct sum $\mathbb{R} \oplus \mathbb{R}$ of two copies of $\mathbb{R}$.

Following this philosophy, the vertices of the Auslander-Reiten quiver are defined to be the *isomorphism classes* of the indecomposable finite-dimensional modules. As one might expect, we will often treat a given vertex as though it is a module by informally identifying it with some representative of the isomorphism class.

The arrows of the Auslander-Reiten quiver

We have seen that the vertices of the quiver encode the finite-dimensional modules and so it stands to reason that the directed edges (or *arrows*) should encode some information about the structure-preserving functions, called homomorphisms, between them. In fact, the homomorphisms give us the sense of orientation in the category that we wish the Auslander-Reiten quiver to record; if there is a non-trivial homomorphism between vertices, then we think of the source vertex as being 'to the left' of the target.

It is important that the Auslander-Reiten quiver only records the non-trivial homomorphisms in the category, or else the picture would soon become overwhelming. For example, between every pair of vertices, there exists a zero homomorphism that maps every element in the source to the zero element in the target. If we were to include these as arrows in our picture, then we would automatically have a connected graph—not to mention a mess.

We have already settled on the idea of identifying isomorphic indecomposable modules and, in a similar vein, we will also ignore the isomorphisms themselves. This leaves us with the task of encoding the non-zero non-isomorphisms between our vertices in such a way that some structure emerges.

As with the vertices, the arrows of the graph will correspond to the smallest possible non-zero non-isomorphisms between the vertices. In this case, it is perhaps better to say that they are as short as possible. A non-zero non-isomorphism $f: M \to N$ between vertices M and N of an Auslander-Reiten quiver is called *irreducible* if there is no factorisation of f of the form

$$M \overset{[g_1 \dots g_n]^T}{\longrightarrow} \bigoplus_{i=1}^{n} L_i \overset{[h_1 \dots h_n]}{\longrightarrow} N \tag{1}$$

with non-isomorphisms g_i and h_i and indecomposable finite-dimensional modules L_i for each $i = 1, \dots, n$.

Since everything in the category is carried out up to isomorphism, two irreducible homomorphisms that are related by scalar multiplication should be identified in our picture. In other words, we only wish to include a linearly independent set of irreducible homomorphisms. Of course, such a statement assumes that we have a vector space $\mathrm{Irr}(M,N)$ of irreducible homomorphisms; this is obtained by taking the quotient vector space $\mathrm{rad}(M,N)/\mathrm{rad}^2(M,N)$ where $\mathrm{rad}(M,N)$ is the vector space of non-isomorphisms and $\mathrm{rad}^2(M,N)$ is the subspace of those with a factorisation as in (1).

We define the arrows in the Auslander-Reiten quiver between vertices M and N to be indexed by the basis elements of $\mathrm{Irr}(M,N)$. In order to think of the arrows as homomorphisms between M and N, we will informally identify them with elements of $\mathrm{rad}(M,N)$ corresponding to a fixed basis in $\mathrm{Irr}(M,N)$.

The faces of the Auslander-Reiten quiver

We are now ready to present our first example of an Auslander-Reiten quiver:

On the left is a quiver Q and on the right is the Auslander-Reiten quiver $\Gamma(KQ)$ of the category of finite-dimensional modules over its path algebra KQ.

As we have seen so far, the vertices represent the finite-dimensional modules. In this case, it can be shown that non-zero non-isomorphisms can also be read off the diagram; they are obtained by finite sums and compositions of irreducible homomorphisms. As we will discuss in the final section, this does not hold in the general case but, for now, we can think of this as the skeleton of the category, consisting only of the unbreakable pieces from which everything else is built.

The definition of the Auslander-Reiten quiver is based on the seminal work of Auslander and Reiten in 1975 and is surrounded by a beautiful theoretical framework rooted in homological algebra and category theory. In particular, the picture above contains more information than it seems at first sight. It is not an accident that it is arranged as a tidy regular pyramid—the triangular 'face' of this configuration of the graph encodes a special short exact sequence called an almost split sequence.

Let us take a quick sidebar to recall the fundamental concept of a short exact sequence. The prototype is given by a pair of composable homomorphisms f and g, where f is the inclusion of the kernel of g and g is the quotient by the image of f. According to our usual principle of up-to-isomorphism, a general short exact sequence is defined to be any pair of composable homomorphisms that is isomorphic to the prototype. Short exact sequences are represented by the notation

$$0 \to K \xrightarrow{f} N \xrightarrow{g} M \to 0 \qquad (2)$$

and are of central importance in homological algebra.

A short exact sequence is an *almost split sequence* when it is built from irreducible homomorphisms. More precisely, the end-points K and M are indecomposable and, after using the Krull-Remak-Schmidt Theorem to decompose N, the homomorphisms f and g can be represented by row and column vectors with entries given by irreducible homomorphisms.

In fact, the relationship between almost split sequences and irreducible homomorphisms is even closer: every irreducible homomorphism can be completed to one of the homomorphisms in an almost split sequence. Together with the following theorem, this explains why the almost split sequences can be seen as the faces of the Auslander-Reiten quiver.

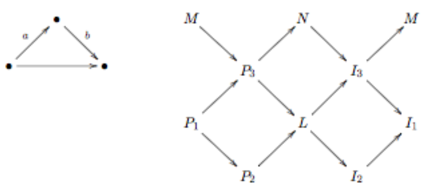
Theorem (Auslander-Reiten 1975). *Let A be a finite-dimensional algebra with underlying quiver Q and let n be the number of vertices of Q.*

(1) All but n of the vertices K of $\Gamma(A)$ begin an almost split sequence of the form (2).

(2) All but n of the vertices M of $\Gamma(A)$ end an almost split sequence of the form (2).

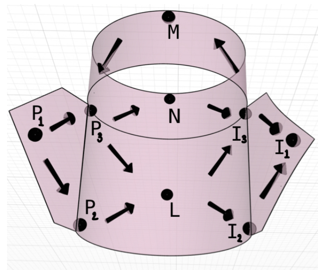
In fact we know exactly which modules do not start or end an almost split sequence: the former are the injective modules and the latter are the projective modules.

We illustrate the theorem with another small example of an Auslander-Reiten quiver.



As before, on the left is the quiver Q that underlies the finite-dimensional algebra that we are considering. Set A to be the quotient KQ/I where I is the ideal in the path algebra KQ generated by ab. The Auslander-Reiten quiver $\Gamma(A)$ is on the right.

