LONDON MATHEMATICAL SOCIETY
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# NEWSLETTER

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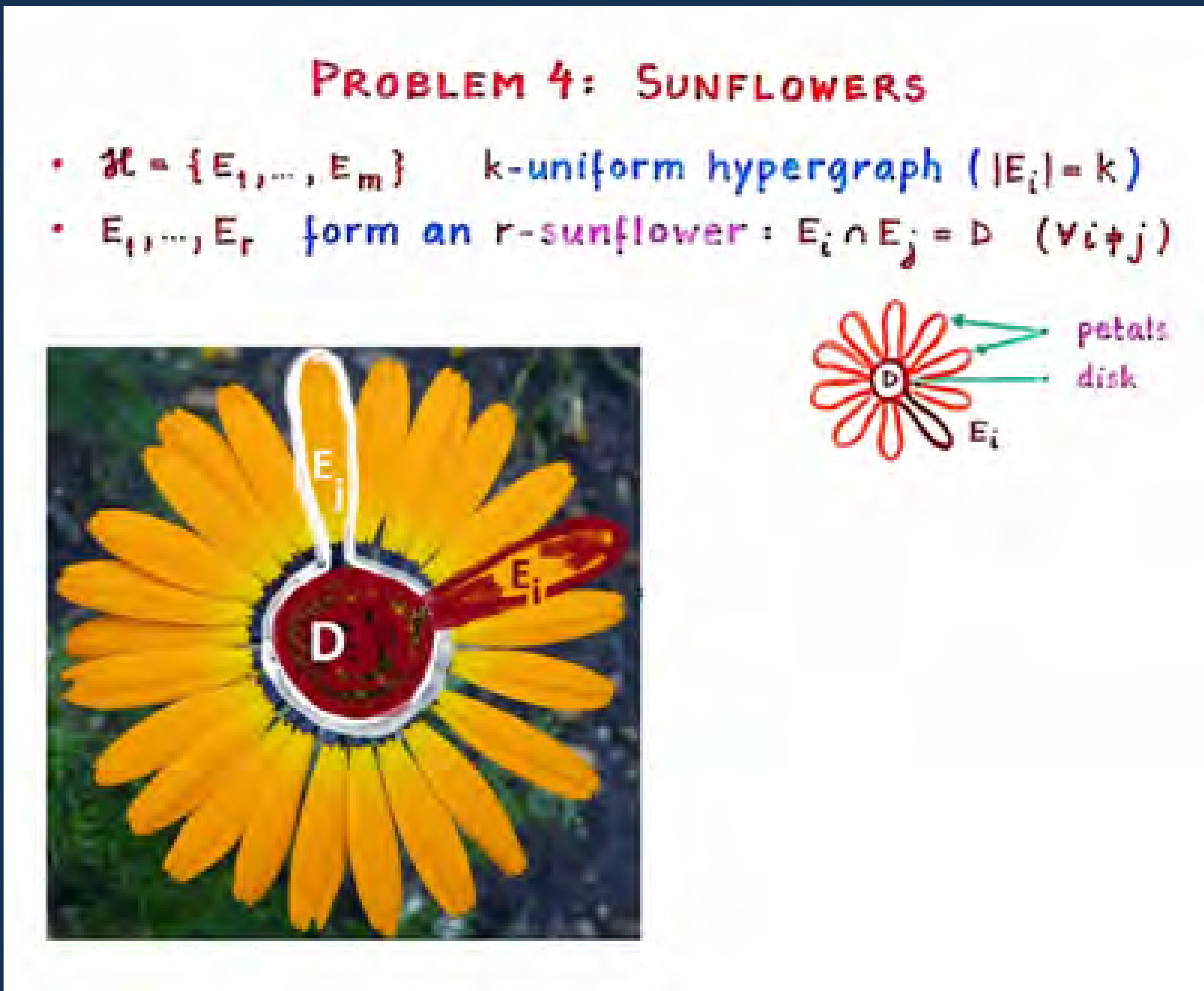


TOPOLOGICAL GRAPH THEORY

HIGHER DIMENSIONAL DOMINOES

A POSTCARD FROM OUR FRONT ROOM

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COVER IMAGE

A screenshot of one of Janos Pach's beautiful slides, from his 8ECM talk *Escaping the Curse of Dimensionality in Combinatorics*. See the review of 8ECM on page 41).

Do you have an image of mathematical interest that may be included on the front cover of a future issue? Email images@lms.ac.uk for details.



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The Newsletter welcomes submissions of feature content, including mathematical articles, career related articles, and microtheses from members and non-members. Submission guidelines and LaTeX templates can be found at lms.ac.uk/publications/submit-to-the-lms-newsletter.

Feature content should be submitted to the editor-in-chief at newsletter.editor@lms.ac.uk.

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# CONTENTS

# Balancing the Books: Changes to the Society's Budget because of Changes in Academic Publishing

One of the Society's charitable objectives is to disseminate mathematical knowledge and make it available worldwide. The Society's academic publishing activities are central to this work. The Society is therefore pleased that, at the time of writing, it is on the point of renewing its contract with its publishing partner John Wiley & Sons for the publication of its core journals: the *Proceedings*, the *Journal*, the *Bulletin*, the *Transactions* and the *Journal of Topology*, and to publish *Mathematika*, which is owned by University College London for another contract term.

The renewal of the Society's contract with Wiley follows a tender process during which the Society invited bids from academic publishers for this contract, and carefully reviewed its publishing requirements and ambitions. The tender process unequivocally demonstrated that, due to major changes in the market including those associated with the move to open access, publishers are no longer able to offer the sort of financial terms the Society has enjoyed in the past for the publication of the core journals. This is not unique to the Society and affects other learned societies and publishers as well. It is clear that open access business models will not replace the income lost from the subscription business model in the short-to-medium term and it is not clear they will do so in the longer term either.

The Society is set to lose about 25% of its existing income as a result of these changes. Although Council is actively exploring other sources of income, it is clear that these cannot generate the necessary replacement income in the short-to-medium term. And it remains to be seen whether new income streams of a sufficient scale can successfully be developed in the longer term. Charity Trustees have a legal duty to manage the charity's resources responsibly, and Charity Commission guidance states that Trustees should act quickly if there is a significant change in the amount of money coming into or going out of the charity. As the Society's Trustees, Council members have therefore had to make difficult decisions about changes to budgets across many areas of the Society's activity in order to put the Society's finances on a sustainable footing for the future.

Over the course of 2021, two dedicated Council discussions have been held to decide how the Society should respond to the income reduction it faces. Council reviewed all the Society's activities in order to identify savings. A number of different factors were taken into account in deciding where savings should be made, and how big those savings should be. These factors included how closely an area of activity aligns with the Society's charitable objectives, what the impact of the activity is and whether there are other organisations that may be better placed than the Society to support a particular area of activity. The output of these discussions was then considered by the Financial and General Purposes Committee, and recommendations were made to Council, which Council accepted at its 2 July meeting.

Council decided overall reductions in budgets for different Committees and areas of activity. Committees and budget holders affected by the reductions have been able in many cases to decide the detail of how the overall savings are made from the various different individual budget lines under their control. Savings will include fewer in-person Committee meetings (the pandemic has shown us that virtual meetings can at times be effective), and a reduction in the total value of grants the Society will award, although the latter area of activity was considered to be one that should be cut least.

Council also decided that, over the course of the 2021/22 financial year, the Society's staff headcount should be reduced by one 'full time equivalent' (FTE) post out of 14 FTE at present. This reflects the reduction in the need for staff because of reduced activity and will also make a contribution to the overall savings that are needed.

Council has established two funds to help smooth the transition to the new reduced budget environment. There is a Covid catch-up fund for the 2021/22 financial year to cover postponed activities and additional funding requests that arise because of the pandemic. There is also a mitigation fund to help alleviate the short-to-medium term impact of the changes to budgets that are being made.

As part of this re-budgeting process, there has been considerable scrutiny of the costs — both financial and environmental — of printing and distributing the *LMS Newsletter*. We know that the *Newsletter* is a treasured benefit of membership and that it helps members stay in touch with mathematical developments and feel connected with the community. Production of the *Newsletter* will of course continue, and it has been agreed that the *Newsletter* will also continue to be printed and distributed during the 2021/22 financial year at least to those members wishing to receive a hard copy. However, the Society will be exploring the extent to which take-up of the electronic version of the *Newsletter* can be encouraged, and whether it is sustainable to continue to produce printed copies of the *Newsletter* in future years.

Professor Simon Salamon
LMS Treasurer

## 2021 LMS Prize Winners

The Society extends its congratulations to the following 2021 LMS Prize Winners and thanks to all the nominators, referees and members of the Prizes Committee for their contributions to the Committee's work this year.

**Professor Ehud Hrushovski** FRS, of the University of Oxford, is awarded a Pólya Prize for his profound insights that transformed very abstract model-theoretic ideas into powerful methods in well-established classical areas of geometry and algebra.

**Professor Tara Brendle** of the University of Glasgow is awarded a Senior Whitehead Prize for her fundamental work in geometric group theory, concentrating on the study of groups arising in low-dimensional topology, and for her exemplary record of work in support of mathematics and mathematicians.

**Professor Endre Süli** of the University of Oxford is awarded the Naylor Prize and Lectureship for his wide-ranging contributions to the study of applied mathematics.

**Dr Ailsa Keating** of the University of Cambridge is awarded a Berwick Prize for the paper 'Dehn twists and free subgroups of symplectic mapping class groups', published in the Journal of Topology. Keating's work sheds light on the global symmetries of symplectic manifolds, by showing that arbitrary products of Dehn twists along two Lagrangian spheres that intersect at least twice never simplify to the identity map.

**Dr Viveka Erlandsson** of the University of Bristol is awarded an Anne Bennett Prize for her outstanding achievements in geometry and topology and her inspirational active role in promoting women mathematicians.

**Dr Jonathan Evans** of the University of Lancaster is awarded a Whitehead Prize for his contributions to symplectic topology and its relation to algebraic geometry. Among his achievements is an innovative study (with Smith and Urzua) of Wahl singularities from the symplectic viewpoint.

**Professor Patrick Farrell** of the University of Oxford is awarded a Whitehead Prize in recognition of his broad, creative and impactful work as a computational mathematical scientist. Farrell's contributions to the general area of the numerical solution of partial differential equations span algorithm development, rigorous analysis, high performance software implementation, and applications in scientific computation.

**Dr Agelos Georgakopoulos** of the University of Warwick is awarded a Whitehead Prize for his contributions to long-standing problems in probability and graph theory, using methods from combinatorics as well as probability, topology and geometry.

**Dr Michael Magee** of the University of Durham is awarded a Whitehead Prize for his deep contributions to a wide range of questions at the interface between number theory and mathematical physics, and in particular to random matrices and to the spectral theory of hyperbolic surfaces.

**Dr Aretha Teckentrup** of the University of Edinburgh is awarded a Whitehead Prize for her incisive research contributions to the foundations of research in uncertainty quantification, at the interface of numerical analysis and probability.

**Professor Stuart White** of the University of Oxford is awarded a Whitehead Prize in recognition of his contributions to the structure and classification theory of nuclear C*-algebras and their interplay with von Neumann algebras.

# 2021 LMS Prize Winners

**Ehud Hrushovski**
Pólya Prize

**Tara Brendle**
Senior Whitehead Prize

**Endre Süli**
Naylor Prize

**Ailsa Keating**
Berwick Prize

**Viveka Erlandsson**
Anne Bennett Prize

**Jonathan Evans**
Whitehead Prize

**Patrick Farrell**
Whitehead Prize

**Agelos Georgakopoulos**
Whitehead Prize

**Michael Magee**
Whitehead Prize

**Aretha Teckentrup**
Whitehead Prize

**Stuart White**
Whitehead Prize

## Hirst Prize and Lectureship 2021

The LMS and British Society for the History of Mathematics have awarded the 2021 Hirst Prize and Lectureship to Professor Karine Chemla (Centre National de la Recherche Scientifique; CNRS), in recognition of her outstanding and innovative work in the history of mathematics. This marks the first year that the Hirst Prize and Lectureship is jointly awarded by the LMS and BSHM.

Karine Chemla's academic career in the history of mathematics has covered some forty years. Throughout virtually all this time she has been based at CNRS, where she is currently Senior Researcher within the research group SPHERE (Science–Philosophy–History, Université de Paris & CNRS). Professor Chemla's research has focused on a number of areas within the history of mathematics, including the history of geometry in France in the nineteenth century, the historiography of non-western mathematics, and the history of mathematics in China. It is particularly in this last area of research that Chemla is widely recognised as a preeminent authority.

See the full citation for Professor Chemla at tinyurl.com/hzhznsfs.

## David Crighton Medal 2021

The LMS and Institute of Mathematics and its Applications are delighted to announce that the 2021 David Crighton Medal is awarded to Professor Caroline Series, of the University of Warwick. This is in recognition of her fundamental and beautiful results connecting geometry and dynamical systems, and her outstanding service to the mathematical community, including her pioneering work to support the careers of women in mathematics.

Professor Series' early work involved coding the dynamics of geodesic flows on surfaces, developing techniques of Rufus Bowen, and led to an invitation to speak at the ICM in 1986. There is now a substantial body of work, carried out by groups around the world, that can be traced back to the theory and connection between areas that was established by Series in this early work.

Subsequently, Series' interests moved to Kleinian groups, and she made major contributions to the study of spaces of certain parametrised families of groups, such as the Maskit slice, the Riley slice and other higher dimensional examples. She showed how the geometry of the group action, and in particular beautiful patterns in its limit set, vary as one moves through the parameter space. Her work has been recognised by numerous awards throughout her career and, in 2016, she was elected a Fellow of the Royal Society.

Alongside her research, Professor Series has provided outstanding service to the mathematical community on numerous committees and panels both nationally and internationally. She served as President of the LMS from 2017 to 2019, having earlier served the Society in numerous ways. Of particular note is the impact of her work to improve gender equality, where she has continually acted as an initiator of new activities at an international level.

See the full citation for Professor Series at tinyurl.com/stttc23n.

## LMS Honorary Members 2021

The London Mathematical Society has elected the following people to Honorary Membership of the Society in 2021:

Professor Bao Châu Ngô, University of Chicago, USA, and Vietnam Institute for Advanced Study in Mathematics; Professor Laure Saint-Raymond, Institut des Hautes Études Scientifiques (IHES), France; Professor Peter Sarnak, Stanford University, USA; and Professor Ya-xiang Yuan, Chinese Academy of Sciences in Beijing, China.

Professor Ngô has made one of the most important contributions to the theory of automorphic forms in the last few decades. In a remarkable work published in 2010 that combines ideas from representation theory, arithmetic geometry, topology, singularity theory, and mathematical physics, he completed a proof of the so-called ‘fundamental lemma’ in the theory of the Arthur–Selberg trace formula. For this work, he was awarded the Fields Medal by the International Mathematical Union in 2010.

Professor Laure Saint-Raymond is an exceptional mathematician, an analyst working on partial differential equations from physics whose landmark contributions include the first rigorous mathematical connexion between non-perturbative solutions of kinetic theory and fluid mechanics, and the first rigorous connexion between the many-body deterministic Newtonian system and Brownian motion.

Professor Peter Sarnak has made fundamental contributions in analysis, number theory, and combinatorics including work on Ramanujan graphs, families of L-functions, quantum unique ergodicity, and sub-convexity of L-functions.

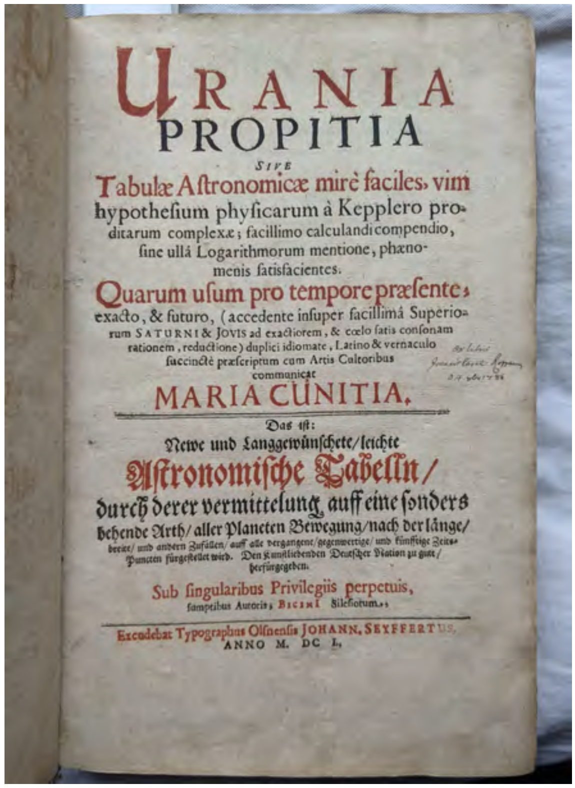
Professor Ya-xiang Yuan is an internationally leading expert in optimization theory and methods. He has had major role in developing optimization algorithms and exhibited leadership of applied mathematics in China and worldwide.

Full citations will appear in the LMS Bulletin at lms.ac.uk/publications/blms.

## *Urania Propitia*

Eagle-eyed readers of the July *Newsletter* will have spotted that Dr A.E.L. Davis bequeathed a rare copy of *Urania Propitia* (Maria Cunitz, 1604/10–1664) to the Society. Fewer than 25 copies of this significant book, published in 1650, are known to exist.

*Urania Propitia* (‘Beneficent Urania’ — Urania is the muse of astronomy) is one of the earliest known books by a female mathematician publishing cutting edge research in her own name. Its long explanatory subtitle immediately focuses us on some of the mathematical issues of the day. It begins, “Wonderfully easy astronomical tables, comprehending the essence of the physical hypotheses brought forth by Kepler, satisfying the phenomena by a very easy, brief way of calculating without any mention of logarithms”. We see at once that Kepler’s laws are new and require explanation and testing against observation, that logarithms are similarly new and viewed as a barrier rather than an aid to calculation, and that there is a great need for ‘easy’ means of astronomical calculation.



URANIA PROPITIA

SIVE

Tabulæ Aſtronomicæ mirè faciles, vim hypotheſium phyſicarum à Kepplero proditarum complexæ; facillimo calculandi compendio, ſine ullâ Logarithmorum mentione, phænomenis ſatisfacientes.

Quarum uſum pro tempore præſente, exacto, & futuro, (accedente inſuper facillimâ Superiorum SATURNI & JOVIS ad exactiorem, & cœlo ſatis conſonam rationem, reductione) duplici idiomate, Latino & vernaculo ſuccinctè præſcriptum cum Artis Cultoribus communicat

MARIA CUNITIA.

Das iſt:

Newe und Langgewünſchete/leichte

Aſtronomiſche Tabellen/

durch derer vermittelung auff eine ſonders behende Arth/ aller Planeten Bewegung/nach der länge/ breite/ und andern Zufällen/ auff alle vergangene/gegenwertige/ und künfftige Zeit-Puncten fürgeſtellet wird. Den Kunſtliebenden Deutſcher Nation zu gutt/ herfürgegeben.

Sub ſingularibus Privilegiis perpetuis, ſumptibus Autoris, BICINI Sileſiorum.

Excudebat Typographus Olſnenſis JOHANN. SEYFFERTUS, ANNO M. DC L.

Cunitz perceived this last need as arising as much from the multitude of calendar and almanac makers, and astrologers scattered across Europe, as from the elite astronomers that we hear of more often. She made her twin audiences clear: the book contains parallel texts in Latin and German and she explained that “the German Nation abounds in those of abilities suited to astronomical practice although often lacking knowledge of Latin”, while the work was also “intended for the universal improvement of the republic of letters”, necessitating Latin [1]. She herself came to astronomy and mathematics through the practical route. She was born in 1604 or 1610 in Schweidnitz, Silesia (now Swidnica, Poland) [1]. Both her parents were well educated. Her father, Heinrich Cunitz, was a physician, while her mother, Maria Schultz, was the daughter of Anton Schultz, a practical mathematician. Astrology was still an integral aspect of medicine and Heinrich Cunitz was skilled in this field, as was Maria’s second husband, Elias von Löwen, whom she married in 1630.