Since Q has three vertices, there are three indecomposable projective modules P_1, P_2 and P_3 and three indecomposable injective modules I_1, I_2 and I_3, up to isomorphism. There are three further vertices of the Auslander-Reiten quiver L, M and N. Notice that M occurs twice in the picture. This indicates that this Auslander-Reiten quiver is better understood in three dimensions:



There are five almost split sequences appearing in the picture:

$$0 \to P_3 \to P_2 \oplus P_1 \to L \to 0$$
$$0 \to L \to I_2 \oplus I_3 \to I_1 \to 0$$
$$0 \to P_1 \to L \oplus N \to I_3 \to 0$$
$$0 \to M \to P_1 \to N \to 0$$
$$0 \to N \to I_3 \to M \to 0$$

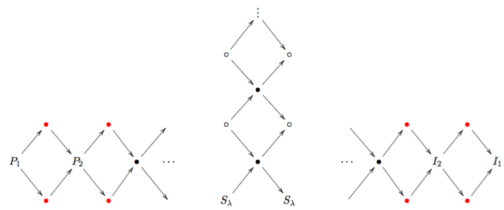
The sequences with a direct sum in the middle term correspond to the three square faces and the remaining two to the triangular ones.

The components of the Auslander-Reiten quiver

The two examples of Auslander-Reiten quivers we have seen so far have the potential to be misleading because there is only one connected component and there are only finitely many vertices. This case is far from typical and we say an algebra associated to such an Auslander-Reiten quiver is of *finite representation type*.

One can prove, however, that any Auslander-Reiten quiver is locally finite, meaning that there are only finitely many arrows adjacent to every vertex. It follows from this that each connected component of the quiver contains only countably many vertices.

This observation places no such bound on the total number of vertices in the whole Auslander-Reiten quiver. For example, one of the simplest examples of an algebra that is not of finite representation type is the Kronecker algebra. That is, the path algebra KQ of the quiver $Q = \bullet \rightrightarrows \bullet$. We will assume that $K = \mathbb{C}$, so that the following is a picture of the Auslander-Reiten quiver of KQ.



In this case, we see that the Auslander-Reiten quiver is infinite and is not connected. Each red vertex should be identified with the one vertically above or below and each white vertex, as well as S_λ, should be identified with the ones horizontally across from it. In other words, each of the three components pictured here looks like an infinite tube. The middle tube in the picture actually represents a family of tubes, each one corresponding to some $\lambda \in \mathbb{P}^1(\mathbb{C})$. This implies that, in fact, there are uncountably many vertices.

Beyond the Auslander-Reiten quiver

Another misleading property of finite Auslander-Reiten quivers is that the arrows in the diagram determine all of the non-zero non-isomorphisms in the category. This does not hold in the general case.

For example, over the Kronecker algebra, there are non-isomorphisms from the component containing the projective modules to each of the other components, but these homomorphisms cannot be seen in the picture. The non-zero non-isomorphisms that cannot be finitely built from the arrows within the components are those contained in the *infinite radical*. We indicate their existence by the way we arrange the components: homomorphisms go from left to right.

To fully answer the problem of communicating the shape of the category, we must determine the correct arrangement of the components of the Auslander-Reiten quiver. The answer to this question is not at all clear but one intriguing strategy is to look at what is between the components.

The Krull-Remak-Schmidt Theorem tells us that the finite-dimensional modules are all accounted for within the components, therefore whatever lies between them must be infinite-dimensional. It is an intimidating prospect to have to deal with arbitrary infinite-dimensional modules but the next theorem, first proved by Krause [2] and later refined by Prest [3], narrows the scope of what we are looking for.

Theorem (Krause 2001, Prest 2005). *Let A be a finite-dimensional algebra and f be a homomorphism between vertices of $\Gamma(A)$. The following statements are equivalent:*

(1) f is in the infinite radical;

(2) f factors through a finite direct sum of indecomposable infinite-dimensional pure-injective modules.

The indecomposable pure-injective modules in the module category of A are of central importance in the model theory of modules and, in fact, constitute the points of a topological space called the *Ziegler spectrum*.

Although the precise connection between the Ziegler spectrum and the arrangement of the components of the Auslander-Reiten quiver remains unclear, perhaps these special infinite-dimensional modules hold the key to discovering the true shape of the finite-dimensional module category.

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Inside Out

STEPHEN HUGGETT

A mathematician who has been an Officer of Council for almost all of this millenium reflects on his time volunteering for the Society.

It feels strange to look at the LMS from the outside, having been an insider for so long (I was Programme Secretary from 2001 to 2011 and then General Secretary from 2012 to 2020). In writing these few reflections I do not claim any particular wisdom, but here are some things I have learned from my experience of working for the Society.

In my view, by far the most important activity of the Society is the small grants schemes. The application procedure has low bureaucracy and a high success rate, and it is clear from all the feedback that these grants make a big difference to research in mathematics. It was a privilege to have chaired the committee awarding them for ten years, during which around £2m was granted to individual mathematicians (or small groups) to support their research.

Furthermore, from the point of view of the Charity Commission these grants are easily the most significant part of the Society's charitable activities. I think they are precious, and to be protected and if possible extended.

The most important asset of the Society is its membership. Without its membership, the Society would be nothing. Well, not quite nothing, because it would still have substantial financial reserves (of £17m or so). But, ultimately, it is the members who decide how to make that money work most effectively for research in mathematics.

A large number of members (between two and three hundred) volunteer to devote a non-trivial amount of time to working for the Society, serving on its various committees. Of course there is a reward: I freely admit that I for one have enjoyed and benefitted enormously from meeting all sorts of fascinating people. But, mostly, volunteering is a gift to the Society, an act of solidarity with other mathematicians.

However, the overall responsibility for the activities of all the committees lies with Council, so it is crucial that the members of the Society are fully engaged with Council's work. Participation in the elections to Council is high (but could be higher). Engagement is not just a question of voting, though: it includes (among other things) nominating people for election.

Nominating Committee was designed as a committee of last resort: its remit is simply to make sure that there is at least one candidate for each vacant seat on Council. Of course it does much more than this, but direct nominations from the membership are also really important. So please nominate, stand for election, and vote!

To summarise, what I love most about the Society is that it is independent, run by research mathematicians, for research in mathematics.

Microtheses and Nanotheses provide space in the Newsletter for current and recent research students to communicate their research findings with the community. We welcome submissions for this section from current and recent research students. See newsletter.lms.ac.uk for preparation and submission guidance.

Microthesis: Nonabelian Cohomology and Cohomotopy of Cosimplicial Groups

MARIAM PIRASHVILI

It is known that various theories of non-abelian cohomology are special cases of the cohomology of cosimplicial groups. In my PhD thesis, I investigated a subclass of non-abelian cosimplicial groups for which there exists a good theory for the second cohomotopy. This has applications in the cohomology and extensions of small categories, and possibly also 2-categories.

Motivation

One of the fundamental objects of study in homological algebra are cochain complexes. These are a sequence of abelian groups and homomorphisms

$$A^0 \xrightarrow{d^0} A^1 \xrightarrow{d^1} A^2 \xrightarrow{d^2} A^3 \xrightarrow{d^3} \cdots$$

such that $d^{i+1} \circ d^i = 0$. This implies that $Im\ d^i \subset Ker\ d^{i+1}$. We can define the n^{th} cohomology group H^n to be $Ker\ d^n / Im\ d^{n-1}$.