Prompted by von Löwen, Cunitz became an early user of Kepler’s Rudolphine Tables, but rapidly concluded that they needed radical transformation if they were to be of extensive practical use. Although the Rudolphine Tables were more accurate than previous tables, their basis in Kepler’s laws and a heliocentric universe presented two new problems of calculation: finding the position angle of a planet for a given time using Kepler’s equation, and combining the heliocentric positions of earth and planet to find its geocentric position [2]. Kepler used logarithmic methods for both, computing the necessary interpolations, but the resultant tables took three times longer to use than earlier ones [1].

Kepler expressed the area traced out by a planet since its aphelion (a term that he introduced and Cunitz adopted) in degrees. Cunitz now undertook the c.30,000 calculations necessary to recast them in terms of time since aphelion, and provide the true motion per day and per hour for interpolation between the entries for days, and simple double-entry tables from which predictions of planetary position and geocentric latitude and longitude could be found by straightforward arithmetic [1]. *Urania Propitia* took Cunitz around 15 years to complete, interrupted by relocations necessitated by the Thirty Years War and the birth of three children.

Even before the book's publication in 1650, Cunitz's fame was spreading. She acquired a reputation in her day akin to that accorded to Mary Somerville 200 years later — intellectually outstanding (she knew Latin, Greek and five modern languages), but "modest", "gentle" and skilled in the feminine accomplishments of needlework, painting and music — although she was accused of neglecting her domestic affairs [1, quoting Herbinius 1657]. Hevelius, a far more elite and famous astronomer, mentioned her in 1646, and began corresponding with her and von Löwen two years later, discussing a wide range of observational phenomena, new methods and instruments, particularly the telescope [3]. In 1655 fire destroyed the couple's library and instruments, leaving few surviving scraps of Cunitz's manuscripts. Maria Cunitz died in 1664, three years after her husband.

*Urania Propitia* was privately printed, and the print run and purchase price are unknown. Moreover, the work of the jobbing astronomers who may have been its main market has been little studied, so it is difficult to judge Cunitz's success. However, it was purchased by James Gregory, first Regius Professor of Mathematics at the University of St Andrews, to equip what would have been Britain's first civic Observatory in 1673 — he did not buy a copy of the Rudolphine Tables [4].

Dr A.E.L. Davis was, herself, a Kepler scholar, as well as a great promoter of studies of female mathematicians. She compiled the Davis database of female mathematics graduates in the British Isles [5], and donated her wide-ranging collection of books by and about women who worked in mathematical areas to the Society as the Philippa Fawcett Collection [6]. *Urania Propitia* is the magnificent and carefully chosen bequest of an expert in her fields.

**References**

[1] Swerdlow, N. M. "Urania Propitia, Tabulae Rudophinae Faciles Redditae a Maria Cunitia Beneficent Urania, the Adaptation of the Rudolphine Tables by Maria Cunitz." A Master of Science History, ed. Jed Z. Buchwald. Springer, 30 [2011], pp. 81–121. doi: 10.1007/978-94-007-2627-7_7.

[2] Gingerich, O. "Johannes Kepler and the Rudolphine Tables." Resonance, 14 [12], p. 1223. doi: 10.1007/s12045-009-0116-3.

[3] Włodarczyk, Jarosław. "'Peripheral' Astronomy in the Correspondence of Johannes Hevelius: A Case Study of Maria Cunitia and Elias von Löwen." Kwartalnik Historii Nauki i Techniki, vol. 2019 [1], pp. 147–55. doi: 10.4467/0023589XKHNT.19.009.10117.

[4] Pringle, R.V. "An edited transcript of the '1687' Catalogue in St Andrews University Library covering the years 1687–1704." bit.ly/3rTFluR; see also M. Pilar Gil, "Collecting Science in St Andrews. A history In Context", M.Litt Dissertation, University of St Andrews, 2016.

[5] The Davis historical archive: Mathematical Women in the British Isles, 1878–1940: bit.ly/3l94k13.

[6] The Philippa Fawcett Collection: bit.ly/3ycdoGf.

**Translation of the title page of *Urania Propitia* (from [1] p86)**:

"BENEFICENT URANIA, or Wonderfully easy astronomical tables, comprehending the essence of the physical hypotheses brought forth by Kepler, satisfying the phenomena by a very easy, brief way of calculating without any mention of logarithms; the concisely taught use of which for the period of time, present, past, and future—adding as well a very easy correction of the superior planets, Saturn and Jupiter, to a more accurate computation and improved agreement with heaven—

MARIA CUNITZ imparts to devotees of this science in a two-fold idiom, Latin and vernacular [German]: That is, new and long-desired, easy astronomical tables, through which the movement of all planets, with regard to longitude, latitude, and other phenomena in all moments of time, past, present, and future, is supplied in an especially handy method, presented here for the benefit of the devotees of science of the German Nation."

Isobel Falconer
University of St Andrews

# LMS Presented with New Turing £50 Bank Note

l to r: De Morgan House staff James Taylor, Andrew Dorward and Caroline Wallace

For its help with the design of the new £50 bank note featuring Alan Turing, the LMS has been presented with a low serial number (AA01 001936) version by the Bank of England. The presentation took place on 23 June, the anniversary of Turing's birth, and the day that the new £50 bank note went into circulation.

The Society contributed to the design of the bank note in 2019 by giving approval and permission for the use on the bank note of two mathematical excerpts from the Turing article *On Computable Numbers, with an Application to the Entscheidungsproblem* (submitted 28 May 1936, published in *Proceedings of the London Mathematical Society* (2) 42 (1937) 230–265) (tinyurl.com/6sbcpmjz).

LMS Executive Secretary Caroline Wallace accepted the note on behalf of the Society at its headquarters in De Morgan House. She said: "I am delighted and humbled to receive this bank note on behalf of the London Mathematical Society. The Society is pleased to have been able to contribute to the successful development of this new design. This bank note continues the process of recognising and celebrating the enormous contribution made by Turing to mathematics, to computer science and to the country".

Alan Mathison Turing OBE FRS (1912–1954) was a mathematician, computer scientist, logician, cryptanalyst, philosopher and theoretical biologist who was instrumental in formalising the concepts of algorithm and computation. Turing worked as a code-breaker during the second world war and is widely accredited with having helped bring an earlier end to the war. The story of his life has had wide implications for changes in political, legal and social attitudes towards human diversity and homosexuality.

The first excerpt is a table from page 240 which provides a schema for succinctly representing Turing machines. The table gives a complete description of how to specify such machines and therefore can be thought of as one of the first examples of a programming language.

| *m-config.* | *Symbol* | *Operations* | *Final m-config.* | |
|---|---|---|---|---|
| $q_i$ | $S_j$ | $PS_k, L$ | $q_m$ | $(N_1)$ |
| $q_i$ | $S_j$ | $PS_k, R$ | $q_m$ | $(N_2)$ |
| $q_i$ | $S_j$ | $PS_k$ | $q_m$ | $(N_3)$ |

The second excerpt, from page 241, is a sequence of Turing machine transitions that helps explain how to encode a Turing machine as a number. The more modern analogue of what Turing describes is how to take an abstract representation of a computer program and convert it into a binary sequence of 0s and 1s so that it can be stored on a disc or in the memory of a computer. The idea that a program can be stored as a number and used as data (by an operating system) in order to execute the program, is hugely important.

$$q_1 S_0 S_1 R q_2;\ \ q_2 S_0 S_0 R q_3;\ \ q_3 S_0 S_2 R q_4;\ \ q_4 S_0 S_0 R q_1;.$$

Turing went on further in his article to describe large classes of real numbers whose binary expansions are computable by his machines, to describe a 'universal machine' that could serve the purpose of an operating system, and to describe the theoretical limits of his machines. Ultimately, Turing showed that there can be no algorithmic method for determining whether or not a given mathematical statement can be proved in a certain axiomatic system. This proved that David Hilbert's famous *Entscheidungsproblem* has no solution (which was also proved independently by Alonzo Church).

Alan Turing is the second mathematician to appear on a Bank of England banknote. A £1 banknote in circulation between 1978 and 1988 depicted Sir Isaac Newton. Famous Britons have featured on the reverse of Bank of England banknotes since 1970.

A collection of Turing's articles in the *Proceedings* is now available to all on Wiley Online Library (tinyurl.com/6ezscvw6).

## LMS Emmy Noether Fellows 2021

Clockwise from top left: Sinead Lyle, Constanze Roitzheim, Mirela Domijan, Jennifer Tweedy

The Emmy Noether Fellowship Selection Panel has recommended the award of four LMS Emmy Noether Fellowships to Dr Mirela Domijan, from the University of Liverpool, Dr Sinead Lyle from the University of East Anglia, Dr Constanze Roitzheim, from the University of Kent, and Dr Jennifer Tweedy who will be based at the University of Bath. Once again this year's applications were of an exceptionally high standard and our warmest congratulations go to the 2021 LMS Emmy Noether Fellows.

LMS Emmy Noether Fellowships are designed to enhance the research, broadly construed, of mathematical scientists who are either re-establishing their research programme after returning from a major break associated with caring responsibilities, or who require support to maintain their research programme while dealing with significant ongoing caring responsibilities. They are sponsored by a generous donation from the Liber Stiftung (tinyurl.com/57ch4t5e).

## Forthcoming LMS Events

**LMS Northern Regional Meeting and Workshop 2021:** 1–10 September, online (tinyurl.com/c5nnwcjd)

**LMS Northern Regional Meeting 2021:** 7 September, online (tinyurl.com/sdrsamve)

**LMS Prospects in Mathematics Meeting 2021:** 9–10 September, online (tinyurl.com/x6khbf9v)

**LMS/IMA Joint Meeting, Maths in Human Society:** 30 September – 1 October, online (tinyurl.com/d4w5bf9b)

**Joint Meeting Black Heroes of Mathematics Conference:** 5–6 October, online (tinyurl.com/4r6dneaw)

**Annual General Meeting & Presidential Address:** 12 November, London (tinyurl.com/yxsvb9rj)

A full listing of upcoming LMS events can be found on page 59.

OTHER NEWS

## IMA Hedy Lamarr Prize

The IMA has awarded the first IMA Hedy Lamarr Prize for Knowledge Exchange in Mathematics and its Applications to Professor Rebecca Hoyle (left). This new IMA prize, which was introduced to the community in 2020, is awarded to a mathematician who demonstrates meritorious knowledge exchange in mathematics and its applications.

Rebecca Hoyle is Associate Dean for Research and Enterprise, Faculty of Social Sciences and Professor of Applied Mathematics at the University of Southampton. Her early work on pattern formation included applications to aeolian sand ripples and chemical catalysis. Her research then moved strongly towards applying mathematics and modelling to issues in the life and social sciences. In collaboration with life scientists she researched two-component system signalling, the dynamics of molecular motors and the evolution of transgenerational effects in response to environmental change. More recently, she has worked on problems in psychology and social science, studying the effect of personality traits on strategy decisions and the evolution of cooperation in social networks, using a variety of mathematical techniques from equation-free methods and agent-based modelling to statistical inference and dynamics on multiplex networks.

Rebecca has used her impressively broad network of contacts to bring University managers, social scientists and epidemiologists to collaborate in online study groups as part of the Virtual Forum for Knowledge Exchange in Mathematical Sciences (V-KEMS), of which she is a founding member, in an effort to develop and sustain a virtual forum for knowledge exchange in the mathematical sciences during the covid-19 pandemic. Leading on from V-KEMS activity, she has also been working with collaborators on modelling the pandemic in higher education settings in the UK. Colleagues have commented on her outstanding ability to guide a group of researchers (of different backgrounds and from different career stages) toward useful outputs that will have the largest possible impact.

Hedy Lamarr was an Austrian-born Hollywood film star, producer and inventor. She co-invented an early version of frequency-hopping spread spectrum communication, originally intended for torpedo guidance and currently employed in Bluetooth and other communications technology.

## Shaw Prize 2021

Jeff Cheeger (left) and Jean-Michel Bismut

The Shaw Prize in Mathematical Sciences 2021 has been awarded jointly to Jean-Michel Bismut (Université Paris–Saclay) and Jeff Cheeger (New York University) for their remarkable insights that have transformed, and continue to transform, modern geometry. They have worked together and are particularly celebrated for their extension of a famous invariant, the so-called eta invariant, from manifolds to families of manifolds, which allowed them to compute explicitly the limit of the eta invariant along a collapsing sequence of spaces. As well as solving long-standing open problems Bismut and Cheeger have introduced important new ideas and built tools that have greatly extended the range of what is possible in modern geometry.

## Mathematician Wins Olympic Gold

The London Mathematical Society is pleased to congratulate cyclist and mathematician Dr Anna Kiesenhofer on her outstanding achievement in winning gold in the Women's Cycling Road Race at the Tokyo Olympics. Dr Kiesenhofer studied mathematics at the Technical University of Vienna before completing a Master's degree at the University of Cambridge. She was awarded her PhD from the Polytechnic University of Catalonia for her work on integrable systems on *b*-symplectic manifolds and she is currently a postdoctoral fellow in Mathematics at the École Polytechnique Fédérale de Lausanne (EPFL).

## Double Gold for UK at International Mathematical Olympiad

Two UK students — Samuel Liew and Yuka Machino, Year 13 students at The West Bridgford School and Millfield School, respectively — have won gold medals at this year's International Mathematical Olympiad. Yuka Machino, 17, came top amongst all female participants for the second year in a row and was the only female participant in the competition to receive a gold medal.

The UK team, which is entered into the competition by the United Kingdom Mathematics Trust and supported by XTX Markets, was also awarded three silver medals and one honourable mention. Over 100 countries took part in the prestigious competition and the outstanding all-round performance meant the UK finished in 9th place. This is the fourth time in six years that the UK has placed in the top 10 at the 'world championship of mathematics'.

More information about the International Mathematical Olympiad is available at imo-official.org.

## Appointment of New CMS Chair

Professor Alison Etheridge

LMS member Professor Alison Etheridge OBE, FRS, FIMA will be the new Chair of the Council for the Mathematical Sciences (CMS) in September 2021 succeeding Professor Sir Ian Diamond. Professor Etheridge is a member of the Mathematical Institute and Head of the Department of Statistics, University of Oxford.

Professor Etheridge said, "It is a huge honour to be asked to contribute to the work of the CMS. Now, more than ever before, new types of mathematical insight are needed to drive forward scientific and industrial innovation, and the role of mathematical science in meeting the challenges posed by the global pandemic has further underlined the value of this ultimate transferable skill. In an evolving landscape, the CMS has a vital role to play in articulating to policymakers (and others) the need to strengthen and grow the people pipeline right across the Mathematical Sciences, and to embed mathematical and computational thinking in all aspects of science, policy, and innovation."

After graduate study split between the Universities of Oxford and McGill (Montreal), Professor Etheridge worked at the Universities of Cambridge, California (Berkeley), Edinburgh and Queen Mary University London before returning to Oxford. Over the course of her career, her interests have ranged from abstract mathematical problems to concrete applications in areas such as population genetics and mathematical finance.

Professor Etheridge has extensive experience of academic leadership nationally and internationally, including working with EPSRC both through the Mathematical Sciences Strategic Advisory Team (SAT) and through the Strategic Advisory Network (SAN) and is a Member of EPSRC Council. She is also the Panel Chair for the Mathematics Unit of Assessment in REF 2021. She brings valuable experience spanning a broad range of the Mathematical Sciences making her an ideal figurehead for the community. The five CMS Societies are delighted that she has accepted this post.

## Appointment of Director of ICMS

Professor Minhyong Kim (left), an LMS member and a member of the LMS Council, has been appointed full-time Director of the International Centre for Mathematical Sciences (ICMS) and Sir Edmund Whittaker Professor of Mathematical Sciences at both Heriot-Watt University and the University of Edinburgh, with effect from the end of September 2021. Professor Kim is the first full-time director of the ICMS and the first joint professor in the Maxwell Institute. More details are available on the ICMS website at https://tinyurl.com/3pmxuh9k .

## Queen's Birthday Honours 2021

- John Aston, Harding Professor of Statistics in Public Life, University of Cambridge, and lately Chief Scientific Adviser, Home Office. Knight Bachelor for services to Statistics and Public Policymaking.
- Ellen Brooks-Pollock, Senior Lecturer, Disease Modelling and Veterinary Public Health, University of Bristol. OBE for services to the Scientific Pandemic Influenza Group on Modelling and SAGE during the covid-19 Response.
- Sarah Anne Caul, Mortality Analysis Team Manager, Office for National Statistics. MBE for services to Health Statistics (Cardiff, South Glamorgan)
- Myer Glickman, Analyst, Office for National Statistics. OBE for services to Health Analysis (Pontypool, Gwent).
- Mark Andrew Jones, Ethical Hacking Services Team, Driver and Vehicle Licensing Agency. OBE for services to Science, Technology and Mathematics.
- Professor Matthew Keeling, Mathematics Institute and School of Life Sciences, University of Warwick. OBE for services to SAGE during the covid-19 response.
- Suzanne Nicola Reeves Co-chair, Disability Group, Office for National Statistics and Chair, Civil Service Disability Network. MBE for services to Disability, to Diversity, and Inclusion.

MATHEMATICS POLICY DIGEST

## Equity in the STEM Workforce

Following an eight-month inquiry, The All-Party Parliamentary Group on Diversity and Inclusion in Science, Technology, Engineering and Mathematics (STEM) published the Equity in the STEM Workforce report in July 2021.

The report highlights five key findings.

(1) The STEM workforce is less diverse than the wider workforce and consistent data collection and sharing is lacking.

(2) There is a need for government to take a multi-pronged approach to drive equity in the STEM workforce.

(3) Intersectional barriers continue from STEM education into the workforce.

(4) There is awareness of structural inequality in some large STEM organisations, but no consensus on solutions.

(5) There is already considerable inequity in STEM but covid-19 is making it worse.

The report has put forward three recommendations for consideration by the sector as a whole and policymakers.

(1) The UK government and STEM organisations, across the private, public and voluntary sectors should commit to leading a STEM Diversity Decade of Action to tackle the historic and systemic underrepresentation of minoritised groups at all levels in the sector.

- The Prime Minister and the UK government should set a bold vision for a diverse and equitable STEM sector at the heart of their ambitions for the UK to become a ‘global science superpower’.
- STEM leaders from organisations across the private, public and voluntary sectors should work together to form and co-fund an ‘Employers’ Coalition for STEM Diversity’ to address the structural inequity in the STEM workforce and drive long-term change.

(2) The UK government must deliver a statutory workforce data strategy and drive forward changes in policy and legislation to support employers to improve equity for minoritised communities in many sectors of the UK workforce, including STEM.

(3) The UK government and STEM organisations must quickly look to address and reverse worsening inequity within the STEM workforce as a result of the pandemic.

The full report is available at tinyurl.com/4kn2xx2w.

Digest prepared by Dr John Johnston
Society Communications Officer

*Note: items included in the Mathematics Policy Digest are not necessarily endorsed by the Editorial Board or the LMS.*

## IMU Centenary

The Centennial Conference of the International Mathematical Union will be held in Strasbourg on 27–28 September, at the same location where the IMU was founded in 1920.The opening session will be held under the patronage of the French President, Emmanuel Macron. All proceedings will be webcast live. For further information see tinyurl.com/49vm6ufk.

## 8ECM

The *8th European Congress of Mathematics* was held in June at Portorož in Slovenia, and was a great success.There were 1766 registered participants, of whom about 200 were able to attend in person. Videos of selected events can be seen on the EMS YouTube channel (tinyurl.com/pf7bexca). A look back on highlights of the conference is available on the ECM page at 8ecm.si/news and there is an interesting write-up at tinyurl.com/2hxnxzwh of the conference by Boštjan Kuzman for the Society of Mathematicians, Physicists and Astronomers of Slovenia.

## EMS Magazine

The second edition of the *EMS Magazine* is now published, available to read free online at euromathsoc.org/magazine. Among the usual wide range of interesting articles is one by Katie Chicot of the Open University and MathsWorldUK on *Creating the UK's first National Mathematics Discovery Centre*, a mission (close to success, we understand) to establish a physical Discovery Centre to match other European Mathematics Discovery Centres that have given helpful guidance, most notably Mathematikum in Giessen and MMACA in Barcelona. MathsWorldUK welcomes further involvement from supporters in Europe and elsewhere.

## CIRM

Centre International de Rencontres Mathématiques (CIRM), located at Luminy (Marseille), now has two auditoriums and several smaller rooms, all fully equipped with videoconferencing systems. It can therefore host two large events simultaneously, or (especially) several small groups. 'Research in residence' and 'workshops' are CIRM programmes for small groups, ranging from 2 to 4 participants for the former and 10–40 participants for the latter. Application for these programmes is open all year round. For more information see tinyurl.com/sabwvsw7. To discover the CIRM in 3 minutes, watch this video: https://tinyurl.com/9ckk62th.

## Czech Mathematical Society

The Czech Mathematical Society has awarded an Honorary Medal for Mathematics to David E. Edmunds, Sussex University, for his long-term and fruitful cooperation with Czech mathematicians.

## EMS–Simons grants for Africa

The next deadline for EMS–Simons for Africa grants is 15 November 2021. The grants are to promote individual career possibilities with the resulting improved global capacity in African academic institutions. The program is open to all areas of pure and applied mathematics and statistics and it is directed to fellows based in Africa. Applications should be submitted online. For details see tinyurl.com/8e72nvuz.

## New Publication: *Memoirs of the EMS*

The *Memoirs of the European Mathematical Society* publish outstanding research contributions in individual volumes, in all areas of mathematics and with a particular focus on works that are longer and more comprehensive than usual research articles. The Editorial Board consists of the Editors-in-Chief of the *Journal of the European Mathematical Society* and the *EMS Surveys in Mathematical Sciences*, along with editors of book series of EMS Press. Submissions are now open. All submitted works will go through a highly selective peer-review process. The *Memoirs* will begin publication in January 2022 and will be available as a subscription as well as for individual purchase. For more information see tinyurl.com/63b6kx6y.

EMS News prepared by David Chillingworth
LMS/EMS Correspondent

*Note: items included in the European Mathematical Society News represent news from the EMS are not necessarily endorsed by the Editorial Board or the LMS.*

OPPORTUNITIES

## LMS Grant Schemes

See lms.ac.uk/grants for full details.

### Research Grants

The next closing date for research grant applications is 15 September.Applications are invited for the following grants to be considered by the Research Grants Committee at its October 2021 meeting. Applicants should be mathematicians based in the UK, the Isle of Man or the Channel Islands. For grants to support conferences/workshops, the event must be held in the UK, the Isle of Man or the Channel Islands:

### Conferences (Scheme 1)

Grants of up to £5,500 are available to provide partial support for mathematical conferences held in the UK.

### Visits to the UK (Scheme 2)

Grants of up to £1,500 are available to provide partial support for a visitor who will give lectures in at least three separate institutions. Awards are made to the host towards the travel, accommodation and subsistence costs of the visitor. Potential applicants should note that it is expected the host institutions will contribute to the costs of the visitor. In addition, the Society allows a further amount (of up to £200) to cover caring costs for those who have dependents.

### Joint Research Groups in the UK (Scheme 3)

Grants of up to £3,000 (£1,500 per year for a two year grant) are available to support joint research meetings held by mathematicians who have a common research interest and who wish to engage in collaborative activities, working in at least three different locations (of which at least two must be in the UK). An additional £1,000 can be applied for by those who would like to organise an Online Graduate Lecture Series. The Society built on its Scheme 3 research networks and expertise to enhance the education of beginning postgraduate students and to support early career researchers. We invite our Joint Research Groups to facilitate a short series of five or six lectures in their subject area to be delivered to a live online audience and to be recorded for future use. The Online Graduate Lecture Series is open to anyone who wishes to facilitate a short series of five or six lectures in their subject area to be delivered to a live online audience and to be recorded for future use. You do not need to be an existing Joint Research Groups (Scheme 3) grant holder.

### Research in Pairs and Research Reboot (Scheme 4)

The Research in Pairs grant is for those mathematicians inviting a collaborator, grants of up to £1,200 are available to support a visit for collaborative research either by the grant holder to another institution abroad, or by a named mathematician from abroad to the home base of the grant holder. For those mathematicians collaborating with another UK-based mathematician, grants of up to £600 are available to support a visit for collaborative research either by the grant holder to another institution or by a named mathematician to the home base of the grant holder. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents.

The Research Reboot grant is for those mathematicians who have found themselves without the time to engage in research due to personal circumstances, illness, caring responsibilities, increased teaching or administrative loads or other factors. Grants of up to £500 are available for accommodation, subsistence and travel to support a two to five day retreat, outside of their usual environment, to help restart research activity. An additional £500 can be applied for to cover Caring Costs for those who have caring responsibilities.

### Collaborations with Developing Countries (Scheme 5)

For those mathematicians inviting a collaborator to the UK, grants of up to £3,000 are available to support a visit for collaborative research, by a named mathematician from a country in which mathematics could be considered to be in a disadvantaged position, to the home base of the grant holder. For those mathematicians going to their collaborator's institution, grants of up to £2,000 are available to support a visit for collaborative research by the grant holder to a country in which mathematics could be considered to be in a disadvantaged position. Applicants will be expected to explain in their application why the proposed country fits the circumstances considered eligible for Scheme 5 funding. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents. Contact the Grants team if you are unsure whether the proposed country is eligible, or check the IMU's Commission for Developing Countries definition of developing countries (tinyurl.com/y9dw364o).

**Research Workshop Grants (Scheme 6)**

Grants of up to £4,000 are available to provide support for Research Workshops. Research Workshops should be an opportunity for a small group of active researchers to work together for a concentrated period on a specialised topic. Applications for Research Workshop Grants can be made at any time but should normally be submitted at least six months before the proposed workshop.

**African Mathematics Millennium Science Initiative (AMMSI)**

Grants of up to £2,000 are available to support the attendance of postgraduate students at conferences in Africa organised or supported by AMMSI. Application forms for LMS-AMMSI grants are available at ammsi.africa.

The next closing date for Early Career Research Grant applications (ECR Travel Grants) is 15 May 2021. Applications are invited for the following grants to be considered by the Early Career Research Committee at its June 2021 meeting:

**Early Career Researcher Grants**

The next closing date for early career research grant applications (Schemes 8, 9 and ECR Travel Grants) is 15 October 2021. Applications are invited for the following grants to be considered by the Early Career Research Grants Committee at its November 2021 meeting. Applicants for LMS Grants should be mathematicians based in the UK, the Isle of Man or the Channel Islands. For grants to support conferences/workshops, the event must be held in the UK, the Isle of Man or the Channel Islands:

**Postgraduate Research Conferences (Scheme 8)**

Grants of up to £2,500 are available to provide partial support for conferences held in the United Kingdom, which are organised by and are for postgraduate research students. The grant award will be used to cover the costs of participants.

**Celebrating New Appointments (Scheme 9)**

Grants of between £450-600 are available to provide partial support for meetings held in the United Kingdom to celebrate the new appointment of a lecturer at a UK University.

**Early Career Researchers Travel Grants**

Grants of up £500 are available to provide partial travel and/or accommodation support for UK-based Early Career Researchers to attend conferences or undertake research visits either in the UK or overseas.

For full details of these grant schemes, and to find information on how to submit application forms, visit the LMS website: lms.ac.uk/content/research-grants. Queries regarding applications can be addressed to the Grants Administrator Lucy Covington (020 7927 0807, grants@lms.ac.uk), who will be pleased to discuss proposals informally with potential applicants and give advice on the submission of an application.

## Cecil King Travel Scholarships 2022: Call for Applications

The LMS administers two £6,000 travel awards funded by The Cecil King Memorial Foundation for early career mathematicians, to support a period of study or research abroad, typically for a period of 3 months. One scholarship will be awarded to a mathematician in any area of mathematics and one to a mathematician whose research is applied in a discipline other than mathematics.

Applicants should be mathematicians in the United Kingdom or the Republic of Ireland who are registered for a doctoral degree or have completed one within 12 months of the closing date for applications. The LMS encourages applications from women, disabled, Black, Asian and Minority Ethnic candidates, as these groups are under-represented in the United Kingdom and the Republic of Ireland mathematics.

To apply, please complete the application form and include a written proposal describing the intended programme of study or research abroad, and the benefits to be gained from such a visit. Further details, including the application form are available at tinyurl.com/cecil2022. The deadline for applications is 23.59 on 15 November 2021.

Shortlisted applicants will be invited to an online interview in January 2022 during which they will be expected to make a short (10-15 minute) presentation on their proposal, followed by 15-20 minutes for questions. Queries may be addressed to fellowships@lms.ac.uk.

## Atiyah UK–Lebanon Fellowships for 2022–23

Set up in 2020 in memory of Sir Michael Atiyah (1929–2019), whose father was Lebanese and who retained strong links with Lebanon throughout his life, the LMS Atiyah UK–Lebanon Fellowships operate in partnership with the Centre for Advanced Mathematical Sciences (CAMS) at the American University of Beirut (tinyurl.com/x5zhzpff).

**Objectives**

The LMS Atiyah UK–Lebanon Fellowships provide for an established UK based mathematician to visit Lebanon as an Atiyah Fellow for a period of between one week up to 6 months, or alternatively for a mathematician from Lebanon of advanced graduate level or above to visit the UK to further their study or research for a period of up to 12 months.

For visits from the UK to Lebanon, the Atiyah Fellowship will cover:

- From the LMS, up to £2,000 towards actual expenses for travel and related expenses, and accommodation and subsistence of £1000 per month pro rata for up to 6 months.
- In addition, CAMS will cover accommodation and provision of office space and logistical support. This will be independent of the host institution.
- Additional subsistence/payment for agreed teaching may be awarded.
- Consideration may be given for additional support to Fellows travelling with a family.

For visits to the UK from Lebanon, the Atiyah Fellowship will cover:

- From the LMS, up to £2,000 towards expenses for travel and related expenses, and accommodation and subsistence of £500 per month pro rata for up to 12 months.
- Additional support will be available for PhD and/or promising MSc candidates in either mathematics or mathematical physics.

**Further information and queries**

Applications open on 1 September 2021. The deadline is 31 January 2022. Further information, including how to apply, is available at tinyurl.com/atiyah2022. Queries should be addressed to fellowships@lms.ac.uk. The Chair of the Fellowship Panel is Professor Caroline Series FRS.

## LMS Early Career Fellowships 2021–22

To support early career mathematicians in the transition between PhD and a postdoctoral position, the LMS, with support from the Heilbronn Institute of Mathematical Research via funding from UKRI-EPSRC, will offer fellowships of 3-6 months to mathematicians who have recently or will shortly receive their PhD. The award will include a monthly stipend plus a travel allowance. The fellowships may be held at one or more institutions but not normally at the institution where the Fellow received their PhD. Fifteen Early Career Fellows were supported through this scheme in 2020-21. A list of current Fellows is available on the LMS website at tinyurl.com/zy957m4y.

Deadline is 14 January 2022. For further details about the Early Career Fellowships, including a sample application form, see tinyurl.com/ktjzfchv. Contact Elizabeth Fisher (fellowships@lms.ac.uk) or call +44 (0)20 7291 9973 with any queries.

## LMS–Bath Mathematical Symposium 2023: Call for Proposals

The LMS is pleased to announce its Call for Proposals for the LMS-Bath Mathematical Symposium to be held at the University of Bath in 2023.

Formerly known as the LMS-Durham Symposia, the LMS-Bath Mathematical Symposia will be held at the University of Bath between 2020 and 2025. Since their foundation in 1974 the Symposia are an established and recognised series of international research meetings that provide an excellent opportunity to explore an area of research in depth, to learn of new developments, and to instigate links between different branches of the subject.

The format is designed to allow substantial time for interaction and research. The meetings are by invitation only and will be held in July/August, with up to 50 participants, roughly half of whom will come from the UK. A novel element of the Symposia is that they will each be complemented either by a summer school, to prepare young researchers such as PhD students, or a 'research incubator', where problems related to the topic of the conference are studied in groups. The entire event, summer school/incubator and workshop, will typically last around two weeks.

Prospective organisers should send a formal proposal to the Grants Team (grants@lms.ac.uk) by 15 December 2021. Proposals are approved by the Society's Research Grants Committee after consideration of referees' reports.

Proposals should include:

- A full list of proposed participants, divided into specific categories: Category A, Scientific Organisers; Category B, Key Overseas Participants; Category C, Key UK-based Participants; Category D, Important Overseas Participants; Category E, Important UK-based Participants.
- A detailed scientific case for the Symposium, which shows the topic is active and gives reasons why UK mathematics would benefit from a symposium on the proposed dates.
- Details of additional support from other funding bodies, or proposed avenues of available funding.
- Indicative plans for the summer school or research incubator.
- Where appropriate, a discussion of the possibility of an 'industry day'.

Proposers are encouraged to actively seek to include women speakers and speakers from ethnic minorities, or explain why this is not possible or appropriate.

For further details about the LMS Mathematical Symposia, visit the LMS website: tinyurl.com/4krypbyd or the Symposia website: bathsymposium.ac.uk.

Before submitting, organisers are welcome to discuss informally their ideas with the Chair of the Research Grants Committee, Professor Andrew Dancer (grants@lms.ac.uk). Further information, in particular regarding available funding, will be published at bathsymposium.ac.uk.

## zbMATH Open: An Invitation to the Mathematical Community

zbMATH has been open access since 1 January 2021. Our database is now freely available to everybody worldwide. We are taking this opportunity to invite the mathematical community to become involved in the future development of the database.

**The zbMATH Open Survey**

We invite you to take part in our online survey https://tinyurl.com/u2awfn78 which will be open until 10 October 2021. We ask you to evaluate the current features of zbMATH Open and to comment on the future direction of the database.

**Interlinking with Other Databases and Community Platforms**

zbMATH Open provides links both to open digital libraries or repositories such as arXiv, ElibM, EuDML, and to external databases including wikidata and the Mathematics Genealogy Project. We aim to integrate as many mathematics open data as possible and interlink them with other databases and services. As zbMATH Open is now open access, it can contribute even more freely to the dissemination of knowledge and to the effective networking of all the different scientific services.

This can be seen with community platforms such MathOverflow. Since August 2017, MathOverflow users have been able to find bibliographical references in zbMATH Open and add them to discussions. The corresponding backlinks are automatically generated. We invite the community to suggest and help us develop further links to other databases and services. Please contact us at editor@zbmath.org.

**Open Data**

zbMATH Open provides free access not only to the database, but also to data which can be downloaded via an API, including all metadata and reviews. We invite you to use these data for research projects. For details see https://oai.zbmath.org/.

**Looking for Reviewers**

The reviews written by experts constitute an essential and extremely valuable part of the database. They provide crucial information on published articles such as the historical background, the relationship to other publications and sometimes also information on problems concerning the validity of published results. Reviewing mathematical works is a valuable service to the mathematical community and is highly appreciated by your colleagues. In addition, your reviews will have a considerable visibility worldwide. Our current reviewer community consists of more than 7,000 experts worldwide. All the same, we are not able to cover as much of the mathematical literature as we would like to. You can provide a valuable contribution by becoming a reviewer: https://zbmath.org/become-a-reviewer/ .

Reviewers receive compensation of EUR 3 for each review, as well as privileges for ordering books from the EMS Press.

With your help, zbMATH Open will improve and become a better mathematics service. Thank you for your contributions!

Klaus Hulek
Editor-in-Chief, zbMath

## LMS Undergraduate Research Bursaries in Mathematics 2022

The Undergraduate Research Bursary scheme provides an opportunity for students in their intermediate years to explore the potential of becoming a researcher. The award provides support to a student undertaking a 6–8 week research project over Summer 2022, under the direction of a project supervisor.

Students must be registered at a UK institution for the majority of their undergraduate degree and may only take up the award during the summer vacation between the intermediate years of their course. Students in the final year of their degree intending to undertake a taught Masters degree immediately following their undergraduate degree may also apply. Applications must be made by the project supervisor on behalf of the student.

For further information contact Lucy Covington (urb@lms.ac.uk). Applications will be open from November 2021, with an application deadline of 1 February 2022.

## LMS Research Schools and Research Schools in Knowledge Exchange 2023

Grants of up to £15,000 are available for LMS Research Schools, one of which will be focused on Knowledge Exchange. The LMS Research Schools provide training for research students in contemporary areas of mathematics. The Knowledge Exchange Research School can be in any area of mathematics.

The LMS Research Schools take place in the UK and support participation of research students from both the UK and abroad. The lecturers are expected to be international leaders in their field. The LMS Research Schools are often partially funded by the Heilbronn Institute for Mathematical Research (heilbronn.ac.uk). Information about the submission of proposals can be found at tinyurl.com/ychr4lwm along with a list of previously supported Research Schools. Applicants are strongly encouraged to discuss their ideas for Research Schools with the Chair of the Early Career Research Committee Professor Chris Parker (research.schools@lms.ac.uk) before submitting proposals. Proposals should be submitted to Lucy Covington (research.schools@lms.ac.uk) by 22 February 2022.

## ICIAM Olga Taussky-Todd Lecture 2023: Call for Nominations

The Olga Taussky-Todd Lecture is held every four years at the International Congress on Industrial and Applied Mathematics (ICIAM). This honour is conferred on a woman who has made outstanding contributions in applied mathematics and/or scientific computation. The lecture is named in tribute to the memory of Olga Taussky-Todd, whose scientific legacy is in both theoretical and applied mathematics, and whose work exemplifies the qualities to be recognised.

The Officers and Board of ICIAM now call for nominations for the Olga Taussky-Todd Lecture, to be given at the ICIAM 2023 Congress, 20–25 August 2023 in Tokyo, Japan.

A nomination will consist of:

- Full name and address of person nominated.
- Web home page if any.
- Justification for nomination (in at most two pages, cite nominator's reason for considering candidate to be deserving, including explanations of the scientific and practical influence of the candidate's work and publications).
- 2–3 letters of support from experts in the field (not mandatory), each a maximum of two pages.
- CV of the nominee.
- Name and contact details of the proposer.

Contact the ICIAM President, Ya-xiang Yuan (president@iciam.org) with any question regarding the nomination procedure. Nominations should be made electronically via the website iciamprizes.org. The deadline for nominations is 30 December 2021.

# Annual LMS Subscription 2021–22

Members are advised that renewal reminders about their annual subscription for the period November 2021-October 2022, including payment for additional subscriptions, will be sent to them either by email or by post in late September/early October 2021. Annual subscriptions become due on 1 November 2021 and payment should be received by 1 December 2021. Note that payments received after this date may result in a delay in journal subscriptions being renewed.

**New Membership Benefit for 2021/22**

The Society is pleased to announce that from 1 January 2022, all LMS members can access the journal *Mathematika*, free of charge. *Mathematika* is published by the Society on behalf of its owner UCL, and free online access will be available through the Society's publishing partner, Wiley & Sons. *Mathematika* publishes both pure and applied mathematical articles and has done so continuously since its founding by Harold Davenport in the 1950s.

Following feedback from the Society's network of LMS Reps that tiered Ordinary memberships rates would be welcomed by the mathematical community Council has agreed to offer three tiers of Ordinary membership rates from 1 November 2021 onwards. Members who pay the Ordinary membership rate can choose which membership rate they wish to pay based on whether their annual professional income falls within the following ranges:

- Above £65,000 per annum: Ordinary (high) member rate.
- Between £35,000-£65,000 per annum: Ordinary (middle) member rate.
- Up to £35,000 per annum: Ordinary (low) member rate

The Society will not collect any data on members' actual professional income nor require proof of earnings. Instead, Ordinary members are asked to advise either via their online member record or the subscription form which tier of Ordinary membership subscription they will be paying. For members who pay by direct debit, we encourage you to update this information by 14 October 2021.

| LMS Membership Subscription Rates 2021-22 | | |
|---|---|---|
| Ordinary Member (high) rate | £120.00 | US$240.00 |
| Ordinary Member (middle) rate | £98.00 | US$196.00 |
| Ordinary Member (low) rate | £80.00 | US$160.00 |
| Reciprocity rate for members based outside the UK and a member of one of the LMS' Reciprocity partners | £49.00 | US$98.00 |
| *Associate (postdoc) rate for early career members whose PhD completion was more than 3 years ago and who are on a non-permanent contract | £49.00 | US$98.00 |
| Associate rate for PhD student members and members whose PhD was completed in the last three years | £24.50 | US$49.00 |
| *Concessionary rate for members working part-time, unemployed or otherwise in hardship | £24.50 | US$49.00 |
| Associate (Undergraduate) rate for undergraduate student members | £12.25 | US$24.50 |
| Senior rate for members who have paid fees for at least 35 years | £0.00 | US$0.00 |

* These rates are by request and subject to agreement by the Treasurer.