These objects appear in various branches of mathematics and are used for computation. One of the main properties of cohomology is that for any short exact sequence of cochain complexes

$$0 \to A^* \to B^* \to C^* \to 0$$

we can define the long exact sequence of cohomology groups

$$0 \to H^0(A^*) \to H^0(B^*) \to H^0(C^*) \to H^1(A^*) \to \cdots$$

For example, in topology, for a space X which is triangulated, A^n is the abelian group of all maps from the n-simplices of X to a given group G. In this context, the cohomology group H^n computes n-dimensional holes. In algebra, for a group G and a G-module M, A^n is the group of all maps $G^n \to M$.

It can be observed that for $n = 0$ and $n = 1$, the definition of $H^n(G, M)$ is meaningful even when M is non-abelian, that is, when M is a G-group. Then the first non-abelian group cohomology $H^1(G, M)$ classifies 'twisted forms' of M, called torsors. These play an important role in Galois cohomology. In geometry, for a space X and a sheaf of groups $\mathcal{G}$, the first cohomology $\mathrm{H}^1(X, \mathcal{G})$ classifies $\mathcal{G}$-bundles in a certain sense. For example, when $\mathcal{G}$ is a sheaf of continuous maps into $Gl(n, \mathbb{C})$ then $\mathrm{H}^1(X, \mathcal{G})$ classifies n-dimensional complex vector bundles over X. Unfortunately, because non-abelian cohomology is only defined up to dimension 1, it lacks the same usefulness as abelian cohomology. One of the main advantages of abelian cohomology are the long exact sequences, which we no longer have in the non-abelian case. Both group cohomology and sheaf cohomology are special cases of a more general construction, known as the cohomotopy of cosimplicial groups. Other special cases of this construction are higher limits, and more generally the cohomology of small categories.

First cohomotopy of cosimplicial groups

A cosimplicial group G^* is a collection of groups G^n, $n \geq 0$ and morphisms $\delta^i : G^n \to G^{n+1}$, $\sigma^j : G^{n+1} \to G^n$, $0 \leq i, j \leq n$ satisfying a number of identities, including

$$\delta^j \delta^i = \delta^i \delta^{j-1}, \quad i < j.$$

In the abelian case, it is possible to define a cochain complex $C^*(G^*)$ by defining a boundary homomorphism $d : G^n \to G^{n+1}$ via

$$d = \sum_{i=0}^{n+1} (-1)^i \delta^i.$$

The cohomotopy of the abelian cosimplicial group can thus be computed as the cohomology of $C^*(G^*)$. For a non-abelian cosimplicial group G^*, we can construct a cohomotopy group $\pi^0(G^*)$ by

$$\pi^0(G^*) = \{x_0 \in G^0 | \ \delta^0 x_0 = \delta^1 x_0\}.$$

We can also construct the cohomotopy set $\pi^1(G^*)$ as the equivalence classes of one-cocycles

$$\{x_1 \in G^1 | \ \delta^2(x_1) + \delta^0 x_1 = \delta^1 x_1\}$$

where $x_1 \sim y_1$ if $y_1 = -\delta^1(x_0) + x_1 + \delta^0(x_0)$ for some $x_0 \in G^0$.

The standard topics of non-abelian cohomology, in particular the relation to torsors and twisting by a one-cocycle, are generalised to the setting of cosimplicial groups.

Second cohomotopy of cosimplicial groups

A natural question arises: under what conditions can we define $\pi^2(G^*)$? For an abelian cosimplicial group, we can define $\pi^n(G^*)$ for all $n \geq 0$, whereas in the general non-abelian case it is only possible to define $\pi^0(G^*)$ and $\pi^1(G^*)$. It is generally accepted that no general theory can exist to define higher homotopies for all non-abelian cosimplicial groups. The aim, therefore, is to find a natural subclass of cosimplicial groups for which π^2 can be defined. The condition introduced in my PhD thesis is as follows. For any $x_n, y_n \in G^n$, we have

$$\delta^0(x_n) + \delta^{n+1}(y_n) = \delta^{n+1}(y_n) + \delta^0(x_n), \quad n \geq 0$$

in G^{n+1}. Such cosimplicial groups are called *centralised*. This is quite a strong condition, and for example implies that G^0 is abelian. In this case, $\pi^0(G^*)$ becomes abelian, $\pi^1(G^*)$ becomes a group and it is possible to define the set $\pi^2(G^*)$ as the equivalence classes of the two-cocycles

$$\{x_2 \in G^2 | \ \delta^2(x_2) + \delta^0(x_2) = \delta^1(x_2) + \delta^3(x_2)\}$$

where $x_2 \sim y_2$ if $y_2 = \delta^1(x_1) + x_2 - \delta^0(x_1) - \delta^2(x_1)$ for some $x_1 \in G^1$. The main body of my PhD thesis consists of extending the exact sequence of cohomotopy as far as can be done in specific cases and analysing the resulting exact sequences of sets.

For the case of a central cosimplicial subgroup A^* of B^*, the exact sequence beginning with $\pi^0(A^*)$ can be extended to $\pi^3(A^*)$. The construction of twisting can be generalised to twisting by a two-cocycle. This new construction serves the function of analysing the fibres of the maps between the higher homotopy sets.

Applications

In the case of group or sheaf cohomology, this condition does not give anything new. The resulting cosimplicial group is centralised if and only if it is abelian. However, a non-trivial example arises in the cohomology of small categories. In this case, the role of the coefficient is played by natural systems. A natural system D on a small category I assigns a group $D(\alpha)$ to each morphism α of I and for each composable pair of morphisms $i \xrightarrow{\alpha} j \xrightarrow{\beta} k$ two homomorphisms are given, $\alpha^* : D_\beta \to D_{\beta\alpha}$ and $\beta_* : D_\alpha \to D_{\beta\alpha}$, which satisfy certain conditions. The resulting cosimplicial group $C^*(\mathrm{I}, D)$ is given by

$$C^n(\mathrm{I}, D) = \prod_{i_0 \xleftarrow{\imath_1} \cdots \xleftarrow{\imath_n} i_n} D_{\imath_1 \cdots \imath_n}$$

with the standard face and coface operations. This cosimplicial group is centralised if the images of α^* and β_* commute in $D_{\alpha\beta}$. Such natural systems arise from 2-categories. For a 2-category T, this is done by taking the underlying category of objects and 1-morphisms of T as the category I and defining the natural system D by assigning to each morphism f of I the collection of 2-automorphisms $\alpha : f \Rightarrow f$ in T. This cosimplicial group is centralised because of the interchange law in 2-categories. In this case, $H^2(\mathrm{I}, D)$ classifies so-called extensions by the natural system D of the category I.

Mariam Pirashvili

Mariam is a postdoctoral researcher at Sorbonne University. Her main research interests are in homological algebra and applied algebraic topology. Mariam was born in Georgia. She obtained her PhD from the University of Leicester, and later worked as a postdoc at the University of Southampton.

The Science of Sci-fi Music

by Andrew May, Springer, 2020, £13.99, US$ 17.92, ISBN: 978-3-030-47832-2

Review by Kim Kenobi

What makes the music of science fiction films sound otherworldly? What tools have composers used to create a sense of the alien or extraterrestrial in sci-fi music? Who are the key figures in this distinctive musical genre? What is the future of sci-fi music?

These are some of the questions addressed in this intriguing and inspiring book. In this age, many of us have access to (a lot of) the world's music via Google Play music (now YouTube music), Apple music or other encyclopaedic music providers. It is refreshing to come across a book that offers many wonderful suggestions for new threads to follow in the global tapestry of what we have at our fingertips in terms of musical entertainment.

The stories of particular people who've played an influential role in shaping the musical soundscapes of science fiction films (and other art forms including opera) leap off the pages of this well-researched volume. The author's description of Schoenberg's development of the 12-tone system as a central part of his exploration of atonality leaves the reader (new to the name Schoenberg) pondering what this mathematical approach to music might actually sound like. At this point, a quick search on YouTube can easily satisfy our curiosity.

It is also interesting to read about the pioneers of the electronic revolution in music, including John Pierce, an electronic engineer, science fiction author and pioneer of computer music. His comments, quoted in the book, about how computers open new ways of thinking about music, new avenues to explore that can be radically different to what has been created before the electronic age, herald the ongoing rise in the use of electronic music in sci-fi settings.

In a section of the book towards the end, we encounter a few examples of 'Extraterrestrial musicians'. Apparently, Stockhausen, a prominent experimental composer of electronic music, claimed to hail from a planet orbiting the star Sirius. In another intriguing story, the author informs the reader about a medium called Rosemary Brown who claimed to channel the music of a range of deceased composers, from Liszt, Beethoven and Bach to Gershwin and John Lennon. The famous Liszt expert Humphrey Searle wrote that one of the 'Liszt' compositions could well have been something he could have written if he had lived another few years.

As well as the many colourful characters who pepper this book, there is also a broad focus on their outputs—the films, books and musical compositions that define the sci-fi genre. The level of the technical detail of the music is spot-on. For example, the quote "The first chord heard in *The Day the Earth Stood Still* is an unresolved polychord consisting of a D minor triad stacked upon an E-flat triad" is preceded by an introduction to the ideas of both triads (in the notation of musical set theory) and polychords. Similarly, in the presentation of Schoenberg's 12-tone system, helpful graphics guide the reader gently into an understanding of the maths and theory behind the musical system being discussed.

Throughout the book the specific details of the films (books, operas, etc.) under discussion are used to illustrate broader themes: mathematical music, the electronic revolution, scientific music and the integration of science fiction in music culture to name a few. Some titles are household names; *2001: A Space Odyssey* features prominently, as does *A Clockwork Orange* and the cantina scene in *Star Wars IV: A New Hope*. Others seem less familiar, more esoteric, such as the 'acoustic astronomy' experiments of astronomers Alex Parker and Melissa Graham (*Supernova Sonata*), using astronomical data from telescopes to create music, or Stockhausen's composition *Mantra*, scored for two ordinary acoustic pianos the sound from which is passed through a ring modulator and projected to the audience through loudspeakers, creating a 'spacey' sound.

Overall, the book offers a highly accessible and engaging introduction to the science of sci-fi music, with technical detail skilfully balanced with fascinating anecdotes about the people and technological developments that have enabled composers to respond to the challenge of creating otherworldly and alien-sounding music.

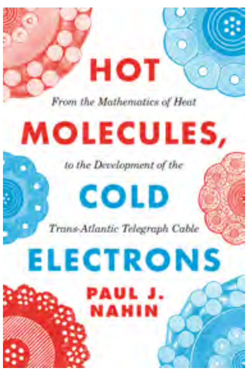
Kim Kenobi

Kim is a Lecturer in Statistics at Aberystwyth University. He has a particular interest in biological statistics and his research includes marine mammals and the effect of climate change on coastal communities both sides of the Irish Sea. He is a keen musician, playing jazz and classical piano, classical clarinet and jazz saxophone.

Hot Molecules, Cold Electrons: From the Mathematics of Heat to the Development of the Trans-Atlantic Telegraph Cable

by Paul J. Nahin, Princeton University Press, 2020, £22.00, US$ 24.95, ISBN: 9780691191720

Review by Isobel Falconer



In 1807 Joseph Fourier published his first work on the heat equation, forming the basis of his monumental *Théorie Analytique de la Chaleur* of 1822. Thirty plus years later, in 1856, William Thomson used Fourier's heat equations in his influential "Theory of the Electric Telegraph", analysing the dispersion of electrical signals in a submarine cable [4]. His theory informed the laying of the first successful trans-Atlantic cable in 1866. Thomson was knighted, and subsequently raised to the peerage as Lord Kelvin.

Paul Nahin's book takes the reader on an enjoyable romp through the mathematics that links these episodes—with miscellaneous bits of history thrown in. Nahin's enthusiasm for the mathematics and its applications is apparent. He has thrown away the famous advice of Simon Mitton (from Cambridge University Press) to Stephen Hawking that every equation would halve his readership; Nahin's book is unashamedly full of equations—solving them is what it is all about.

The cover blurb assures us that these "mathematical explanations can be easily understood by anyone with a basic knowledge of high school calculus", and the book begins innocuously enough with a derivation of the standard integral $\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$. But soon the assurance proves to be a sleight of hand. Nahin frequently "uses nothing but AP calculus". A bit of googling reveals that "Advanced Placement" calculus is accredited university level work studied in the top year of U.S. high schools by only the most able students. Indeed, by Chapter 2 we are on to problems that my first year undergraduate students (all with at least grade A at A level), are currently struggling with even when assisted by a tutor.

The book proceeds, via Euler's and De Moivre's formulae, to develop Fourier series and transforms, and Dirichlet's integral, but interestingly fails to mention that Dirichlet was only 17 when the *Théorie Analytique* appeared and did not publish his work until seven years later; it can have had little influence on Fourier. The heat equation occupies Chapter 3, and its solutions come in Chapter 4. Nahin treats these using "AP calculus" rather than the Laplace transforms of electrical engineering, justifying his decision on the grounds that he is using "the sort of mathematics available to Fourier and Thomson". While this statement is true for the modern "Laplace transform", it deserves explanation in a work that purports to portray the historical development of the mathematics. Indeed, it comes as a bit of a shock, since Nahin has just detailed the "Fourier transform" that Fourier himself developed [2] in response to Laplace's work on transforms—work that ultimately developed into our modern version.