### LMS Member Benefits

Members are reminded that their annual subscription entitles them to a range of benefits, including voting in the LMS Elections (see below), free online access to selected journals for personal use only; a 50% discount on European Mathematical Society (EMS) membership and discounted subscription to the Journal of the EMS, and more. For a full list of member benefits, see lms.ac.uk/membership/member-benefits.

### Renewal and payment

Members can log on to their LMS user account (lms.ac.uk/user) to make changes to their contact details and journal subscriptions, and to make payment either by card via WorldPay or by setting up a direct debit via GoCardless, under the "My LMS Membership" tab. Members can also renew their subscription by completing the subscription form and including a cheque either in GBP or USD. We regret that we do not accept payment by cheques in Euros.

Please email any queries to the LMS membership team (membership@lms.ac.uk).

Elizabeth Fisher
Membership & Grants Manager

## Maximising your LMS Membership: Voting in the LMS Elections 2021

The LMS has around 3,100 members who form a vibrant international mathematical community. The engagement of all its members is essential for the functioning of the Society. One of the best ways to get involved and influence the decisions that the charity makes is to take a part in the annual elections to Council and Nominating Committee that will be held during October and November 2021. The members' active participation in voting keeps the Society's governing body fresh, accountable, and credible; it brings diversity of opinions and gives members a chance to connect with its leadership.

The Society promotes a fair and transparent election that results in the general welfare of the mathematical community and portrays a positive public image of its members. Competitive elections allow the Society to find and form a strong governing body that will be responsible for the general control and management of its administration, strategies and plans and the charity's financial operations.

The slate of candidates for this year's LMS Elections for Council and Nominating Committee can be found at our website tinyurl.com/kktzncfb and all members are invited to an online discussion forum which is available at tinyurl.com/pxvsprkk.

Instructions on how to vote will be sent to members by e-mail or post by Civica Election Services, the organisation that administers the elections, on 8 October 2021, the day voting opens. Members are encouraged to check that their contact details are up to date at lms.ac.uk/user.

The results of the Council and Nominating Committee elections will be announced at the Annual General Meeting that will be held on Friday 12 November 2021 at 3.00 pm.

Valeriya Kolesnykova
Accounts and Membership Assistant, LMS

## LMS Council Diary — A Personal View

Council met via video conference on the morning of Friday 2 July 2021, before the General Meeting and Society Lecture later in the day. The meeting began with the President's update, which included the fact that Professor Alison Etheridge FRS had been appointed as the new Chair of the Council of Mathematical Sciences, and brief reports on the second Council Strategic Retreat, where the Society's budget had been discussed, and a recent meeting of the Presidents of European mathematical societies.

Following a report on the Covid Working Group, Vice-President Gordon introduced a paper on sustainability matters including discussion of whether the Society should be seeking to reduce its environmental impact, the issue of indirect carbon dioxide emissions via travel and investments, and whether sustainability targets should be set. It was agreed, after discussion, that specific recommendations on sustainability would be brought to the next Council meeting.

Council then heard a very interesting presentation from the public affairs company Connect PA, which is engaged to promote the Pure Mathematics campaign. Vice-President Hobbs gave an update on publications contract negotiations, which are ongoing, before the Treasurer briefed Council on the third quarter financial review together with the income and expenditure budget for 2021–22 and planning figures 2022–24, which had been informed by discussions during the two Strategic Retreats of Council that had taken place earlier this year.

We also discussed committee representatives, Society membership, and the recent LMS Representatives Day, where the importance of LMS Regional Meetings and networking opportunities had been discussed. The Education Secretary led a discussion on a draft Code of Practice of the Teaching Mathematics as a Career (TeMAC) subcommittee of the Education Committee, which was approved. This subcommittee seeks to address the shortage of mathematics teachers through both supply and retention.

The meeting concluded with the President thanking everyone for their contributions and noting that he looked forward to seeing Council Members again later in the day for the General Meeting and Society Lecture.

Elaine Crooks
Member-at-Large

## LEVELLING UP

The latest updates about the *Levelling Up: Maths* scheme being developed by the LMS, made possible by a generous donation from Dr Tony Hill. The scheme seeks to widen participation of those who are under-represented in mathematics. It is part of a broader *Levelling Up: STEM* project which also covers Physics and Chemistry.

**Hamilton Commission Report and Levelling Up**

In 2020 Sir Lewis Hamilton and the Royal Academy of Engineering set up the Hamilton Commission, aimed at 'improving the representation of Black people in UK motorsport'. The Commissioners were tasked with 'examining research findings and helping to identify the key challenges and opportunities facing young Black people entering STEM careers, particularly in UK motorsport'. The Commission Report, *Accelerating Change: Improving Representation of Black People in UK Motorsport* was published in July 2021, available at tinyurl.com/ras5xrda.

While the report focuses predominantly on the progression of Black people into engineering and motorsport there are implications for the wider STEM community. The report recommendations can be effective not only in providing routes for a greater number of Black people to enter engineering but more generally through more Black children studying and achieving in STEM subjects, leading to exciting and fulfilling careers in a wide variety of industries. Such opportunities also need to be extended to those from a wider set of minority ethnic backgrounds and other under-represented groups entering into STEM education and employment.

The Commission identified ten recommendations, including:

- the development of best practice guidance for STEM inspiration and outreach activities to enable inclusive engagement with Black students in schools, and with those who influence them.
- Supporting the piloting of new approaches to increase the number of Black teachers in STEM subjects that lead to careers in engineering, namely mathematics, physics, design and technology, and computing.

*Levelling Up: STEM* has been recognised in the Hamilton Commission Report as an important part of the equation in widening participation of under-represented groups in STEM, and the Commission was particularly impressed with the tailored tutoring offered to participating students. The report states:

"A new programme, *Levelling Up: STEM*, sets out to address challenges of academic attainment of young people from low-income families alongside offering mentoring and support around university applications. As with other schemes, the programme provides pastoral support and peer mentoring from students with university applications through universities' 'widening participation' programmes. However, what makes Levelling Up: STEM stand out is the academic tutoring that is put in place specifically designed to improve grades in science and mathematics A-levels."

Professor Jon Keating, President, London Mathematical Society, said: "The Society is delighted to be part of the *Levelling Up* Scheme, and is extremely grateful to Tony Hill for his most generous support. Levelling Up has got off to an excellent start and we are confident it will have a major impact on a problem that is deeply concerning to us. It is particularly encouraging that organisations such as the Institute of Physics are partnering with us, and that the Royal Academy of Engineering has recognised the Scheme's potential."

**Become Part of a Successful Scheme**

The LMS has worked initially with Durham University and the University of Leicester to develop tutorial material for students in under-represented groups in their local areas to help improve grades and enable them to study for a STEM degree. The first cohort of students began tutorials in March 2021.

The Scheme is now established at Durham and Leicester and moving forward the aim of the Scheme is to engage with a wider number of university partners. As noted in the previous Levelling Up update, expressions of interest are currently being sought from a range of universities and applications are welcomed from all universities offering courses in Mathematical Sciences, and in particular from those universities in areas where HE participation is low and where community engagement is strong.

As the *Levelling Up* Scheme aims to increase the diversity of students pursuing STEM degrees and, eventually, careers, it is important that potential university partners can show a commitment to equality, diversity and inclusion. Ideally the Scheme will form part of a university partner's current programme of widening participation with the aim of dismantling the barriers to entry faced by under-represented groups. University partners are being asked to commit to the programme in several ways in terms of providing an 'academic champion' to coordinate the institution's involvement in the scheme, provision of funding of around £500 per student and pastoral engagement and training undergraduate tutors.

It is important that everyone who wants to study mathematics is given the opportunity, regardless of gender, ethnicity or economic status, for the continued health of the subject and for the benefit of society, providing a richer, more diverse mathematical community. The expansion of the Levelling Up: Maths Scheme has the potential to provide such opportunities to a large number of students.

*Working with other Learned Societies on Levelling Up*

As we work towards the aim of bringing in more university partners we are increasingly looking to work closely with other societies. Discussions with the IMA about how we can work together on Levelling Up: Maths are in progress. Dr Nira Chamberlain, President, Institute of Mathematics and its Applications, commented: "I welcome the comprehensive findings of the Hamilton Commission and its endorsement of the Levelling Up: Maths Scheme. I do not wish to underestimate the significant milestone achieved here. However, we also must recognise that this is very much the start of us building a strong foundation to achieve true equal opportunity for all. The IMA is looking forward to joining the work on the Levelling Up scheme."

More information about Levelling Up: STEM is available at levellingupstem.co.uk.

John Johnston
Society Communications Officer

## Report: LMS Popular Lectures 2021

Popular Lecturers Aoife Hunt and James Maynard

On 24 June 2021 the London Mathematical Society hosted its Popular Lectures online for the first time. The audience of nearly 100 enjoyed an evening of mathematics with a journey into how mathematics is used to understand crowd movement and how prime numbers are the 'fundamental building blocks of arithmetic'.

Aoife Hunt from Movement Strategies presented the first lecture, *The Maths of Crowd Behaviour*, looking at how many people can fit into a football stadium or a music festival and how quickly they can all get out. These are important questions for event organisers, and they need mathematicians to answer them. In her presentation Aoife described the mathematics behind crowd flow and how also in the current pandemic the need to look at models to recalculate venue capacities to adhere to social distancing rules has become increasingly important.

Simulation models create a physical environment digitally that provides a test bed to look at how people move. These can be used for example to model large crowds at venues such as Wembley stadium and Glastonbury to ensure the safety and security of the crowds, and also to make sure that those who attend have a positive experience. Flow rates can be calculated simply by the number of people moved divided by time taken and models are used to calculate evacuation times in various settings including buildings and stadiums.

During the covid-19 pandemic social distancing has posed problems for venues and other situations where people need to be spaced further apart. Here circle packing models and Pythagoras' Theorem is used to work out how far people need to be away from each other to optimise seating plans at venues.

The second lecture, *Patterns in Primes*, was presented by James Maynard (University of Oxford) asking the question why anyone should care about primes. He outlined that, although prime numbers appear to be mathematical oddities, they play a key role in both pure and applied mathematics. James also looked at how various different, difficult problems in science lead to seemingly very simple questions about prime numbers and the patterns within the primes.

James described four areas in particular: primes as building blocks where large complicated objects are understood by breaking them down into smaller components; primes being used in the real world applications of internet cryptography where information, e.g. credit card details, are turned into a prime number and encrypted; primes in pure mathematics, where he discussed Fermat's Last Theorem and the proof by Andrew Wiles as well as the work of Sophie Germain. In his last section James discussed how progress can still be made on 100-year old problems, in particular the Prime Number Theorem, the Riemann Hypothesis, and the Twin Prime Conjecture.

The lectures were a striking example of the breadth of mathematics and the audience engaged enthusiastically with the presentations.

The lectures are available on the LMS YouTube Channel at tinyurl.com/mzdn45hn.

John Johnston
LMS Communications Officer

## Report: LMS Midlands Regional Meeting

On Wednesday 2 June 2021, the Charlotte Scott Centre for Algebra, University of Lincoln, hosted the London Mathematical Society Midlands Regional

Meeting as an online event. This meeting was followed by an online workshop. This is the first time such an LMS Regional Meeting has been held at Lincoln, and surely it will not be the last.

The meeting began with the welcome talk by Iain Gordon, Vice President of the LMS. Then, chaired by Evgeny Khukhro, this afternoon event consisted of three talks. The first speaker was Delaram Kahrobaei from the University of York. She spoke on *Cryptographic Multilinear Maps via Pro-p Groups*. After a virtual coffee break with an informal group discussion, Simon Smith from the University of Lincoln spoke on *A Local-to-Global Complement to Bass-Serre Theory*. The third speaker of the day was Alexandre Borovik from the University of Manchester, who spoke on *Black Box Algebra and Homomorphic Encryption*. Each talk was concluded with some interesting questions, and there were also good discussions among the participants.

Lastly but not least, after the official end of the meeting, a poster was presented by Simone Blumer, a PhD student from the University of Milano-Bicocca. His clearly-written poster was on Galois Theory, specifically on examples of groups that may or may not occur as absolute Galois groups of fields, and on certain Galois theoretic conjectures for a family of groups. After the poster presentation, a smaller group of speakers and participants stayed on for the virtual drinks reception.

All in all, it was a very successful meeting, with around 30 participants, over 15 of whom were from outside the United Kingdom. The Midlands Regional Meeting was followed by a 2-day workshop on Profinite Groups and Related Aspects, which was an equally enjoyable event with over 40 participants, and from even more continents. This workshop brought together many aspects of, and relating to, profinite groups, such as finite groups and Lie theory, and the workshop promoted new connections between these aspects.

The speakers for the workshop were Gunnar Traustason (University of Bath), John Wilson (University of Cambridge and University of Leipzig), Nadia Mazza (Lancaster University), Henry Bradford (University of Cambridge), Rachel Camina (University of Cambridge), Pavel Shumyatsky (University of Brasilia), Alejandra Garrido (Autonomous University of Madrid), Anastasia Hadjievangelou (University of Bath) and Colin Reid (University of Newcastle, Australia). The talks from the Regional Meeting also tied in nicely with the workshop talks. More information on the talks delivered can be found on the meeting and workshop website: profinitelincoln2021.wordpress.com/.

Anitha Thillaisundaram
Organiser

## Records of Proceedings at LMS meetings
## Ordinary Meeting: 2 June 2021

This meeting was held virtually on Microsoft Teams, hosted by the University of Lincoln, at the LMS Midlands Regional Meeting. 25 members and visitors were present for the Society meeting session.

The Society meeting began at 1.30 pm BST on 2 June with the LMS Vice-President, Professor Iain Gordon, in the Chair. Professor Gordon welcomed guests and thanked the organising parties. No new LMS members were elected at this meeting.

Due to the online nature of the meeting, no members signed the Member's Book and were admitted to the Society.

Professor Gordon then handed over to Professor Evgeny Khukhro to introduce Professor Delaram Kahrobaei (University of York) who spoke about *Cryptographic Multilinear Map via Pro-p Groups*.

After the break Professor Khukhro introduced a presentation given by Dr Simon Smith (University of Lincoln) on *A Local-to-Global Complement to Bass-Serre Theory*. Professor Khukhro then introduced Professor Alexandre Borovik (University of Manchester) who gave a presentation on *Black Box Algebra and Homomorphic Encryption*.

Professor Gordon concluded the meeting by thanking the speakers, organisers and meeting attendees on behalf of the LMS.

## Records of Proceedings at LMS meetings
## Ordinary Meeting: 24 May 2021

This meeting was held virtually on Zoom, hosted by ICMS, at the LMS Mary Cartwright Lecture. 55 members and visitors were present for the Society meeting session.

The Society meeting began at 2.00 pm BST on 24 May with the LMS Vice-President, Professor Catherine Hobbs, in the Chair. Professor Hobbs welcomed guests and thanked the organising parties. There were no elections to LMS Membership at this meeting.

Due to the online nature of the meeting, no members signed the Members Book and were admitted to the Society.

Professor Hobbs handed over to Dr Eugenie Hunsicker (Chair, LMS Committee for Women and Diversity in Mathematics) and Professor Anne Davis (Chair, LMS Good Practice Scheme Steering Group) to introduce Professor Claudia de Rham (Imperial), who presented her lecture titled *Analyticity in the Sky with (causal) Diamonds*.

Dr Hunsicker (on behalf of LMS Vice-President, Professor Hobbs) thanked the speaker for their excellent lecture and concluded the meeting by thanking the organisers and meeting attendees on behalf of the LMS.

# Topological Graph Theory Through Matroid Theory

IAIN MOFFATT AND STEVEN NOBLE

A standard statement in undergraduate graph theory is that 'deletion and contraction are dual operations'. But this statement is only partially true, and completing it will take us on a route through graphs in surfaces to the world of delta-matroids. Along the way we'll see some of this general setting's advantages and challenges.

## Some classical graph theory

Let's begin with some undergraduate graph theory. Suppose we have a graph $G = (V, E)$ and an edge $e$ of $G$. We can *delete* the edge $e$ by simply removing it. The resulting graph is denoted by $G \backslash e$. We can also *contract* $e$ by removing $e$ and then identifying its endvertices, resulting in a graph denoted by $G/e$.

The operations of deletion and contraction are not entirely independent of each other. A graph $G^*$ is an *abstract dual* of $G$ if it has the same edge set as $G$, and a set of edges in $G$ forms a spanning tree (a tree in $G$ that includes all vertices of $G$) if and only if its complement forms a spanning tree in $G^*$.

Deletion and contraction are dual operations, and related through the beautiful identities

$$(G/e)^* = G^* \backslash e \quad \text{and} \quad (G \backslash e)^* = G^*/e. \qquad (1)$$

However, there is a catch: these identities are not valid for all graphs. In fact the situation is worse: not all graphs have abstract duals!

So in (1) do we have a fundamental graph theoretic result that is not truly a result about graphs? If so, in what setting does it properly reside? To find out there are two routes we can follow: an algebraic route and a topological route.

## Following the topological route

Only some graphs have abstract duals and there is a topological characterisation of those that do: they can be drawn on the plane. A connected graph $\mathbb{G}$ is said to be a *plane graph* if it has been drawn on the plane in such a way that its edges don't intersect, and is *planar* if it admits such a plane drawing.

Plane graphs have another type of dual. If $\mathbb{G}$ is a plane graph, then its *geometric dual* $\mathbb{G}^*$ is the plane graph obtained by placing a vertex in each face of $\mathbb{G}$ and joining two of these vertices by an edge whenever the corresponding faces share an edge in $\mathbb{G}$.

In 1933, H. Whitney proved that a graph has an abstract dual if and only if it is planar. Moreover, the geometric and abstract duals of a plane graph (essentially) coincide. It follows that (1) is valid for planar graphs. Can we drop planarity?

An *embedded graph* $\mathbb{G}$ comprises a graph drawn on *any* closed surface so that its edges don't intersect and its faces are discs (so cutting the surface open along the graph results in a collection of discs). We can form the *geometric dual* $\mathbb{G}^*$ of an embedded graph $\mathbb{G}$ just as we did for plane graphs. We can also delete and contract edges.

If $e$ is an edge of $\mathbb{G}$ then $\mathbb{G} \backslash e$ is formed by removing $e$ from the drawing. This may create redundant handles in the surface. If it does, remove them so the faces remain discs. And $\mathbb{G}/e$ is formed by contracting the edge $e$ to a point which forms a new vertex of the graph. This may result in a pinch-point (a point whose neighbourhood looks like a diabolo). If it does, resolve it by splitting the vertex and the pinch-point in two.

This gives our topological completion of (1):

$$(\mathbb{G}/e)^* = \mathbb{G}^* \backslash e \quad \text{and} \quad (\mathbb{G} \backslash e)^* = \mathbb{G}^*/e, \qquad (2)$$

identities that, unlike in (1), apply to $\mathbb{G}$ without any restriction. Now let's consider the algebraic route.

## Following the algebraic route

Only planar graphs have abstract duals: if $G$ is a non-planar graph, then there is no graph $H$ whose spanning trees are the complements of the spanning trees of $G$. But there is a combinatorial structure having 'spanning tree like' objects that are the complements of the spanning trees of $G$.

The edge sets of the spanning trees of a graph satisfy an *exchange property*: if $T_1$ and $T_2$ are spanning trees of a graph $G$ and $e$ is an edge of $T_1 \backslash T_2$, then there is an edge $f$ of $T_2 \backslash T_1$ so that $T_1 \backslash \{e\} \cup \{f\}$ is also a spanning tree of $G$. This exchange property should look familiar from linear algebra as the bases of a vector space satisfy the same property.

In 1935 Whitney introduced matroids in an attempt to find a combinatorial abstraction of the exchange property. A *matroid* is a pair $(E,\mathscr{B})$ comprising a finite set $E$ and a non-empty collection $\mathscr{B}$ of subsets of $E$, called *bases*, that satisfy the exchange property. So examples of matroids come from graphs and from vector spaces. In the (connected) graph case, $E$ is the edge set of the graph and $\mathscr{B}$ comprises the edge sets of its spanning trees.