The historical anomalies are beginning to pile up. The most egregious is implied by the title itself: *Hot Molecules to Cold Electrons*. Neither concept was known to the corresponding protagonist. The idea that heat comprised the vibrational energy of "hot" molecules, described by Nahin at the beginning of his account of Fourier's work on the heat equation, did not develop until 40 years later. (Incidentally, Thomson was one of the main architects of that development, but that is a story that Nahin chooses not to mention.) Similarly, the concept and physics of electrons developed in the 1890s, 40 years after Thomson's analysis of the electric telegraph. Neither man embedded in their mathematics, or seeded the ideas, attributed to them by the title. Indeed, Fourier's main aim was to follow Newton in feigning no hypothesis, and to develop a mathematical description of heat that was couched purely in terms of the observables, temperature, length, and time [1, p.1]; it could thus withstand changes in ideas about the underlying nature of heat. To associate "hot" vibrating molecules with heat in his work misses the point of what Fourier was trying to do.

The discussion of the use, by both men, of the heat equations to estimate the age of the Earth, is fascinating but could be more accurately contextualised. Nevertheless, the book contains a few little-known historical gems, such as Henry Wilbraham, apparently the first to plot a Fourier series, in 1848 [5], and Jan Ingenhousz's wax-coated wire method of comparing thermal conductivities [3]. Unfortunately, poor referencing deters the reader from pursuing these, and other historical nuggets.

I wonder, then, quite where the market for the book lies. It is written in an enjoyable, lively style, has a jazzy dust jacket and reassuring blurb, but is too mathematical for the general public. The tone is casual, the commentary of one giving a chalk and talk undergraduate lecture, but while the initial development of the Fourier series might be of use to students, the decision not to use modern transform methods precludes the rest of the book from being of any practical use to them or their teachers. There is a vicarious pleasure in seeing the equations built up from a relatively elementary starting point which might encourage a few enthusiastic students to pursue the topic further, or engage professional mathematicians who are a bit rusty on their physics and do not mind historical anachronism. Perhaps the most likely use is as a handy reference to heat equations for historians of physics, who may appreciate a break from 19th century language and will discount the history.

FURTHER READING

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[3] J. Ingenhousz, Sur les métaux comme conducteurs de la chaleur, Observations et Mémoires sur la Physique, sur l'Histoire Naturelle et sur les Arts et Métiers, etc., 34 (1789) 68–69.
[4] W. Thomson, On the theory of the electric telegraph, Proceedings of the Royal Society of London, 7 (1856) 382–399.
[5] H. Wilbraham, On a certain periodic function, Cambridge and Dublin Mathematical Journal, 3 (1848) 198–201.

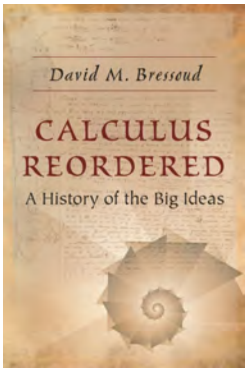
Isobel Falconer

Isobel is Reader in History of Mathematics at the University of St Andrews. Her research focuses on interactions between mathematics and physics in the 19th century. In between times, she tries to keep her husband, daughter, and cat sane, to visit her laidback son, and to stay fit enough to tackle 100 mile continuous walks.

Calculus Reordered: A History of the Big Ideas

by David M. Bressoud, Princeton University Press, 2019, £25.00, US$ 29.95,
ISBN: 978-0691181318

Review by Ciarán Mac an Bhaird



Reading *Calculus Reordered: A History of the Big Ideas* by David Bressoud, was a thoroughly enjoyable experience. While the book is not targeted at a general audience, it should be suitable for anyone with mathematical experience and an interest in the development of mathematics. I would recommend that those involved in teaching calculus, analysis or history of mathematics courses read it. It would also be useful for students starting to study mathematics in higher education.

The book gives a historical overview of the origins of many of the main ideas that led to the development of calculus, covering several millennia and multiple cultures. The first four chapters tell the stories of Accumulation, Ratios of Change, Sequences of Partial Sums and The Algebra of Inequalities. Bressoud's approach highlights that the order in which these ideas developed is often quite different to how to they are taught in modern calculus courses. From my own experience, the order is usually to tackle limits and continuity first, then differentiation, integration and finally infinite series. Throughout the book, Bressoud weaves recurring arguments supporting a significant revision of the arrangement of the traditional calculus curriculum, advocating that it be more in line with the historical development. He brings these arguments together in the appendix.

There is, as you would expect, considerable attention paid to Newton and Leibniz in this book, their fundamental contributions to calculus, and the inverse relationship between differentiation and integration. But the story of calculus doesn't begin or end with Newton and Leibniz, and this is very clearly laid out in the book. The first chapter on Accumulation or integration guides us across almost two thousand years of mathematics. It features the incredible insights of Archimedes and the paradoxes of Zeno from the ancient Greek world. We follow key ideas through the Islamic Middle East and India, to 16th century Europe. There the significant contributions of Cavalieri, Descartes, Fermat, Galileo, Kepler and many more are described and lead us naturally to Newton and Leibniz.

The second chapter, focuses on rates of change or derivatives, starting with Hipparchus of Rhodes but bringing attention to the contributions of astronomers from South Asia and their determination of how changes to one variable impact on another. We are introduced to the importance of logarithms, through the work of Napier and Briggs, before two important sections on 'The Emergence of Algebra' and 'Cartesian Geometry', both of which had a significant influence on how the ideas of calculus could be understood and represented. The second part of the chapter centres on how derivatives developed from Fermat, through Newton and Leibniz, and on to the work of several Bernoullis. Euler's masterful contributions, and especially his advancement of differential equations are then highlighted, before we encounter wave and field theory and Maxwell's equations.

Chapter 3 discusses sequences of partial sums. The dangers of infinite series are introduced, and we see early work in this area in the 17th century, such as Taylor series. Elsewhere in this chapter, we are reminded of the important role these series have had in facilitating the differentiation and integration of functions. Euler and his contributions feature prominently in this chapter, and appear initially via his remarkable, if non-rigorous, solution to the Basel problem. Euler's breath-taking work on the infinite expansions of continued fractions, products and series are highlighted. The chapter closes by looking at the contributions of Cauchy, D'Alembert, Lagrange and Fourier.

In Chapter 4, we arrive at limits. This is one of the most challenging ideas for students to master, and derivatives, integrals, tests for the convergence of series and many other concepts are all defined using limits. The chapter opens with a comment from Abel in 1825, deploring the lack of rigour in relation to infinite series, and continues with a brief overview of the work of those before Cauchy. However, Cauchy is the main star of this chapter and we are guided succinctly through the development of epsilon and delta, completeness, continuity, uniform convergence and finally integration. This new concept of a limit led directly to the expansion of analysis, the topic of the fifth and final chapter. Here, the author considers briefly the work of Riemann and Riemann integration, broaches several related issues and describes how they were tackled. He continues with sections on the advances of Weierstrass, Dedekind and Cantor, to name but a few.

As we begin to think of a new academic year of teaching, I will recommend that my colleagues teaching calculus consider this book and I will also add it to the reading list for my history of mathematics course.

Ciarán Mac an Bhaird

Ciarán is an Assistant Professor in Mathematics at Maynooth University and Director of the Mathematics Support Centre. His current research interests are mostly in mathematics education, but he also conducts research in the history of mathematics and, occasionally, in algebraic number theory. Ciarán tries to play sport on a regular basis and helps his parents on the small family farm in Co. Monaghan.