Every matroid $M$ has a *dual* $M^*$, formed by taking the complements of all the bases. So a non-planar graph does not have a dual, but its matroid does. Moreover, there are notions of deletion and contraction in matroids, which are consistent with the definitions for graphs: for *all* matroids

$$(M/e)^* = M^*\backslash e \quad \text{and} \quad (M\backslash e)^* = M^*/e, \qquad (3)$$

and we have a second way to complete (1).

## All roads lead to Rome

Starting with graphs we can move to embedded graphs or to matroids. Are these really two different directions, or artefacts of some higher theory?

The key to answering this question is to think of spanning trees in a different way. Take an embedded graph, choose a spanning tree $T$ and imagine standing just to one side of one of its edges. We can walk along the side of this edge until we reach a vertex, continue walking round the side of this vertex to the next edge, then walking along that edge. By continuing in this way, we end up walking around the boundary of the spanning tree and will return to our starting place having visited every vertex. In a plane graph, the only subgraphs for which this is possible are the spanning trees, but for embedded graphs in general, there will be other subgraphs with this property. These subgraphs are the *spanning quasi-trees* of an embedded graph and they satisfy a modified version of the exchange property.

The pair $D(\mathbb{G}) := (E,\mathcal{F})$, where $E$ is the edge set of $\mathbb{G}$ and $\mathcal{F}$ the edge sets of its spanning quasi-trees then gives an abstraction of an embedded graph that turns out to be an example of a *delta-matroid* (see "The definition of a delta-matroid").

As spanning trees are just plane spanning quasi-trees, delta-matroids arise through the topological route of dropping the planarity condition when forming the matroid of a graph. There's also an algebraic route to delta-matroids: it's not hard to see that a matroid is a delta-matroid in which every set in $\mathcal{F}$ has the same size. Delta-matroids arise by loosening the requirements so that bases are no longer forced to have the same size. In fact, one can use Euler's formula to show that these two approaches agree.

## Compatibility and minors

Many results in graph theory, not just the duality result in (1), turn out to be special cases of results in matroid theory. This is beneficial in two ways. First, graph theory can serve as an excellent guide for studying matroids. W. Tutte famously observed that, "if a theorem about graphs can be expressed in terms of edges and circuits alone it probably exemplifies a more general theorem about matroids." Second, insights from matroid theory can lead to new results about graphs. Both areas have benefited from this relationship (see [2] for some examples).

A similar relationship, proposed in [1], holds between topological graph theory and delta-matroid theory. Such a relationship is hinted at by observing that basic parameters associated with each type of object agree. For example a delta-matroid $D = (E,\mathcal{F})$ is said to be *even* if the sizes of the sets in $\mathcal{F}$ all have the same parity. A delta-matroid being even corresponds to an embedded graph being orientable. Similarly the genus of an embedded graph is given by the difference between the sizes of the largest and smallest sets in $\mathcal{F}$.

### The definition of a delta-matroid

A *delta-matroid*, $D = (E,\mathcal{F})$, is a pair consisting of a set $E$, and a non-empty collection $\mathcal{F}$ of its subsets that satisfies the *Symmetric Exchange Axiom*:

$(\forall\, X,Y \in \mathcal{F})\ (\forall\, u \in X \triangle Y)\ (\exists\, v \in X \triangle Y)$ such that $(X \triangle \{u,v\} \in \mathcal{F})$.

Here $X \triangle Y := (X \cup Y)\backslash(X \cap Y)$ is the symmetric difference of sets.

Three groups introduced Delta-matroids independently in the 1980's: A. Bouchet in 1987; R. Chandrasekaran and S. Kabadi in 1988, under the name of pseudo-matroids; and A. Dress and T. Havel in 1986, under the name of metroids. Each group had a different motivation.

Possibly the most important relations come from deletion, contraction and duality. These may be extended from matroids to delta-matroids in a way that is consistent with their meaning in embedded graphs. In other words, if $D(\mathbb{G})$ denotes the delta-matroid associated with an embedded graph $\mathbb{G}$, then for every edge $e$ of $\mathbb{G}$,

$$D(\mathbb{G}/e) = D(\mathbb{G})/e, \quad D(\mathbb{G}\backslash e) = D(\mathbb{G})\backslash e$$

and

$$D(\mathbb{G}^*) = D(\mathbb{G})^*.$$

This last duality relation can be strengthened. Rather surprisingly, there is a way to form the geometric dual $\mathbb{G}^*$ of an embedded graph one edge at a time. This leads to the *partial duals* $\mathbb{G}^A$ of $\mathbb{G}$ in which the geometric dual of $\mathbb{G}$ is only formed with respect to a subset $A$ of edges of $\mathbb{G}$ (so $\mathbb{G}^E = \mathbb{G}^*$ and $\mathbb{G}^\emptyset = \mathbb{G}$). This unexpected extension of duality arose from knot theory and is due to S. Chmutov, appearing in 2009.

On the other hand, in 1987 A. Bouchet introduced a fundamental operation on a delta-matroid called a *twist*. Given a delta-matroid $D = (E, \mathcal{F})$ and a subset $A$ of $E$, the twist $D * A$ is formed by replacing each set in $\mathcal{F}$ with its symmetric difference with $A$. Since $D * E = D^*$ a twist can be considered as a 'partial dual' of a delta-matroid. Again the two concepts of partial duals align: $D(\mathbb{G}^A) = D(\mathbb{G}) * A$.

The identities above enable the use of geometric insights from embedded graphs to study delta-matroids and vice versa. Below we give an illustration of how bouncing between the areas can advance them both. Our examples revolve around the concept of minors, which is key in structural graph and matroid theory. If $D'$ can be obtained from $D$ by a sequence of deletions and contractions, then $D'$ is called a *minor* of $D$. Minors for graphs and embedded graphs are defined similarly, but isolated vertices may also be deleted.

But before going further let's pause to address the natural question of whether embedded graphs and delta-matroids are just two descriptions of the same thing. Although every embedded graph gives rise to a delta-matroid, most delta-matroids do not arise in this way. Furthermore, it is possible that different embedded graphs give rise to the same delta-matroid. This mismatch between the areas is well understood, but it does mean that care must be taken when moving between them. Results in one area may not directly give results in the other, rather, as we will see, they point you in the right direction.

### The Tutte polynomial

Many graph parameters satisfy recursive *deletion–contraction relations* (i.e., linear relations involving $G$, $G\backslash e$, and $G/e$). For example, if $a(G)$ is the number of ways to direct the edges of a graph so that it contains no directed cycles, then $a(G) = a(G\backslash e) + a(G/e)$. The *Tutte polynomial*, $T$, is a function from graphs to $\mathbb{Z}[x,y]$. It associates a 2-variable polynomial $T(G;x,y)$ to a graph $G$. It is a universal deletion–contraction invariant in the sense that any graph parameter with deletion–contraction relations can be obtained from it (for example, $a(G) = T(G;2,0)$). Because of this it has applications in a diverse variety of topics including codes, network reliability, chip-firing and the sandpile model, knot theory, and statistical physics.

The Tutte polynomial can be defined through a recursive deletion–contraction relation. A consequence of this and (1) is that it satisfies a duality relation,

$$T(G;x,y) = T(G^*;y,x), \tag{4}$$

where $G^*$ is the abstract dual. This identity is surprisingly important in the theory of the Tutte polynomial. But here, we shall use it as another springboard into delta-matroids.

### Two extensions of the Tutte polynomial

Once again, since not all graphs have abstract duals, in (4) we find ourselves in the situation where we have a fundamental graph theoretic identity that does not apply to all graphs. But once again, we can complete the result by taking either an algebraic route or a topological route.

For the algebraic route, the Tutte polynomial can be extended to matroids (as was done by H. Crapo in 1969), and the identity $T(M;x,y) = T(M^*;y,x)$ holds for *all* matroids. The definition of the Tutte polynomial of a matroid is, more or less, a word for word lifting of its definition for graphs. Such an approach does not work in the topological setting.

Finding a version of the Tutte polynomial for embedded graphs is a long-standing problem and one that is not entirely settled. The situation is surprisingly subtle, several candidates have been proposed, and it's not completely clear what the correct definition should be. Some of the latest research shows that there is a family of Tutte polynomials for embedded graphs, each arising from a slightly different notion of an 'embedded graph'. Moreover, all of the previously defined candidate polynomials can be recovered from members of this family.

In this family the 'Tutte polynomial' for the type of embedded graph we consider here is a two-variable polynomial $R(\mathbb{G}; x, y)$, commonly called the *ribbon graph polynomial*. It is a specialisation of a well-known polynomial of B. Bollobás and O. Riordan from 2001.

The ribbon graph polynomial has many properties analogous to those of the Tutte polynomial. Like the Tutte polynomial, it has a recursive deletion–contraction definition, so from (2) we can deduce the topological analogue of (4): for *any* embedded graph $\mathbb{G}$ we have $R(\mathbb{G}; x, y) = R(\mathbb{G}^*; y, x)$.

### Completing the polynomial picture

Again the topological and algebraic routes have taken us to two different generalisations and we are left asking if we can complete the picture by showing that they both arise from a common object. Again we can answer this by appealing to delta-matroids.

The above relationships between matroids, embedded graphs and delta-matroids enable us to translate between the three settings. Applying this to the graph polynomials results in a *Tutte polynomial for delta-matroids*, denoted by $R(D; x, y)$ for a delta-matroid $D$. Again this has a recursive deletion–contraction definition, and $R(D; x, y) = R(D^*; y, x)$. And again, since the polynomials coincide when $D$ comes from an embedded graph or is a matroid, our two approaches are united by delta-matroids. Let's push the delta-matroid theory to see what it tells us about our various polynomials.

### Connectivity and separability

Take two connected graphs, pick a vertex of each and merge these vertices together to form a single larger graph. Graphs arising in this way are called *separable*. The new graph is connected, but in a sense its matroid is not. Taking the union of spanning trees of the original graphs yields a spanning tree of the new graph and every spanning tree of the new graph arises in this way. Intuitively, the two parts of the matroid corresponding to the original graphs have no influence on each other. More generally we say that a matroid $M$ is the direct sum of matroids $M_1$ and $M_2$ (with disjoint sets of elements) if the bases of $M$ are precisely the sets that may be formed by taking the union of a basis of $M_1$ and a basis of $M_2$. Matroids arising from a non-trivial direct sum are also called *separable*. These ideas extend *mutatis mutandis* to the quasi-trees of an embedded graph and to delta-matroids.

### Splitter theorems and inductive tools

Chain and splitter theorems are useful tools in inductive proofs in structural matroid theory. Chain theorems allow us to delete or contract various elements of a matroid without reducing the connectivity; splitter theorems tell is that if $N$ is a connected minor of a connected matroid $M$, then we can move from $M$ to $N$ by deleting and contracting elements without reducing the connectivity. All the key concepts extend from matroids to delta-matroids suggesting that these results may be more widely applicable to delta-matroids and indeed they are.

Bouchet proved if $D$ is an even, non-separable delta-matroid with element $e$, then either $D/e$ or $D\backslash e$ is non-separable. As a corollary one obtains that if $G$ is embedded in an orientable surface, then for each edge $e$ of $G$, either $G/e$ or $G\backslash e$ is non-separable.

By extending Bouchet's result to a larger class of delta-matroids, in 2017 C. Chun, D. Chun and S. Noble extended the corollary to all embedded graphs and also proved that if $D$ is a non-separable even delta-matroid with non-separable minor $D$, then for every $e \in E(D) - E(D')$ either $D/e$ or $D\backslash e$ is non-separable and contains $D'$ as a minor.

### Irreducible graph polynomials

The Tutte polynomial of a separable graph factorizes into the product of the Tutte polynomials of its constituent graphs. That this is really an artefact of the separability of the corresponding matroid was confirmed by C. Merino, A. de Mier and M. Noy who proved in 2001 that a matroid is non-separable if and only if its Tutte polynomial is irreducible. Again, all of the key concepts in this result have counterparts in embedded graphs and delta-matroids and we can prove that the separability of an even delta-matroid corresponds to the irreducibility of its Tutte polynomial.

A key step in the proof involves using the chain theorem to show that if $D$ is an even delta-matroid with at least two elements, then the coefficient of $x$ in $R(D)$ is non-zero if and only if $D$ is non-separable. One direction follows easily from the facts that the constant term of $R(D)$ is zero and that $R(D_1 \oplus D_2) = R(D_1)R(D_2)$. The polynomial $R(D)$ has a recursive definition akin to that of the Tutte polynomial, but unlike the Tutte polynomial, some of the coefficients may be negative. However, one can show that in the recursive definition of $R$ no cancellation involving the coefficient of $x$ occurs. But if $D$ is non-separable, then at least one of $D/e$ and $D\backslash e$ is non-separable, giving a simple inductive proof.

### Series-parallel graphs

Using the splitter theorem, we can say more about the coefficient of $x$ in $R(D)$, generalizing another result on the Tutte polynomial of a graph. A plane graph is *series-parallel* if it can be built by starting from a cycle with two edges and then repeatedly subdividing or adding an edge in parallel with an existing edge.

T. Brylawski proved that $G$ is series-parallel if and only if the coefficient of $x$ in $T(G)$ is one. Again, this is really a result about embedded graphs and delta-matroids. We say that an embedded graph is series-parallel if it is the partial dual of a plane series-parallel graph and that a delta-matroid is series-parallel if it is the delta-matroid of a series-parallel embedded graph.

We can prove that an even delta-matroid $D$ is series-parallel if and only if the coefficient of $x$ in $R(D)$ is $\pm 1$. In one direction this follows from the fact that $D$ contains an element $e$ so that one of $D/e$ and $D\backslash e$ is separable and the other is series-parallel. The other direction follows from the splitter theorem, because any even delta-matroid that is not series-parallel has a minor belonging to a small class of specific delta-matroids. For each of these the coefficient of $x$ in $R(D)$ is $\pm 2$.

### Is this the whole picture?

We started this article with the partial results about graphs given by (1) and argued that its incompleteness is an artefact of a higher theory. We then illustrated the benefits of the resulting relationship between topological graph theory and delta-matroid theory, showing that each area can be advanced by bouncing between the two. (Our example considered the Tutte polynomial, but that's not too relevant. We could have chosen a different one.) However, an astute reader may have noticed that we also ended the article with examples that were themselves partial results — they were restricted to even delta-matroids, or, correspondingly, orientable surfaces.

It turns out these partial results are themselves artefacts of a higher theory. Completing them requires a move into objects called multi-matroids. At this level of generality, the hierarchy seems to stop. But the picture is more complicated than it first appears. Some results in topological graph theory seem to genuinely reside in the world of delta-matroids, while others belong to multi-matroids. We do not yet understand why and when this split happens, and understanding it and its implications in topological graph theory is an active area of research.

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# Higher-Dimensional Dominoes and $k$-Rank Graphs

SAM A. MUTTER

Higher-rank graphs generalise graphs in a way that is well-suited for applications in analysis and K-theory. They are traditionally categorical objects, but we use the geometry of dominoes to cook up new examples. Viewing higher-rank graphs geometrically has opened new connections to physics and statistical mechanics.

### Higher-rank graphs

A *directed graph* is a set of vertices and a set of arrows between them. A *path* (of length $n$) in a directed graph is a list of arrows $e_1 e_2 \cdots e_n$ where the target vertex of $e_k$ is the same as the source vertex of $e_{k+1}$. We can learn everything about the properties of a directed graph by considering all possible paths around it: paths of length $1$ are just the arrows, and paths of length $0$ can be thought of as the vertices.

Now, let $G$ be a $k$-*coloured graph*, that is, a directed graph where each arrow is assigned one of $k$ colours. We can list all the possible paths around $G$ as before, but instead of writing down the length of a path, we tally up the number of times each colour appears in it. For example, in the $2$-coloured graph

$$\begin{tikzcd} v_0 \arrow[r] \arrow[dr, dashed] & v_1 \arrow[r] \arrow[dr, dashed] & v_2 \arrow[r] \arrow[dr, dashed] & v_3 \arrow[dr, dashed] & \\ & v_4 \arrow[r] & v_5 \arrow[r] & v_6 \arrow[r] & v_7 \end{tikzcd}$$

the path $v_0 \to v_1 \dashrightarrow v_5 \to v_6 \to v_7$ comprises three pink and one blue (dashed) arrows. By assigning the degree $(1,0)$ to pink arrows and $(0,1)$ to blue arrows, we can say that this path has *length* $(3,1)$. In this particular graph, it turns out that any path from $v_0$ to $v_7$ will have length $(3,1)$. In a general $k$-coloured graph, each path will have some length in $\mathbb{N}^k$.

In the above graph, the paths $v_0 \to v_1 \dashrightarrow v_5$ and $v_0 \dashrightarrow v_4 \to v_5$ form a *square*: a pair of $2$-coloured paths with two arrows of opposite colouring, which join the same two vertices. We call a general $k$-coloured graph $G$ a $k$-*rank graph* if we can find a set of squares for it such that:

**G1:** If $u \to v \dashrightarrow w$ is a $2$-coloured path in $G$, then there is a unique path $u \dashrightarrow v' \to w$ with opposite colouring which completes a square, and

**G2:** Whenever $v_0 \to v_1 \dashrightarrow v_2 \dashrightarrow v_3$ is a $3$-coloured path in $G$, there is a uniquely-determined set of six squares which fit together as in Figure 1.

Our example above with $k = 2$ has the properties of a $2$-rank graph. **G1** says that, whenever we have a pink–blue path, we can assign it a unique blue–pink path between the same two vertices, and vice versa. **G2** is only valid for $k \geq 3$.

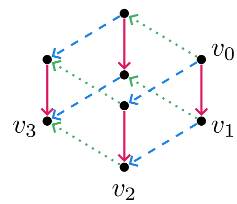


Figure 1. This path from $v_0$ to $v_3$ determines all the other paths.

Kumjian & Pask first studied $k$-rank graphs as a means to generate new and interesting C*-algebras [4]. The graphs were originally defined using the language of categories, but since then (e.g. in [1]) there has been interest in a geometrical approach, something which is totally natural when we deal with ordinary directed graphs. Indeed, a directed graph can be thought of as a $1$-rank graph.

### What can $k$-rank graphs do?

Though it's usually considered taboo to ask a pure mathematician why their field is 'useful', it's always nice to attempt providing context for new research.

In this case, it's worth knowing that we can associate to each $k$-rank graph a *graph C*-algebra*, generated by functions labelled by all the paths in the graph. In certain situations (including ours!), these algebras have very nice properties which makes them eligible for *Kirchberg–Phillips Classification*. This means that there is an invariant (K-theory) which distinguishes the algebras, and hence the graphs, from each other. Playing with this invariant gives us insight into how the algebras are related to the geometry of the graphs, thus building new bridges between the fields of K-theory and geometry.

### Higher-dimensional dominoes

Let $A = \{a_1, a_2, \ldots, a_m\}$ and $B = \{b_1, b_2, \ldots, b_n\}$ be sets of *labels*. A 2*-D domino* is a square whose horizontal edges are labelled by some element of $A$, and whose vertical edges are labelled by some element of $B$. We say that a domino $Y$ is *vertically-adjacent* to a domino $X$ if, without spinning or flipping either of them, $Y$ can be stacked on top of $X$ (Figure 2). Likewise, $Y$ is *horizontally-adjacent* to $X$ if it can be stacked to its right.

Likewise, a 3*-D domino* is a cube with parallel edges labelled from one of three sets, and two dominoes can be adjacent in three directions (on top of, to the right of, and behind, for example). In general, a $k$*-D domino* is a $k$-dimensional cube with edges labelled from $k$ sets. We'll focus on low dimensions here to make drawing pictures easier.

Suppose we have a bunch of 2-D dominoes: we'll call this set $\mathscr{V}$. We can make a 2-coloured graph using $\mathscr{V}$ as the vertices by drawing a pink arrow from $X \in \mathscr{V}$ to $Y \in \mathscr{V}$ whenever $Y$ is horizontally-adjacent to $X$, and a blue arrow if they are vertically-adjacent.