CMI-HIMR Dynamics and Geometry Summer School 2021, Online

CMI Heilbronn Institute for Mathematical Research University of BRISTOL School of Mathematics

Date: 21 June – 2 July 2021
Website: https://eur.cvent.me/L8oBz

This will be the third in a series of UK postgraduate mathematics summer schools, jointly funded by the Clay Mathematics Institute and the Heilbronn Institute for Mathematical Research.

Dynamics and Geometry are two intertwined areas of mathematics that have seen revolutionary breakthroughs in recent years. In this online summer school world-leading experts will speak about some of these developments, alongside problem sessions and other opportunities for discussion and interaction.

Confirmed lecturers:
Francisco Arana-Herrera, Stanford University
Alex Eskin, University of Chicago
Elise Goujard, Bordeaux Mathematics Institute
Ursula Hamenstadt, University of Bonn
Yair Minsky, Yale University
Amir Mohammadi, UC San Diego
Barak Weiss, Tel Aviv University

See the summer school website for more information and to register.

Obituaries of Members

Patrick D. Barry: 1934–2021

Professor Patrick D. Barry, who was elected a member of the London Mathematical Society on 21 December 1961, died on 2 January 2021, aged 86.

Finbarr Holland writes: At the age of thirty in 1964 Barry was appointed Professor of Mathematics at University College, Cork (UCC), one hundred years after the death of George Boole, Cork's first Professor of Mathematics. To mark the centennial Cork University Press issued *George Boole: A Miscellany,* a booklet of essays edited by Barry. His appointment was followed shortly afterwards by that of President Donal McCarthy (1967–78), who set about modernising the College. Barry was central in McCarthy's ambitious plans. Following a sabbatical at Imperial College, London, his peers elected Barry to UCC's Governing Body. As UCC's first Vice-president (1974–76) he led the way in streamlining the appointments system and also helped to acquire degree-awarding status for Limerick's Teacher Training College.

His tenure as Head of Department (1964–99) was one of harmony and collegiality. Even-tempered, thoughtful and non-confrontational by nature, he always managed to coax consensual decisions at meetings he chaired. Having to cater for a broad range of student ability, interest and class size, he appointed staff with a diverse range of specialisms, and gave each the freedom to develop his or her own course

Towards the end of his role as HoD he introduced an innovative part-time two-year postgraduate degree course in Mathematical Education for secondary-school mathematics teachers. This was offered on only two occasions in a ten year period, and was availed of by about forty teachers drawn from all over Munster. It raised their profile, earned them an extra salary increment and was hugely beneficial to their students.

Barry had a life-long passion for geometry and loved to teach it. Following his retirement in 1999 he prepared for publication a rigorous account of Euclid's geometry suitable for teachers of school Mathematics, based on Birkhoff's approach. A first edition of his *Geometry with Trigonometry* was published in 2001 and a second edition in 2015; it is the foundation of the geometry section of the current Mathematics syllabus for the Leaving Certificate. Simultaneously, he wrote up notes under the heading *Generalizations of Geometry* which he had almost completed before his health began to decline. These notes form the basis of his third book *Advanced Plane Geometry* published in 2019.

Patrick Denis Barry — Paddy to everybody who knew him — was born in Ballynacargy, Co. Westmeath, Ireland, in 1934. His father was a Garda sergeant, his mother a Primary School teacher. He was the fifth in a family of three boys and four girls. When Paddy was two the family moved to Co. Cork. There, he received his secondary education at the Patrician Academy, Mallow, his performance in the Leaving Certificate examination in 1952 winning him a university scholarship from Cork County Council. In the same year, he achieved first place in the UCC Entrance Scholarship examination, and second place in the examination for the Irish Civil Service. During his schooldays Paddy played cricket, soccer, tennis and badminton, the latter a sport at which he was particularly skilful, and continued to play late in life.

With two scholarships Paddy became the first pupil from his school to go to university, graduating in 1955 with a first class honours BSc in Mathematics and Mathematical Physics. That same year he represented Ireland at badminton at Under-21 level.

He remained in UCC for a further two years studying for his master's degree, winning as well the prestigious Travelling Studentship in Mathematical Science from the National University of Ireland. He declined this award, having already accepted the position of Research Assistant to Walter Hayman FRS at Imperial College, London. He earned a Diploma from IC and a PhD from London University for a thesis entitled *On the Minimum Modulus of Integral Functions of Small Order*, based on his own solution to his own problem, which was typical of him. Indeed, writing in *My Life and Functions*, Hayman says that "Barry was the only student I ever had who came to me with a PhD already prepared".

On receipt of his doctorate Paddy spent a year at Stanford University as a Mathematics Instructor before returning to his alma mater in 1961 to become first a lecturer in the Mathematics Department, and later Professor and Head of Department in 1964, positions he occupied until his mandatory retirement in 1999.

Shortly after returning to Cork, Paddy met and married Frances King, a vivacious young woman from Belfast, whose sister's Belfast boyfriend had a post in UCC's English Department, and was Paddy's flatmate! Fran became a secondary teacher of Mathematics, later acquiring a PhD in group theory to become one of the few Irish secondary teachers with a doctorate.

Paddy loved cooking and liked to show off his culinary skills at dinner parties hosted by Fran and himself, producing a variety of exotic dishes, made — according to his children — with mathematical precision! His speciality was a delicious cinnamon-infused apple pie.

Paddy died in a Dublin nursing home, from covid-19, Fran having pre-deceased him by fifteen years. They are survived by their children: Conor, a film maker in Dublin, Una, who practises general medicine in Calgary, Canada, and Brian, a surgeon in Cork.

Isadore M. Singer: 1924–2021

Professor Isadore M. Singer, who was elected an Honorary member of the London Mathematical Society on 18 June 2004, died on 11 February 2021, aged 96.

Nigel Hitchin writes: Isadore was a long-standing collaborator of Michael Atiyah and, largely because of that, a frequent visitor to the UK. The Atiyah–Singer Index Theorem is their most notable achievement and won the two authors the Abel Prize in 2004.

Born in Detroit in 1924 to Polish immigrant parents, he went through hard times in the Depression but shone academically and went on to study physics at the University of Michigan. When war came, he served as a radar officer in the Philippines and then on his return turned to mathematics at the University of Chicago, following this with postdoctoral work at MIT, where he stayed for most of his working life.

It was at the Institute for Advanced Study in Princeton in 1955 that he first met Atiyah. This was a particularly fertile period where Bott, Hirzebruch, Serre, Kodaira and others were redefining the relationship between algebraic geometry, topology and differential geometry.

And then, in 1961, Singer made contact again. As he recalled later to Sir Michael,

> "...family considerations required a change in my sabbatical plans. Based on our friendship at IAS, that special year 1955–56, I felt comfortable calling you to ask whether the Maths Institute in Oxford might have a place for me beginning January 1962. Although you had just yourself come to Oxford, you simply said "Come; we'll take care of it". I'm still grateful for that instantaneous response.
>
> Early in January, on my second day at the Maths Institute you walked up to the fourth floor office where I was warming myself by the electric heater. After the usual formalities, you asked "Why is the $\hat{A}$ genus an integer for spin manifolds?" "What's up, Michael? You know the answer much better than I". "There's a deeper reason", you said.
>
> ...the question kept haunting me as I wandered through the beautiful gardens of the Oxford Colleges. In March I knew what the answer must be..."[1]

This was the index theorem, which occupied the two of them for more than 20 years. Singer's work till then was largely in functional analysis as a consequence of having Irving Segal as thesis supervisor, but ever since being exposed to Shiing-Shen Chern in Chicago his enthusiasm resided more on the differential geometry side.

Together with Atiyah's expertise in topology and algebraic geometry the fusion of ideas produced a huge generalisation of the classical Gauss–Bonnet and Riemann–Roch theorems in the context of elliptic partial differential operators on manifolds.

In the mid-1970s, Singer's interests were directed towards theoretical physics, partly through conversations with Jim Simons and Chen-Ning Yang of Yang–Mills fame in Stony Brook. In 1976-77 another extended visit to Oxford brought these ideas of gauge theory to Atiyah's group of researchers and a rapid development of one of them — finding all

[1] ST Yau (ed.), *The Founders of Index Theory*, International Press, Somerville, Mass. (2005) ISBN 978-1571461377

self-dual solutions of the Yang–Mills equations — took place. The index theorem had an important role here and the papers of Atiyah, Singer (and myself) on the subject laid the foundations for Simon Donaldson's later spectacular work.

From this point on, Singer's mathematical papers virtually all had links to physics, sometimes with physicists as joint authors. He held highly influential joint seminars in the two disciplines in MIT, and also in Berkeley where he spent the years 1979–83. He was called on for many roles in determining science policy in the USA, in particular as a member of the White House Science Council in the Reagan years.

Professor Singer is survived by his wife Rosemarie, three children, two stepchildren, and four grandchildren.

Stephen J. Pride: 1949–2020

Professor Stephen Pride, who was elected a member of the London Mathematical Society on 13 October 1978, died on 21 October 2020, aged 71.

Jim Howie writes: Steve was born in Melbourne on 8 January 1949. He graduated first class in Mathematics from Monash University in 1971 and obtained a PhD in Group Theory at the Australian National University in 1974, under the supervision of Mike Newman. He thrived in the research culture at ANU: by the time his prize-winning thesis *Residual Properties of Free Groups* was completed his research had already been published in three journal articles.

Steve moved to the UK in 1974, initially to a research fellowship at the Open University, then in 1978 to a temporary lectureship at King's College London, and in 1979 to a permanent lectureship at the University of Glasgow where he would spend the rest of his career. Over this period he embarked on a research programme that would bring him to prominence in the field of geometric group theory. His early work made significant contributions to small cancellation theory and to the theory of one-relator groups. Notable among the latter was his solution of the isomorphism problem for two-generator one-relator groups with torsion. This is the most difficult of the three algorithmic problems proposed by Dehn in 1910, and known to be unsolvable in full generality; its solution even for a small class of groups was a remarkable achievement.

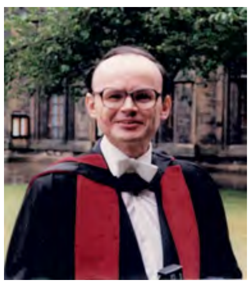
A collaboration with Benjamin Baumslag was to prove immensely significant. Their short note in the Journal of the London Mathematical Society in 1978 entitled *Groups with Two More Generators than Relations* would become Steve's most-cited publication. In it they sowed the seeds of the concept of 'largeness', proving that the groups in the title were large. This concept was codified and developed by Steve and his student Martin Edjvet in a sequence of later papers; today it is a standard topic in the field.

Steve was one of a group of young, enthusiastic mathematicians appointed to Glasgow around the same time, which led to a blossoming research environment there. His reputation as a research mathematician and teacher grew apace. Promotions to Reader and then Professor followed in short order, and he was elected Fellow of the Royal Society of Edinburgh in 1992. He developed a global network of collaborators, many of whom spent prolonged visits in Glasgow. These included leading names in the field, but also many younger mathematicians for whom Steve played a mentoring role. He travelled widely and frequently, taking part in mathematical conferences all around the globe.

Steve also contributed to the mathematics environment via work for EPSRC, the Commonwealth Scholarships Committee and editorial positions on various journals – including a 10-year spell on the editorial board of the LMS.

Steve was an inspirational supervisor of graduate students. He supervised around 20 in total, from his time at the OU until after his formal retirement in 2011. He is survived by his two sisters Jocelyn Pride and Beverley Hulme.

Thomas A. Whitelaw: 1943–2021



Dr Thomas Whitelaw, who was elected a member of the London Mathematical Society on 16 November 1967, died on 21 January 2021, aged 77.

Gordon Blower writes: Thomas Andrew Whitelaw, popularly known as Tommy, was born in April 1943 and grew up in the Ayrshire coastal town of Troon. He was a pupil at Marr College, becoming Dux in 1960. By his own account, his life then underwent a huge quantum jump when he started at the University of Glasgow, and he duly graduated with a BSc Maths.-Nat. Phil. in 1964.

Following advice from Professor Ian Sneddon, he entered Trinity College Cambridge as a PhD student of J.A. Todd. His thesis covered sporadic finite simple groups, specifically Mathieu's group of degree 12 and Janko's simple group. He supervised undergraduates who were studying courses on number theory and projective geometry. Garth Dales recalls "Tommy was the best-organized and clearest supervisor that I had at Cambridge".

In this era, several academic staff in the Department of Mathematics at Glasgow followed a similar career path. Whitelaw returned to Glasgow in 1967 as assistant lecturer, and ultimately retired as a senior lecturer in 2007. As a lecturer, he had an inspiring and highly individual style, which enlivened every topic. His course on Linear Algebra is well captured in his textbook, which features the uncluttered notation of the lectures. His pedagogical approach was to view subjects from the perspective of a learner, and to present material developmentally. He invariably wore a gown when lecturing; reportedly because a head of department had once recommended this attire and no subsequent head had rescinded the order. While emphasizing teaching, he disapproved of Quality Assurance, which he dismissed as 'Quackery'.

Even when teaching large classes, Whitelaw took an interest in the progress of individual students, and addressed them by name in the style Mr X or Miss Y. In 1975, he began his role as Advisor of Studies, which he took very seriously. Indeed, he maintained correspondence with all his former advisees, who numbered more than 600 by the time he fully retired in 2011. His Family of Former Students enjoyed various gatherings, especially Whitelaw's 70th birthday celebration. In addition to social networking, his lengthy epistles to FFS provided a well-informed and entertaining account of academic life in the University of Glasgow and cultural life in Ayrshire.