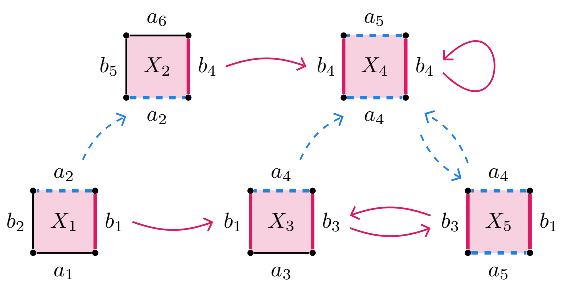


Figure 2. A 2-coloured graph of dominoes: $X_4$ is adjacent vertically to $X_3$ and $X_5$, and horizontally to $X_2$ and itself.

For the example in Figure 2 to be a 2-rank graph, we only need condition **G1** to be satisfied, since **G2** only makes sense with three or more colours. So, every path $X \rightarrow Y \dashrightarrow Z$ has to determine a *unique* path $X \dashrightarrow Y' \rightarrow Z$, and vice versa. The path $X_1 \rightarrow X_3 \dashrightarrow X_4$ completes a square with $X_1 \dashrightarrow X_2 \rightarrow X_4$ for example, but to get from $X_4$ to $X_3$ there is a blue–pink path via $X_5$, but no corresponding pink–blue one. Hence this set of 2-D dominoes doesn't give us a 2-rank graph.

It's a fun, and not altogether difficult, challenge to build a set of dominoes which have the **G1** property, but once we increase the dimension and bring **G2** into play, it becomes much harder to find dominoes which obey the rules of $k$-rank graphs (c.f. [2]). One method involves looking at the *corners* of the dominoes.

A $k$-dimensional domino has $2^k$ corners, each incident to $k$ edges: one edge from each labelling set. We can encode any corner by writing a $k$-tuple which lists the edges incident to it. We have to take care to fix a basepoint and orientation for each domino at this point—this can be decided arbitrarily but must remain consistent across all the dominoes. In two dimensions, it's most convenient to read labels anti-clockwise and starting from the south-west corner, e.g. $X_2$ from Figure 2 has corners $(b_5, a_2)$, $(a_2, b_4)$, $(b_4, a_6)$, $(a_6, b_5)$. Once we've fixed our dominoes' orientations we can talk of the possible *positions* of corners, e.g. the corners occurring in the north-east position in Figure 2 are $(b_1, a_2)$, $(b_4, a_6)$, $(b_3, a_4)$, etc. Names of positions could get very inventive when the dimensions increase. Now we have a theorem:

#### Theorem

Suppose $A_1, A_2, \ldots, A_k$ are sets of labels, each with even size. Let $\mathscr{V}$ be a set of $k$-dimensional dominoes with the property that each possible tuple $(x_1, x_2, \ldots x_k)$, where $x_i \in A_i$, occurs exactly once in each of the $2^k$ possible positions, somewhere in the set of dominoes. Then the adjacency graph of the set $\mathscr{V}$ induces a $k$-rank graph.

In other words, if each possible corner occurs exactly once in each possible position, then the set of dominoes gives us a graph with properties **G1** and **G2**. In [5], an algorithm for finding such sets of dominoes in arbitrary dimension was written. We've turned this into a Python program which takes as an input $k$ lists of labels, and outputs a $k$-rank graph made of dominoes with those labels. Now we have an exciting new (infinite) pool of $k$-rank graphs to play in, and have already computed some K-theoretical facts about their *graph C*-algebras*, which have a more intricate structure than the graphs themselves.

It's worth noting that the above theorem is not the only criterion for designing dominoes with the $k$-rank graph properties—there are sets of 2-D dominoes which arise from so-called *tile complexes*—but for most constructions, fulfilling condition **G2** proves rather formidable whenever we try to generalise to dimensions above 2.

Another quirk of using dominoes is the ability to study the cube complex $\Delta(\mathscr{V})$ formed by gluing all adjacent dominoes from a set $\mathscr{V}$ together along their common faces. Usually, a domino will be adjacent to more than one other domino, so these complexes will have lots of branching and self-intersection, making them impossible to embed happily in Euclidean space (try doing this with the example in Figure 2). In fact it is this branching business which is most interesting from a geometer's point of view: it is characteristic of a *building*, resulting as the quotient of an action on the product of $k$ infinite trees.

### The domino group

The size of each labelling set $A_i$ needs to be even, because we want to match up pairs of elements in order to define the following group.

Suppose $A_1, \ldots, A_k$ are sets of labels of even size, labelling a set of dominoes $\mathscr{V}$ which induces a $k$-rank graph. Suppose that each set is endowed with a fixed-point-free involution which we'll denote by $a \mapsto \overline{a}$.

We define the *domino group* $\Gamma(\mathscr{V})$ to be the group with presentation:

$$\langle A_1, \ldots, A_k \mid a\overline{a} = 1, x_1y_1x_2y_2 = 1 \rangle,$$

whenever $x_1, y_1, x_2, y_2$ labels the boundary of some square (2-cell) in the cube complex $\Delta(\mathscr{V})$, i.e. a square which appears as the $2$-face of some domino.

Indeed, suppose that $T(m)$ is a regular tree of valency $m$, that is, an undirected graph with no cycles, and whose vertices each have exactly $m$ adjacent edges. If $A_1, \ldots A_k$, of respective size $2m_1, \ldots, 2m_k$, label a set of dominoes $\mathscr{V}$, then the *domino group* $\Gamma(\mathscr{V})$ acts on the Cartesian product of trees $T := T(2m_1) \times \cdots \times T(2m_k)$. The quotient of $T$ by this action is precisely the cube complex $\Delta(\mathscr{V})$ formed by gluing together the dominoes, and is a $k$-rank affine building. The building $\Delta(\mathscr{V})$ is *thick* (has branching) whenever $|A_i| > 2$ for all $i$, that is, whenever at least two dominoes are adjacent to any given domino in each direction.

In two dimensions, the complexes $\Delta$ have had their geometry pretty well mapped out. The groups $\Gamma$ which act on the products of trees have been described as *Burger–Mozes(–Wise) groups*, and the sets of dominoes variously as *VH-data* or *VH-complexes*, coming from the *Vertical–Horizontal* axes of adjacency.

Certain geometrical invariants (the *homology groups*) of the $2$-D domino buildings have been directly linked to the anatomy of the C*-algebras, by means of their K-theory (a good place for the interested reader to start is [3]). We're currently working on analogous results in higher-dimensions. Since higher-rank graph C*-algebras have connections to inverse semigroups, Thompson's groups, braid groups and elsewhere, finding some easier ways of characterising them would indeed open a lot of doors. Moreover, sets of $k$-D dominoes have recently been used to find solutions to the *Yang–Baxter Equation*, which pops up in fields such as statistical mechanics and knot theory, so this work could advance understanding in these areas as well.

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Sam is a PhD student at Newcastle University, UK. He mainly works in topology and geometric group theory, with special interests in cube complexes and buildings (the maths kind). He was awarded the Newcastle PG Prize for Outstanding Research in Pure Mathematics, and Student Talk Prize at the LMS Graduate Student Meeting in 2021. Outside of maths, he also likes learning languages and eating.

# Happy Birthday, Arthur Cayley

TONY CRILLY

Tony Crilly reflects on the life and works of the prolific mathematician Arthur Cayley.

16 August 2021 marks the 200th birthday of Arthur Cayley (1821–1895). A staunch supporter of the London Mathematical Society, he was its third President (following Augustus De Morgan (1806–1871) and James Joseph Sylvester (1814–1897)) and the first recipient of the Society's De Morgan Medal. Cayley was the foremost British pure mathematician of the Victorian age.

Cayley's childhood was spent in pre-Revolutionary Russia, where his father, a British merchant, operated in St Petersburg, a city then described as 'the City of London in miniature'. When the family returned to England in around 1830 they were returning to a country undergoing rapid social change. Industrialisation was well under way, and the Great Reform Act of 1832 went some way to widening the suffrage.

Education remained the privilege of the upper and middle classes, and London was provided with privately funded schools. Cayley went to a clergyman's school in Blackheath (the same one attended by Benjamin Disraeli), and from there to King's College London for his secondary school education. The Anglican-sponsored King's in the Strand was founded by Royal Charter in 1829 as a counterweight to the secular University College London in Gower Street. Ahead of the normal age of entry Cayley was enrolled in 1835 in the senior department, and this schooling was followed by Cambridge University. In the nineteenth century Cambridge was a beacon for the mathematically inclined student on account of its famed Mathematical Tripos; Cayley entered Trinity College in 1838, the year after Victoria became Queen.

At Trinity, mathematics was gaining ground as a living subject, one ripe for new developments. With the energy and foresight of Scots Archibald Smith and Duncan Farquharson Gregory, the Cambridge Mathematical Journal was launched at Trinity in 1837, and Cayley was a beneficiary of this fresh spirit for research.

Arthur Cayley's graduation portrait (1842)

Cayley's coming of age in mathematics came through a paper published in the Cambridge journal in May 1841 while he was only a third-year undergraduate. Authored by a 'Correspondent' and merely signed 'C', it has since become significant in the history of mathematics. In it he devised a spatial notation for determinants with the now-familiar vertical lines, and he added novelty with his use of determinants to express relationships between arbitrary points in Euclidean space. It was a new treatment of an old problem, but the geometer Julian Coolidge noted that Cayley had lighted on a 'singularly fruitful bough.' The contribution has since been recognised as a foundation of 'distance geometry', a subject revisited by Karl Menger ninety years later.

Cayley's teacher at Cambridge was the famed coach William Hopkins. Hopkins suggested an exercise in potential theory to Cayley, which furnished the material for his second paper, written out before he took the January Senate House examination and published soon after. In this, Cayley had found inspiration from Lagrange's *Mécanique Analytique* (1811).

From the 1842 Tripos Examination Cayley emerged as the Senior Wrangler, a position for which the annual champion had demonstrated they could work at speed on a wide range of topics, both pure and applied. The group in the early 1840s actually distinguished themselves and were not just clever schoolboys. They went on to scientific achievement — Robert Leslie Ellis (1840), G. G. Stokes (1841), and John Couch Adams (1843). Some students accepted the rigours of the examination system better than others. Ellis hated the relentless examinations and the way it inculcated competition but Cayley adapted himself to 'the great writing race', as De Morgan called it. Heading off from Cambridge for the long vacation in 1839, Ellis wrote in his diary: "Off at ten. Cayley in the coach — the great man of the freshmen. He has my pity — yet probably needs none".

In the historical literature Cayley's name is frequently paired with James Joseph Sylvester, who had graduated from Cambridge a few years earlier. At the time of Cayley's first paper, Sylvester was the Professor of Natural Philosophy at University College London. Sylvester lent his support for the new journal with a paper also on determinants but one applied to algebra. This appeared in the same issue, just next to Cayley's, though it appears they were not acquainted with each other at this stage. The London-Cambridge axis actually featured in each of their lives. Sylvester was writing from 22 Doughty Street (Charles Dickens lived at number 48), a short step from the present day De Morgan House and within the same geographical compass of the Cayley family home in York Terrace, Regent's Park.

In the long summer vacation, Cambridge dons frequently took groups of students on country retreats for extra tuition. On graduation in 1842, Cayley took part in one to Scotland. Francis Galton was a member and pronounced Cayley a 'brick', that being high praise for a person who conveyed reliability and solidity. From Galton's account and letters, Cayley took more to tramping over Rannoch Moor than teaching logarithms to unreceptive students. In May and November of 1842 he contributed further papers to the Cambridge journal, and began to take an active interest in its running. He advised the editor on the suitability of submitted articles in pure mathematics, and this activity gave him another avenue for making contact with mathematicians. In this capacity he discovered the Rev. Thomas Kirkman, and generously helped him to progress in mathematics.

On return from Scotland there was preparation for a further batch of written examinations at Trinity, this time in the competition for Fellowships. Cayley was elected at the first attempt and being on the Foundation, even as a minor Fellow, he enjoyed a share of the annual Trinity Dividend. During 1843 he visited Switzerland (for mountain climbing) and Scandanavia (for hiking), and with the Napoleonic Wars retreating into the historical distance, he was one of the new breed networking with European mathematicians. He led Cambridge out of hibernation by publishing work in both Liouville's and Crelle's journals — and was once described as overpowering the editor August Crelle with 'oceans of symbols' (in the mid 1850s he monopolised volume 50 of this journal with seven papers).

Inspired by George Boole in Lincoln, Cayley took up invariant theory with alacrity, and discovering the subject wrote to Boole that 'it really appears inexhaustible.' He expressed the wish to visit him and 'should so much enjoy talking over some things with you not to mention the temptation of your Cathedral. I think I must contrive it some time in the next six months, in spite of there being no railroad, which one begins to consider oneself entitled to in these days.' Cayley lived and breathed mathematics; Boole wrote of him being 'the most favoured of the Nymph Mathesis.'

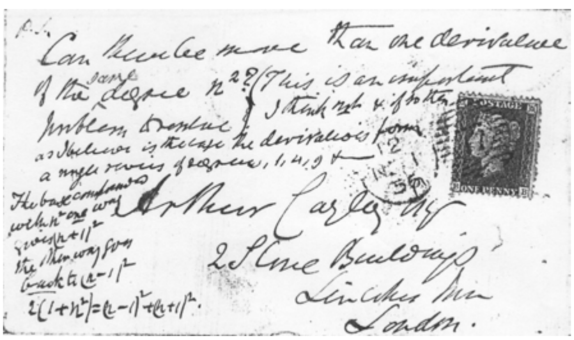
5 Montpelier Row, Blackheath: the Cayley family home 1855–75

In 1846 Cayley entered the Lincoln's Inn legal chambers of Jonathan Christie, a man who held the dubious distinction of being the last man to fight a duel on English soil. Christie allowed Cayley full rein to pursue mathematics and rather enjoyed having such a celebrity as the Senior Wrangler on his books. In 1849 Cayley was called to the bar.

Now Cayley resided more in London. In 1851 the Great Exhibition took place in Hyde Park, a great showplace for the products of the world and demonstrating Britain's place in it. In 1852, Cayley's Trinity Fellowship expired but in the metropolis his career in mathematics was on an upward spiral. In that year he was elected FRS and by the following year was on the Council of the Royal Society. In 1857 he was elected a member of the Royal Astronomical Society, yet another haven for mathematicians, which also included Augustus De Morgan as a member. In this institution Cayley brought his legal skills to its service by drafting a new constitution.

Cayley's work in the law consisted in drafting legal documents. While he did not neglect his legal duties (he contributed to Davidson's *Precedents*, a collection of legal templates), he could choose which legal briefs to accept. During this period he sustained a remarkable publishing record in mathematics.

In mathematics, the rapidity of Cayley's responses is a distinguishing mark of his style of working. After the discovery of quaternions by Sir W. R. Hamilton on 16 October 1843, Cayley quickly made a contribution on quaternions treated as rotations, and this led to a sequence of papers on skew symmetric determinants. His initial quaternion paper was quickly followed by his discovery of the 'Cayley numbers'. This discovery was much to the chagrin of John Graves, who had the same idea — but did not publish immediately. When Kirkman's 'school girl problem' became known in London in 1850, it was Cayley who first published a solution, and one distinct from all the others.




A letter from Sylvester to Cayley, 1856. Sylvester issues a query and Cayley's calculation follows

From the late 1840s Sylvester and Cayley were united in promulgating invariant theory and this began their lifelong quest together. Cayley wrote from '2 Stone Buildings' in Lincoln's Inn with Sylvester nearby in Lincoln's Inn Fields, but their full cooperation was hampered by 'ownership' a notion derived from the prized idea of individuality and being the 'first'. No jointly written papers are in evidence, and there was demarcation on what each did separately.

In the 1850s Cayley set in train the work that established his reputation, notably the *Ten Memoirs on Quantics* (1854–1878), quantics being his neologism for 'algebraic forms'. The long Cartesian expressions for invariants and covariants were what Cayley understood and he avoided shortened forms. In geometric applications, variables of the algebraic forms referred to coordinates. In this vein he was an opponent of quaternions as a means of treating geometry — they were a pocket map, 'a capital thing to put in one's pocket', which must be unfolded (to an algebraic form) to be properly understood. Thus Cayley's calculations are expressed as extensive algebraic forms. In this and other work (on elliptic functions, for example) he was an outstanding calculator, and though he used human calculators when necessary, he never shied away from long handmade calculations himself.

Cayley's papers take various forms, ranging from 'bulletins from the front' to pieces for the *Educational Times*, and from Encyclopedia articles to the composition of numerical tables. He also adopted the strategy of setting up connected series of papers. He is nowadays known for his trilogy of papers in the 1850s on group theory, now seen as a first contribution to abstract group theory in which he combined the notion of a permutation group with the calculus of operations. Many see his series on matrix algebra (1858) as founding the theory of matrix algebra. In writing to Sylvester, he momentarily forgot his natural restraint on discovering the 'Cayley–Hamilton Theorem'.

Indeed, Cayley is most widely known for not proving the theorem but merely verifying it in the cases of $2 \times 2$ and $3 \times 3$ matrices. The importance for him lay in the theorem's discovery. It is not widely known that Cayley posited a more general theorem in terms of 'binary quantics' with which he was familiar — that if $f(x,y) = \det(xA - yB)$ then $f(B,A) = 0$ if $AB = BA$. Yet Cayley did not publish it.

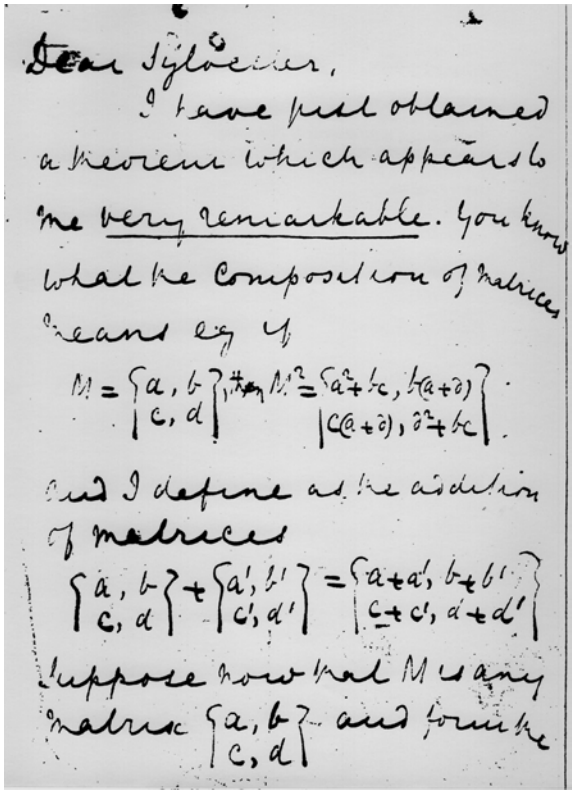
In 1863 Cayley was elected as the Sadleirian Professor of Pure Mathematics at Cambridge, a newly created Chair. When the idea of a mathematical society in London was first proposed in 1864, he gave it his full backing. He was himself elected to its membership in June 1865. His presidency in 1869–70 and the presidency of the Cambridge Philosophical Society ran concurrently, and was followed by a two-year presidency of the Royal Astronomical Society. It was rare for a mathematician to be president of the British Association for the Advancement of Science but in 1883 he was elected to this so-called 'parliament of science'.

Cayley's professorship at Cambridge was funded by rents from the Sadleirian estates and during the agricultural depression of the 1870s his income was saved by a transfer from an honorary Trinity fellowship to an ordinary one. His lifelong association with Trinity College was complete. On his passing he joined other luminaries with a plaque in the College Chapel with the engraved eulogy given by its Master, Henry Montagu Butler, which read:

> Arthur Cayley, glory of our College and of the University, was a straightforward, modest and lovable man, prudent in counsel. He stood out so much among mathematicians that he uncovered areas of study that were new even to the greatest of them.[1]

There is much mathematical history to discover in the thirteen volumes of Cayley's *Collected Mathematical Papers*. For the mathematician there are both fruitful ideas and mathematics that has fallen out of fashion. His mode of pursuing invariant theory, for example, quickly fell out of favour in the succeeding generation, and today pages of invariants of the quintic hardly stir the modern mathematical imagination. But, as has so often been pointed out, 'one never knows'. Cayley's variants of the ordinary determinant surely met the same fate, and one — the Pfaffian — must appear the most arcane of them all. Yet in modern graph theory the Pfaffian crops up in the counting of perfect matchings.

There is another recent historical instance that throws a spotlight on the way Cayley's early work is now seen as relevant. Cayley's little paper 'On the theory of elimination' was published in the *Cambridge Mathematical Journal* in 1848, but when Cayley edited his *Collected Mathematical Papers* he did not single it out as meriting a note.


Dear Sylvester,
I have just obtained a theorem which appears to me very remarkable. You know what the composition of matrices means e.g. if

$$M = \begin{vmatrix} a, & b \\ c, & d \end{vmatrix}, \text{ then } M^2 = \begin{vmatrix} a^2+bc, & b(a+d) \\ c(a+d), & d^2+bc \end{vmatrix}.$$

and I define as the addition of matrices

$$\begin{vmatrix} a, & b \\ c, & d \end{vmatrix} + \begin{vmatrix} a', & b' \\ c', & d' \end{vmatrix} = \begin{vmatrix} a+a', & b+b' \\ c+c', & d+d' \end{vmatrix}$$

Suppose now that M is any matrix $\begin{vmatrix} a, & b \\ c, & d \end{vmatrix}$ and form the

The Cayley–Hamilton Theorem in prospect

In *Discriminants, Resultants, and Multidimensional Determinants* (Birkhäuser, 1994), I. M. Gelfand and his research colleagues identified this paper as laying the foundations of modern homological algebra and republished it. In the preface they wrote: "We were happy to enter into spiritual contact with this great mathematician". Indeed, one never knows.

## Tony Crilly

Tony Crilly was Reader in Mathematical Sciences at Middlesex University before retirement in 2008. He has written a biography *Arthur Cayley: Mathematician Laureate of the Victorian Age* (Johns Hopkins, 2006) and a populariser *Fifty Mathematical Ideas You Really Need to Know* (Quercus, 2008). He is currently completing a biography of the mathematician Thomas Penyngton Kirkman (1806–1895).

[1] I am grateful to Chris Stray for the translation of Montagu Butler's eulogy and to Dale Johnson for his reading of my original draft.

# A Postcard From Our Front Room: The Eighth European Congress Of Mathematics

MARIANNE FREIBERGER AND RACHEL THOMAS

The authors report from the European Congress of Mathematics, held online on 20–26 June 2021.

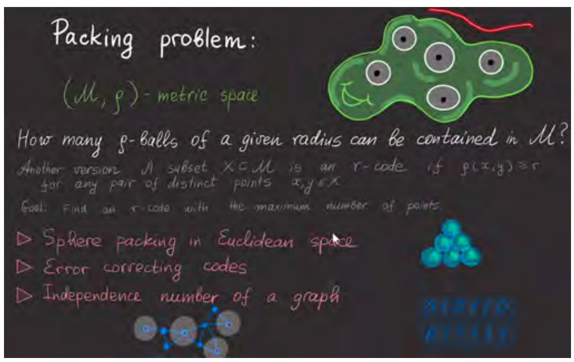
The eighth European Congress of Mathematics (ECM) was due to take place in the summer of 2020, but like so many other international events it was postponed owing to the pandemic. So we were very pleased to be able to attend the Congress in June this year. We had been looking forward to travelling to the beautiful city of Portarož, Slovenia, to attend the Congress in person, but we, along with most participants, were attending online and only got to travel to our front rooms. Nevertheless, we clicked our Zoom links full of admiration for the organisers, who have had to deal with this unprecedented challenge.

Despite being online, the familiar 'first day of a conference' frisson quickly returned as we reviewed the online programme and book of abstracts to plan our days. There were over 1000 contributions to choose from, organised across thirty parallel sessions and sixty mini symposia. On top of the ten plenary talks, thirty invited lectures, twelve prize winners' lectures, and five public talks, this was a welcome opportunity to dive into a whole sea of mathematics.

With so much on offer it's impossible to be exhaustive, so we will focus on a few personal highlights. Perhaps fittingly for a time focused so much on human health, the first plenary of the conference explored mathematical models of the human heart. In a talk called *The Beat of Math*, Alfio Quarteroni highlighted the vast scale of this undertaking: it can take two days on a supercomputer, and 60 million hexahedra, to simulate a single heartbeat. Non-linear differential equations describe the electrical, mechanical, and fluid dynamical processes occurring in the heart, and machine learning will have a role to play in using mathematics to optimise therapy and surgery, and provide clinicians with non-invasive tools. The simulations shown in this talk were strikingly beautiful.

In terms of pure mathematics, we very much enjoyed the talks by Kaisa Matomäki and Maryna Viazovska. Both received an EMS prize, awarded to European 'young researchers not older than 35 years in recognition of excellent contributions in mathematics'. It's not easy to pitch a talk at the kind of general mathematical audience the ECM attracts, but Matomäki did so expertly. Starting with the definition of prime numbers, she went on to describe the significance of the Riemann zeta function and the associated hypothesis with admirable clarity. She then told us what is known about the distribution of primes in small intervals, and what can be said when instead of primes you consider almost primes or the Möbius function. She explored recent results in the area, and also gave us a sense of how such results are proved.

Maryna Viazovska stunned the world in 2016 by solving the sphere packing problem in dimension 8 and (with collaborators) dimension 24. The problem is a higher-dimensional version of Kepler's conjecture, which remained open for nearly 400 years until Thomas Hale provided a proof in 1998, famously and controversially relying on a computer. Viazovska's proofs required no such help (they come to no more than around 20 pages each) and single out two very special objects: in dimension 8 the densest sphere packing is defined by the $E_8$ lattice (with a density of $\pi^4/384$) and in dimension 24 it's defined by the Leech lattice (with a density of $\pi^{12}/12!$). Viazovska explored what makes these two dimensions so special and the mystery of why they host these magic lattices.

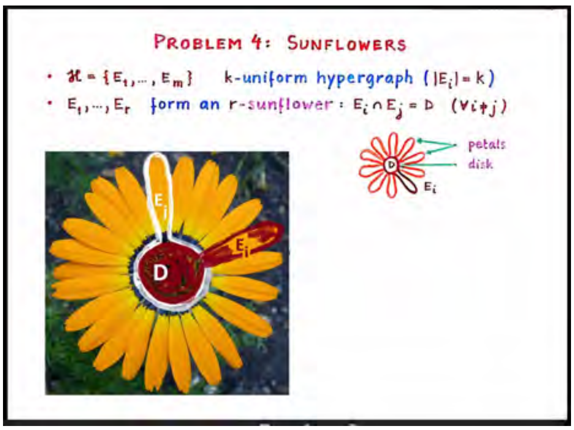



One of Maryna Viazovska's beautiful slides

Another highlight of Viazovska's talk was her beautiful slides which, in our opinion, made her joint winner for the best-hand-written-slides-ever prize. The other recipient of this esteemed prize would have to be Janos

Pach with the slides for his talk, *Escaping the Curse of Dimensionality in Combinatorics*.

Pach began by saying that traps and curses lie in higher dimensions, but that must mean that lower dimensions come with certain blessings. His first example of such a curse was determining the independence number of a graph (the cardinality of the largest subset of vertices which do not share any edges) — an NP-hard problem. But for certain classes of graphs — string graphs — you can do much better. The reason this is possible is the blessing of separator theorems that allow a divide and conquer approach. It's been known since the 70s that you can separate every planar graph on $n$ vertices into two disconnected parts by removing at most $2\sqrt{n}$ vertices, but only recently have Pach and colleagues generalised these results to string graphs. Pach led us from questions first asked in the mid-twentieth century by botanists and biologists, chip designers and neurologists to notoriously difficult combinatorial problems for graphs and hypergraphs today. His talk was scattered with pictures of beautiful flowers, and also included our favourite definition of the Congress: an $r$-sunflower is a collection of $r$ sets (the petals) where the intersection of any two petals is the same (the central disk of the sunflower).




A screenshot of one of Janos Pach's beautiful slides, featuring our favourite definition of the Congress

As Betül Tanbay noted on the first day, the ECM mathematically captured hearts and minds. That day began with Quarteroni's modelling of the heart, and ended with the excellent public lecture by Kathryn Hess from EPFL, Switzerland, on the insights topology brings to understanding the structures of the brain. Public lectures were a prominent element at this ECM, sharing the excitement and beauty of mathematics with the wider community, hopefully mathematically capturing their hearts and minds too.

Hess' public lecture was an intriguing introduction to her work with the Blue Brain Project, whose aim is to digitally reconstruct the rat brain based on real biological data. Hess focussed on directed simplices in the network representation of the reconstructed neural microcircuits, as these subnetworks captured the directional nature of the flow of information through the synapse connections in the brain. There are far higher numbers of these directed simplices in the reconstructed neural microcircuit than in a comparable Erdős–Rényi network, or a more general biological network, and the reconstruction also contains higher-dimensional directed simplices. The work suggests that to enable the necessary coordinated firing behaviour of the neurons, the neurons need to be part of bigger structures such as these higher dimensional simplices. Further results show similar topological parameters of the network faithfully reflect biological reality. Hess' lecture showed the power of mathematics in reducing the complexity of the tangled web of neurons through the clarity of topology.

Of course, given the unpredictable and difficult times we are all living through there were a few technical glitches in this very unique Congress. But we felt that overall the eighth ECM was a great success, and we would like to extend a huge thanks to the organisers. Nothing can replace meeting real people over real coffee, eating real biscuits (or better, wine and canapes), but there are also advantages to the online format. We enjoyed the feeling of being invited into speakers' homes and the high quality slides many had produced for their online talks. A large amount of carbon was not emitted as a result of people staying at home. And importantly, it was presumably the online format that enabled an unprecedented number of people to attend: there were 1771 participants from 77 countries, including 350 students. As EMS president Volker Mehrmann pointed out at the closing ceremony, the mathematical community needs to continue to nurture and support diversity in all its forms. Improving accessibility to future ECMs by exploiting the online format, building on what has been learnt from this ECM, might be a part of that.

The next ECM is due to take place in Seville in 2024. A little guiltily with regards to the carbon emissions, we still hope to be there in person next time around. It's not far from Granada and the wallpaper groups in the Alhambra won't count themselves. See you all there!

If you'd like to revisit the ECM, many of the lectures are now available at the 8ECM YouTube channel tinyurl.com/mwb7u388.

Notes of a Numerical Analyst

# Random Smoothies

NICK TREFETHEN FRS

In 1872 Weierstrass presented his example of a continuous function that is nowhere differentiable. This was defined by a lacunary Fourier series,

$$f(x) = \text{Re} \sum_{k=-\infty}^{\infty} a_k e^{ikx}, \qquad (1)$$

where most of the coefficients $\{a_k\}$ are zero but the nonzero ones decay more slowly than $|k|^{-1}$. Fifty years later continuous, nowhere-differentiable functions began to take on a new importance with the development of the mathematical theory of Brownian motion. Wiener showed that a Brownian path can be written as a Fourier series (1) where $\{a_k\}$ are independent normal variates with mean 0 and standard deviation proportional to $|k|^{-1}$ [3].

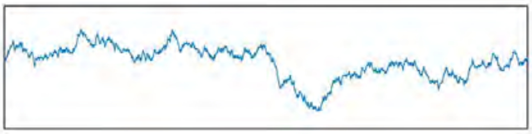


Fig. 1. A Brownian path, continuous but nowhere differentiable, can be defined by a random Fourier series with coefficients $O(|k|^{-1})$.

My students and I have been having fun with continuous functions at the other end of the smoothness spectrum [4]. What if $f$ is $C^\infty$ but nowhere analytic? With apologies to the fruit beverage industry, we call such a function a *smoothie*. Smoothies too can be defined by Fourier series, the key now being that the coefficients decrease at a rate faster than the reciprocal of any polynomial but slower than exponential. For example, a random smoothie can be defined by taking $\{a_k\}$ to be independent normal variates with mean 0 and standard deviation $\exp(-|k|^{1/2})$.

There is a small literature on $C^\infty$, nowhere-analytic functions beginning with du Bois-Reymond, Cellérier and Borel in the Belle Époque [1, 5].

If you differentiate a smoothie, the result is another smoothie. Similarly if $f$ is a smoothie and $g$ is analytic, then $f + g$ and $fg$ are smoothies, provided $g$ is not identically zero. Adding two independent random smoothies gives another random smoothie, with probability 1. On the other hand the sum of two arbitrary smoothies need not be a smoothie, since, for example, one might be the negative of the other.

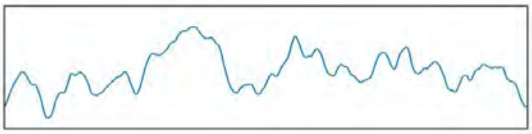


Fig. 2. A random smoothie, $C^\infty$ but nowhere analytic, can be defined by a random Fourier series with coefficients decaying root-exponentially.

In Chebfun, the command `smoothie` generates a random smoothie on an interval, just as `randnfun` generates a smooth random function defined by a finite random Fourier series [2]. Apart from fun, the importance of smoothies for me is educational. My mission is to make sure every student knows the difference between $C^\infty$ and analytic.

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**Nick Trefethen**

Trefethen is Professor of Numerical Analysis and head of the Numerical Analysis Group at the University of Oxford.

# Mathematics News Flash

Jonathan Fraser reports on some recent breakthroughs in mathematics.

### Groups with decidable word problem that do not embed in groups with decidable conjugacy problem

**AUTHORS:** Arman Darbinyan
**ACCESS:** https://arxiv.org/abs/1708.09047

It is shown that there exists a torsion free group with decidable word problem which does not embed in a group with decidable conjugacy problem. The *word problem* asks if two elements of a group are equal and the *conjugacy problem* asks if two elements of a group are conjugate. In particular, the word problem is easier than the conjugacy problem since proving that $x = y$ is the same as proving that $xy^{-1}$ is conjugate to the identity. One can consider a sort of converse to this implication by asking whether a group with decidable word problem necessarily embeds in a group with decidable conjugacy problem (even if it does not have decidable conjugacy problem itself). More than 50 years ago Macintyre showed that this is false in general, but the counterexamples exhibited necessarily had elements of finite order (torsions). Order is preserved by conjugation and so one can see how torsions could play a central role in such a construction.

It subsequently became a famous problem if counterexamples could be constructed which were 'torsion free' (all elements have infinite order). This paper, published in *Inventiones Mathematicae* in 2021, solves this problem in the affirmative.

### How many vertices does a random polygon have?

**AUTHORS:** Anna Gusakova and Christoph Thäle
**ACCESS:** https://arxiv.org/abs/2011.04563

Consider a convex polygon $P$ and let $Q_n \subseteq P$ be a random polygon generated by taking the convex hull of $n$ randomly selected points in $P$. Rényi and Sulanke proved that the expected number of vertices of $Q_n$ is

$$\left(\frac{2}{3} v \log n\right)(1 + o(1))$$

where $v$ is the number of vertices of $P$. This work led to more refined analysis. For example, Groeneboom showed that the vertices of $Q_n$ cluster near the vertices of $P$ and the behaviour near different vertices is (essentially) independent. Moreover, the growth of the number of vertices satisfies a central limit theorem. The clustering phenomenon means it is useful to study neighbourhoods of vertices of $P$ in isolation and this can be reduced to studying the case when $P$ is a right-angled triangle. Gusakova and Thäle published a paper in *Mathematika* in 2021 which studies this 'triangular problem' and obtains surprising new results on the associated probability generating function.

### Rado's criterion over squares and higher powers

**AUTHORS:** Sam Chow, Sofia Lindqvist and Sean Prendiville
**ACCESS:** https://arxiv.org/abs/1806.05002

Given a finite colouring of the positive integers, Schur's theorem guarantees the existence of a monochromatic solution to the equation $x + y = z$. Various analogues of this problem have been considered over the years. A particularly famous open problem posed by Erdős and Graham asks if one can always find a monochromatic solution to $x^2 + y^2 = z^2$. This was solved in the affirmative in the case of a 2-colouring of the positive integers in a behemoth computer aided proof of Heule, Kullmann and Marek.

The paper of Chow, Lindqvist and Prendiville, published in *Journal of the European Mathematical Society* in 2021, proves (in only 73 pages!) that, given any finite colouring of the positive integers, one can always find a monochromatic solution to

$$v^2 + w^2 + x^2 + y^2 = z^2.$$

**Jonathan Fraser** is a Professor of Mathematics and Statistics at the University of St Andrews and an Editor of this Newsletter. His research interests centre on fractal geometry.

Microtheses and Nanotheses provide space in the Newsletter for current and recent research students to communicate their research findings with the community. We welcome submissions for this section from current and recent research students. See newsletter.lms.ac.uk for preparation and submission guidance.

# Microthesis: Automorphisms of Groups and Actions on Trees

NAOMI ANDREW

Group actions on trees — when they exist — provide powerful tools for understanding the group. I use these actions to extract information about the group in order to investigate its automorphisms.

## Groups and Actions

Groups arise as symmetries of various mathematical objects: the symmetric group on $n$ points tells you the permutations of those points, a dihedral group gives symmetries of a regular polygon (see Figure 1), the orthogonal group $O(n, \mathbb{R})$ encodes the isometries of Euclidean space which fix the origin.

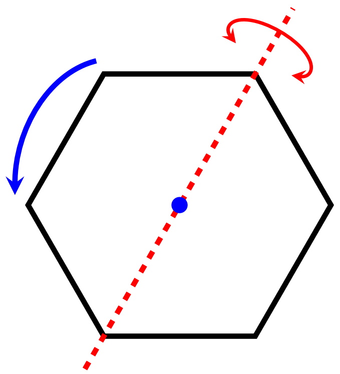

Figure 1: Two of the symmetries of a hexagon

Groups appear throughout mathematics: whenever we have an object, we can ask which transformations keep its structure. The collection of all these symmetries is called its automorphism group.

In group theory, we take the opposite perspective: given an abstract group, we realise it as symmetries of some object. This realisation is known as a group action. We choose objects with properties which highlight aspects of the group — for example, vector spaces in representation theory, and spaces with nice geometries in geometric group theory.

## Actions on trees

The geometry of trees is particularly nice: for example, since they have no loops, there is a unique path between any two points. When a group acts on a tree, we can associate a graph labelled by a collection of subgroups (the stabilisers) to the action. The combinatorial information in this 'graph of groups' recovers the original group up to isomorphism.

The basic examples — with one edge — correspond to the notions of an HNN extension and an amalgamated free product. More complicated examples can be obtained by applying these constructions one edge at a time.

For an HNN extension (Figure 2), we have the group $G$, and two isomorphic subgroups $H$ and $K$ of $G$. The construction adds a new element $t$ (encoding the loop), and declares that conjugating an element of $H$ by $t$ gives the corresponding element in $K$.

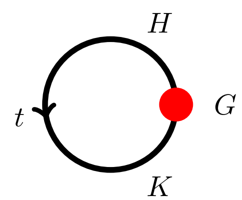


Figure 2: an HNN extension

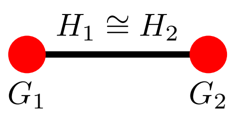


Figure 3: a free product with amalgamation

In an amalgamated free product (Figure 3) there are two groups $G_1$ and $G_2$, with isomorphic subgroups $H_1$ and $H_2$. We glue $G_1$ and $G_2$ together by identifying an element of $H_1$ with the corresponding element of $H_2$.

### Automorphisms of Groups

Groups themselves have symmetries, their automorphisms. One obvious kind of automorphism is an 'inner' automorphism, which arises from conjugating the group by one of its elements. Quotienting by the inner automorphisms leaves a group called the outer automorphism group.

The properties of automorphism groups and outer automorphism groups depend on the group but it is not always obvious how. The family of Baumslag-Solitar groups provides examples with both small and large outer automorphism groups: on the one hand BS(2,3) only has finite outer automorphism group, while for BS(2,4) it is not even finitely generated.

### Serre's Property FA

Given a group, we can ask how and even whether it acts on trees. If it has no non-trivial actions on trees, a group is said to have Property FA. Groups with Property FA include all finite groups, $\mathrm{SL}(3,\mathbb{Z})$ and $\mathrm{Out}(F_3)$.

I investigated in [1] whether the automorphism group of a free product admits actions on trees or if it has Property FA. After some assumptions on the factor groups, this depends only on the number of times each factor group appears: the automorphism group admits an action on a tree when some group appears only two or three times or two groups appear exactly once.