Whitelaw was an only child and his father died when he was 12 or 13. In later years, he looked after his mother, as they stayed in a flat overlooking Troon bay. Whitelaw was a stalwart supporter of various organizations in Troon, which appreciated his generous and friendly manner. He was an elder in Troon Old Parish Church, serving as roll-keeper, and sang bass in the choir. Another choir was Troon Chorus, for which he became president. He wrote some music, including a carol, published in a collection, and he attempted a four-movement piano sonata. His assiduous record keeping of Troon weather led to a Fellowship of the Royal Meteorological Society. While less mobile in his final years he persisted with these many interests; he died peacefully at Ayr University Hospital.

Death Notices

We regret to announce the following deaths:

- Lucien P. Foldes, formerly Emeritus Professor of Economics at the London School of Economics, who died on 10 February 2021.
- Peter D. Robinson, formerly of Bradford University, who died on 16 February 2020.
- Rabe R. von Randow, formerly University of Bonn, who died on 6 March 2021.

LMS Midlands Regional Meeting

Location: University of Lincoln
Date: 2 June 2021
Website: tinyurl.com/2ch6wn3y

The meeting forms part of the Midlands Regional Workshop on *Profinite Groups and Related Aspects* on 3–4 June. The speakers will be Delaram Kahrobaei, Simon Smith and Alexandre Borovik. Funds are available for partial support to attend the meeting and workshop.

Random Matrices and Integrable Systems

Location: Online
Date: 16–18 June 2021
Website: tinyurl.com/358pvcp5

The past decade has seen enormous progress in understanding the behaviour of large random matrices and interacting particle systems. A large variety of complementary methods have emerged to study these systems, and in particular methods originating from integrable systems.

Python for A-Level Maths and Beyond

Location: Online
Date: 16–17 July 2021
Website: tinyurl.com/9pd7ssnc

This is a hands-on workshop and will introduce delegates to the freely available, open-source and general-purpose programming language Python, which is one of the most popular programming languages in the world. Delegates need no prior knowledge of programming to benefit from this workshop.

Statistics at Bristol: Future Results and You 2021

Location: University of Bristol
Date: 16–17 September 2021
Website: tinyurl.com/y5xlmxkx

In 1871, Florence Nightingale published the second of two seminal papers on the preparation of hospital data. These are still widely regarded as some of the most important works in the history of statistics. Exactly 150 years later, with challenges from climate change to epidemics, unlocking the potential of data is more important than ever. Where will you fit in?

Young Researchers in Mathematics 2021

Location: Online, hosted by Bristol University
Date: 6–9 June 2021
Website: tinyurl.com/2fn4js75

This conference is for all PhD students. Attendees will have the opportunity to (virtually) meet researchers from all areas in a friendly environment. The plenary talks will showcase a wide range of mathematics. Visit the website for further details on the event. Register at tinyurl.com/2j66a767.

Harmonic Analysis and Partial Differential Operators

Location: Loughborough University
Date: 16 July 2021
Website: tinyurl.com/y3d3cb47

In recent years there have been spectacular advances in Euclidean harmonic analysis, with wide-ranging applications in particular to constant-coefficient partial differential operators. This event will highlight the interaction between harmonic analysis and PDE.

Mathematics of Robotics IMA Conference

Location: Online
Date: 8–10 September 2021
Website: tinyurl.com/4aat9694

This conference aims to bring together researchers working on all areas of robotics which have a significant mathematical content. The idea is to highlight the mathematical depth and sophistication of techniques applicable to robotics and to foster cooperation between researchers working in different areas of robotics. Organised in cooperation with SIAM.

Mathematics in Times of Crisis

Location: Online
Date: 18–20 November 2021
Website: tinyurl.com/tdnjp65x

This is the 32nd Novembertagung History and Philosophy of Mathematics annual international conference for graduate students. The theme is Mathematics in Times of Crisis. A full call for papers can be found at the conference website. The deadline for submissions is 1 July 2021. Keynote lectures will be given by Amir Asghari, Juliet Floyd and Tilman Sauer. This event is supported by an LMS Postgraduate Research Conference Grant.

Covid-19: Owing to the coronavirus pandemic, many events may be cancelled, postponed or moved online. Members are advised to check event details with organisers.

Society Meetings and Events

May 2021

14 LMS Spitalfields History of Mathematics Meeting: Educational Times Digital Archive Launch, online (493)
24 Mary Cartwright Lecture 2021 (online)
26 LMS Good Practice Scheme Workshop (online)

June 2021

2-4 Midlands Regional Meeting and Workshop, Lincoln (494)
22 Society Meeting at the 8ECM, Portorož, Slovenia
24 Summer Graduate Student Meeting (online)

July 2021

2 Summer General Society Meeting (online)

September 2021

6-10 Northern Regional Meeting, Conference in Celebration of the 60th Birthday of Bill Crawley-Boevey, University of Manchester

January 2022

4-6 South West & South Wales Regional Meeting, Swansea

Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website (www.lms.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

May 2021

26 Korteweg–de Vries Equation, Toda Lattice and their Relevance to the FPUT Problem, University of Lincoln (493)

June 2021

6-9 Young Researchers in Mathematics 2021, University of Bristol (494)
16-18 Random Matrices and Integrable Systems, University of Bristol (494)
20-26 8th European Congress of Mathematics, Portorož, Slovenia (492)
21-2 Jul Dynamics and Geometry Summer School, University of Bristol (493)
28-2 July Modelling in Industrial Maintenance and Reliability (online) (493)
28-2 July Dynamics and Geometry Summer School, University of Bristol (494)

July 2021

7-9 Nonlinearity and Coherent Structures, Loughborough University (492)
12-16 New Challenges in Operator Semigroups, St John's College, Oxford (490)
16 Harmonic Analysis and Partial Differential Operators, Loughborough University (494)
16-17 Python for A-Level Mathematics (494)
19-23 Rigidity, Flexibility and Applications, Lancaster University (492)

21-23 Research Students' Conference in Population Genetics, University of Warwick (493)

26-30 Young Geometric Group Theory X, Newcastle University (493)

August 2021

12-14 Young Functional Analysts' Workshop, Lancaster University (493)

16-20 IWOTA, Lancaster University (481)

18-20 Young Researchers in Algebraic Number Theory III, University of Bristol (492)

September 2021

1-3 Scaling Limits: From Statistical Mechanics to Manifolds, Cambridge (493)

8-10 Mathematics of Robotics Conference (online) (494)

9-10 Heilbronn Annual Conference 2021, Heilbronn Institute (493)

16-17 Statistics at Bristol: Future Results and You 2021, Heilbronn Institute (494)

19-24 8th Heidelberg Laureate Forum, Heidelberg, Germany

21-23 Conference in Honour of Sir Michael Atiyah, Isaac Newton Institute, Cambridge (493)

November 2021

18-20 Mathematics in Times of Crisis, online (494)

July 2022

24-26 7th IMA Conference on Numerical Linear Algebra and Optimization, Birmingham (487)