Using my results, if $G$ satisfies the assumptions, the group $\mathrm{Out}(G * G * G)$ acts on a tree since the only factor group appears three times. In fact it is a free product with amalgamation, and if $G$ is finite then so are all the stabilisers in this decomposition.

### Free-by-cyclic groups

Every automorphism $\varphi$ of the free group $F_n$ defines a semidirect product $F_n \rtimes_\varphi \mathbb{Z}$, where conjugating by the new generator acts as $\varphi$. Their properties depend on the defining automorphism: for example they are hyperbolic if and only if the automorphism has no periodic conjugacy classes. The difficulty is that there are groups which can be expressed as free-by-cyclic groups in several different ways.

While there is no standard way to write a free-by-cyclic group as a semidirect product, in [2] we found that in many cases they admit actions on trees which are invariant under twisting by automorphisms of the group.

For example, with $\varphi$ taking $a \mapsto a, b \mapsto ba, c \mapsto bcb^{-1}$ the graph of groups corresponding to the canonical tree is shown in Figure 4.

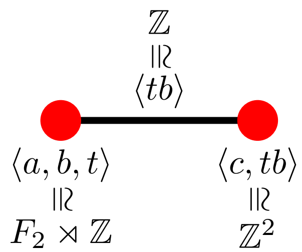


Figure 4: A graph of groups encoding a canonical action on a tree for $F_3 \rtimes_\varphi \mathbb{Z}$.

If a group has a canonical action on a tree, then its automorphism group can be understood by analysing the quotient graph of groups. Using the trees we constructed, we showed that the outer automorphism groups of certain free-by-cyclic groups are finitely generated.

Going further, these canonical actions are conjectured to reveal more properties of free-by-cyclic groups as well as their automorphisms. Similar techniques will also apply to other families of groups.


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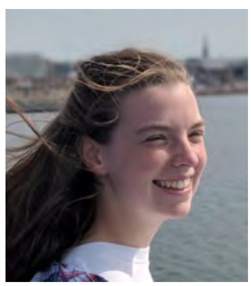


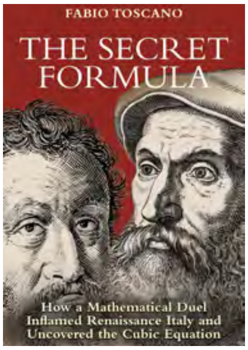
### Naomi Andrew

Naomi is finishing a PhD in geometric group theory at the University of Southampton. When not thinking about actions on trees, she enjoys going for long bike rides, playing the trumpet, and spending time in the mountains.

# The Secret Formula

by Fabio Toscano, Princeton University Press, 2020, £20.00, US$24.95,
ISBN: 978-0691183671

Review by Patrick Love




Fabio Toscano clearly believes in the historical importance of the discovery of the solutions to cubic equations, and in *The Secret Formula* we see his account of the story behind this mathematical work. The solutions are at the centre of a centuries-old priority dispute which is Toscano's focus throughout the book, and he discusses it in a dramatic, story-like style. He focuses on the priority dispute, but this is done against a backdrop which provides a great insight into pre-symbolic mathematics. Between its narrative structure and good mathematical explanations, it is suitable for all who are interested in the history of mathematics without requiring prior experience in the field.

The "formula", as Toscano often refers to it, was in fact a series of solutions to different cases of cubic problems, discovered by several mathematicians over the first half of the 16th century, notably Scipione del Ferro, Niccolò Tartaglia, and Gerolamo Cardano. The discoveries were widely well received and many modern historians believe they had a profound impact on the mathematics of the day [3, 6].

The cubic case written today as $x^3 + bx = c$ was solved first, independently by both del Ferro and Tartaglia, but neither published their work. Later, Cardano learned of Tartaglia's solution and went on to publish it himself, in his 1545 *Ars Magna* [1]. Cardano justified this by claiming he published del Ferro's (identical) solution, not Tartaglia's, but this still angered Tartaglia and led to an unsolvable dispute on who of the three could claim ownership of the solutions. All sides of the dispute have merit and, as is often the case, there is no simple answer to this question of priority [2, 3]. Toscano explores the dispute in great depth, adding a sense of intrigue and drama to the story. He starts simplistically, implying that there will be a single "right" answer, but as the book continues his view softens, and he goes on to conclude that credit should be shared between all three discoverers [4].

Toscano focuses heavily on this dispute throughout *The Secret Formula*, but in doing so he inevitably explores and explains wider historical mathematics. Unlike some historians, he does not skip over the today-alien rhetorical and syncopated algebra used prior to the end of the 16th century, nor does he ignore the geometrical underpinnings of much historical mathematics. Instead his explanations are very good at guiding the reader to an understanding of the algebra as it was done in the past. As such, he provides great context for the dispute, and a good exploration into the cubic "formula" as it was when it was first discovered [4, 5].

While his mathematical explanations are generally very good, the narrative style of the book forces a quite jumpy and twisty discussion of the discoveries. *The Secret Formula* is at times a book on how different civilisations have approached mathematics, and at others it is a mystery drama exploring the priority dispute and who should get credit. These two sides don't always gel, with sometimes harsh jumps backwards and forwards to go between story and mathematical context. In this way, while building intrigue in the reader is a good idea, it does lead to a hard-to-follow structure for the book [4].

Similarly, Toscano makes some very misleading statements early in the book, implying that Tartaglia was robbed of an entirely unique discovery and that del Ferro, "the mystery man", did not exist. On continuing though, Toscano seems to give Cardano more and more credit, before revealing del Ferro almost as a surprise twist. In this way, throughout the book Toscano's opinions seem to vary wildly, and it appears to be a deliberate choice to create a narrative; he chooses when to present certain facts and opinions to ensure they have maximum impact and create drama for the reader [4]. While this does make for compelling reading, it feels difficult to trust a source that is, at points, so misleading of simple facts, such as the implied non-existence of del Ferro (who was a prominent mathematician) [3].

*The Secret Formula* paints a picture not only of the history of the cubic formula, but also of the wider state of mathematics in the 16th century and the mathematical customs of the time [4]. Toscano explores this through the lens of the duel between Tartaglia and Cardano. Through their correspondence we see how mathematicians in this period approached problems, what they deemed necessary in their solutions, the institutions and positions they fought to associate themselves with, and we see how mathematical debate was conducted in this very different time. The duel allows Toscano to explain by example how mathematics was conducted in the past, and it makes the read an extremely interesting one, as well as being informative.

By presenting the history in the light of the Tartaglia–Cardano duel, Toscano is able to provide real insight into 16th century western mathematics without having to dwell too much on the mathematics itself for non-technical readers. In this way, the book is targeted well at its audience, and by providing the information in quite a narrative structure it is also an entertaining read. It is let down somewhat by its over-dramaticism and at times misleading points, but in the end Toscano is able to provide a realistic and accurate view that captures the complexity of the story of the cubic formula and the very different mathematical practices of this time. Anyone interested in learning about the history of mathematics will likely find it an interesting and informative read.

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[6] T. R. Witmer, Ars Magna or The Rules of Algebra, translation of [1], Dover Publications Inc., New York, 1993 — unabridged republication from MIT Press, 1968.

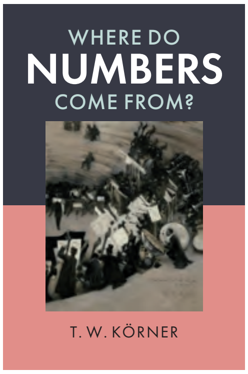
**Patrick Love**

Patrick works as a software developer and graduated from Oxford University in 2019 having studied the history of mathematics — in particular in relation to the *Ars Magna* and its impact in the development of early symbolic algebra. He splits his spare time between rowing and building robots.

# Where Do Numbers Come From?

## by T. W. Körner, Cambridge University Press, 2020, £24.99, ISBN: 978-1108738385

Review by Julia Collins




Where *do* numbers come from? The title of this book conjures questions about the history of the world's earliest mathematicians and their process of numerical discovery, or philosophical questions of whether numbers are invented or discovered. But this book is neither focused on history nor philosophy; it is a rigorous axiomatic approach to the construction of the real numbers, aimed at university mathematics students who want a deeper understanding of the properties of the real numbers, how these properties have been abstracted from other number systems, and the reasoning as to why it is these properties in particular that mathematicians have chosen as the foundation of their subject.

Körner's book is not designed to be a textbook and is not written as such. Yet it is also not a relaxed read: the person buying this book should be prepared to pause often, to try the exercises and get their hands dirty with the mathematical material. Fans of Körner's other books, like *The Pleasures of Counting*, will appreciate the conversational style of this book, which provides plenty of amusing footnotes, historical context, quotations and poems, while never relenting on the rigour of the mathematics.

The book begins with a short chapter on how early mathematicians could have abstracted the properties of the natural numbers, such as commutativity and associativity of addition and multiplication, the multiplicative identity 1, and the notion of an order $>$. Körner then immediately dives in deep in Chapter 2 with an axiomatic construction of the rational numbers using the concept of equivalence classes of natural numbers, followed by proofs that the desired properties of associativity, commutativity and order still hold for this new class of numbers. This sets up the structure of the rest of the book: the axiomatic definition of a class of numbers, abstracting from the previous one, followed by proofs that the desired properties are still there. Along the way, the constructions yield new desirable properties — like the introduction of multiplicative inverses for the rational numbers.

In this way the book progresses through constructions of the integers (as a subset of the rationals), finite fields such as $\mathbb{Z}_p$ for $p$ prime, extension fields such as $\mathbb{Q}[\sqrt{2}]$, the real numbers (as equivalence classes of Cauchy sequences), the complex numbers (as ordered pairs of real numbers with rules for addition and multiplication), the integral domain of real polynomials (which creates an alternative construction of the complex numbers via an equivalence relation), and finally the quaternions (ordered quadruples of real numbers). The climax of the story is the construction of the reals, and Körner is particularly keen to emphasise the importance of the reals being the unique ordered field satisfying the fundamental axiom of analysis (that every increasing sequence bounded above has a limit). Without this axiom, we would not be able to justify our use of calculus.

Along the path of this story we learn about related concepts like the principle of induction, prime numbers, irrational numbers, modular arithmetic, the Peano axioms, set theory and Russell's paradox, limits, continuity, infinity and Gödel's theorem, and the fundamental theorem of algebra (i.e., the completeness of the complex numbers). Occasionally a philosophical question will arise, such as "Is this the correct way to construct these number systems?" or "Are real numbers real?", to which Körner provides his opinion but leaves plenty of room for debate on the matter. There are also several interesting digressions (for example, to cryptography, Hamming codes and magic tricks), leaving one with the sense that we are only scratching the surface of this fascinating subject.

Overall the book is a solid introduction to the concepts underlying the real numbers, covering many different important topics and addressing key questions arising from them. Yet, at the same time, the treatment of the material feels very traditional for a book published in 2020, with little thought about making the concepts accessible to someone encountering them for the first time. For example, the abstract construction of the reals as equivalence classes of Cauchy sequences is never related back to the concept of decimal numbers, which is how most people intuitively understand the reals. Nor is there much of a hint of the notion that real and complex numbers have geometric interpretations as well as algebraic ones. The focus on rigour means that some truly mind-bending concepts, like Cantor's proof of different sizes of infinity, get lost among the symbols unless one knows to look out for them.

The book is best suited to a confident and dedicated student who already has a good sense of the mathematical big picture and is looking to fill in the detail, rather than someone who has seen some of the detail and wants the bigger picture. Readers are expected to be familiar and confident with mathematical notation, reasoning and proofs, so the ideal audience will be people who have already completed a year or two of university mathematics, and who are intrigued to delve further into the concepts of number theory and real analysis.

### Julia Collins

Julia Collins is a lecturer in mathematics and computer science at Edith Cowan University in Perth, Australia. Her scholarly interests are in mathematical education and outreach, while her mathematical background is in knot theory. She is the author of two popular maths books: *Numbers in Minutes* and *Get Smart: Maths*. When not teaching or writing, Julia will be found knitting mathematical objects, folding origami, or designing puzzles for her smart Kelpie puppy Kali.

# A First Journey through Logic

## by Martin Hils & François Loeser, American Mathematical Society, 2019, US$55, ISBN: 978-1-4704-5272-8

Review by Carl-Fredrik Nyberg Brodda


This aim of this book, in the words of its authors, is "to present a broad panorama of mathematical logic to students who feel curious about this field, but have no intent to specialise in it". The book most certainly succeeds in doing so. It is a *tour de force* of logic, set theory, and model theory, presenting deep and technical results in a way which remains accessible and very pleasant to read.

The chapters are to some extent independent, and the reader only interested in a single chapter will for the most part not find it difficult to read only this, with only minor jumps back in the text required. Chapter 1 covers the basics of ordinals and cardinals. Chapter 2 covers an introduction to first-order logic, which includes a proof of Gödel's Completeness Theorem via Henkin witnesses. Chapter 3 covers model theory, including introducing the notion of quantifier elimination and proving it in detail for algebraically closed fields. This is applied to prove the Lefschetz Principle and Ax's Theorem. Chapter 4 covers basic recursion theory, ending with the Halting Problem and Rice's Theorem. In Chapter 5, more "negative" results are proved, including Tarski's theorem on the non-definability of truth, and Church's theorem on the undecidability of arithmetic. This culminates in proofs of Gödel's First and Second Incompleteness Theorems, both very clearly written. Finally, in Chapter 6, set theory is formalised using the Zermelo–Fraenkel axioms (ZF) and the Axiom of Choice (AC). The equivalence of Zorn's Lemma, AC, and the well-ordering principle is proved. The Axiom of Foundation (AF) is studied in detail, and its relative consistency is proved. Some more advanced topics, like inacessible cardinals, club sets, and a brief discussion on the independence of the continuum hypothesis, giving a glimpse of some of the powerful results proved more recently.

Where the book really shines is in its extensive and deep exercises. Beyond many instructive and concrete exercises which ensure a proper working understanding is developed (e.g., Exercise 4.8.1: "Describe a Turing machine which computes the addition function $\lambda xy.x + y$"), the exercises also include many longer, step-by-step exercises introducing deeper ideas. These are all exceptionally well crafted, and introduce the reader to a host of new topics for each chapter, making the task of further reading and curiosity all the easier. For example, the list of exercises in Chapter 1 includes an introduction to Goodstein sequences, proving Solovay's theorem, ultrafilters, proving the continuum hypothesis for closed subsets of $\mathbb{R}$, proving Kőnig's Lemma, as well as an appendix with Hindman's remarkable theorem. The list of exercises in Chapter 2 includes proving the infinite version of Ramsey's Theorem, Beth definability, Craig Interpolation, Herbrand normal forms, and the Omitting Types Theorem. In Chapter 3, we find exercises proving Loś' Theorem, Vaught's Criterion, quantifier elimination for divisible ordered abelian groups, and indiscernible sequences. Chapter 5 has an exercise proving the decidability of and quantifier elimination for Presburger arithmetic and on proving Tenenbaum's Theorem. The final Chapter 6 introduces Mostowski collapses, the Reflection Principle, and proving the relative consistency of AC. The exercises are, in addition, clearly crafted with an exceptional level of care.

Although there are quite a few misprints scattered throughout (e.g., on page 42, on which the word "simultaneous" is simultaneously misspelled in two different ways), and although certain difficult formulae and expressions are at times ill treated by line and page breaks, this does not detract from the overall presentation. At times, the book could have benefitted from some additional historical and philosophical context, especially given the many interesting such connections existing in logic and set theory, as well as given the stated aim to give a broad panorama for those who feel curious about the field. Nevertheless, the choice not to include such context is more than understandable, given the slim style of the exposition of ideas given within. No reader can pick up this book and read it without getting a feeling for the depth of the many wonderful areas of mathematics that it treats. For the reader wishing to get a working knowledge of some or all of the ideas presented therein, this book will doubtlessly prove invaluable.

## Carl-Fredrik Nyberg Brodda

Carl-Fredrik Nyberg Brodda is a final-year Ph.D. student at the University of East Anglia. His main research interests are in the interactions between algebraic structures, logic, and computability, as well as the history of all these areas (and others!). Being Swedish, he enjoys visits to the (comparatively) tropical climate of the north Norfolk coast, and long walks along the same.

# Obituaries of Members

## Brian J. Philp: 1942 – 2021

Brian Philp, who was elected a member of the London Mathematical Society on 16 May 1975, died on 28 June 2021, aged 79.

*Tony Gardiner writes:* Brian was born in April 1942, and attended Moseley C of E Primary School in Birmingham and Moseley Grammar School, before moving to Coventry at the age of 15. He was an undergraduate in Leeds, and did an M.Sc. and two years of a PhD in logic/set theory under the supervision of Frank Drake, taking a lectureship in Birmingham in 1967. The then Mason Professor, Murray MacBeath, had taken a degree in Belfast, served in Bletchley Park from age 19 to 21, and completed his PhD in Princeton under Emil Artin after the war. The idea that beginners might need 'supporting' probably never occurred to him, and it was certainly not the fashion at the time; nor was Brian the sort of person to press. So, while Brian gave high level courses on Logic, always tried to keep up to date, and helped to nurture a number of subsequently impressive mathematicians, he became one of the foot-soldiers of the department — taking on numerous responsibilities over the years. Despite never becoming a prolific 'researcher', Brian retained a profound love for mathematics, and was an enthusiastic participant in the termly Midlands Universities Pure Mathematics Seminars. He was always available (and often effective) in helping colleagues who had questions in logic or set theory. And his approachable and caring nature made him a popular and conscientious personal mentor to generations of undergraduates and postgraduates — many of whom remember him with affection: one 2011 PhD thesis captures this by singling him out with the words: "Particular thanks should go to Brian Philp, who while I was an undergraduate, always had time to answer my questions."

Brian died unexpectedly at the end of June, two days after playing a round of his beloved golf. He leaves his wife Val, four children and seven grandchildren.

## Lynne H. Walling: 1958 – 2021

Dr Lynne Walling, who was elected a member of the London Mathematical Society on 24 October 2007, died on 28 May 2021, aged 62.

*Thomas Jordan and Misha Rudnev write:* Roughly a month before Lynne died she was diagnosed with terminal cancer. On the evening of Monday 10 May, after having seen a consultant on that day, she called a friend and said, "I'm gonna die. It sucks. I don't have much time. I want a party on Friday." She passed away eighteen days later at her Ashley Down home, surrounded by her friends and her pets.

Lynne completed her PhD in Number Theory at Dartmouth College under the supervision of Thomas Shemanske in 1987. She credited her supervisor with also teaching her many useful practical skills, such as roofing. After her PhD she took a position at St. Olaf College, Minnesota, where she lived in an old farmhouse which did not have indoor plumbing. She solved this problem by building a hand-operated pump, which also enabled her to install a bathtub in the kitchen, next to a wood-burning stove.

In 1990 she became an Assistant Professor at the University of Colorado, Boulder. She received tenure there in 1995 and in 2000 became a full Professor. She came to Bristol as a Reader in Mathematics in 2007. Lynne herself defined her mathematical ambition as solving hard and interesting problems and constructing elegant proofs, success as an expositor and a teacher, enriching and contributing to the community, happiness and fulfilment.

She was a number theorist of renown. The centrepiece of her scholarship was the study of theta series. Theta series have a long and glorious history that has engaged mathematical giants from Bernoulli, Euler and Jacobi to

Hilbert, Siegel and Weil, and they remain mathematical objects of intense study. They provide a systematic way for constructing continuous mathematical quantities, called modular or automorphic forms, which encode information about discrete arithmetic problems, e.g. in how many ways can one express an integer as the sum of squares. Automorphic forms arise in number theory, algebraic geometry, as well as mathematical physics. By studying these quantities and their properties, information about the original arithmetic question is gleaned. And then, following the magic of mathematics, the newly formed objects take on a life of their own and an enveloping theory often leads to yet unimagined places.

Lynne's work on theta series began in her thesis *Theta Series Attached to Lattices of Arbitrary Rank*. Her interest in them extended throughout her career. The word 'explicit' occurs frequently in the titles of her roughly 40 papers. Many of these papers are single authored, delivering a musing and unhasty narrative, evincing the author's thoughtfulness and intimacy with her subject. This substantial body of work has helped mathematicians understand spaces of theta functions and their structure, and explained how that understanding translates into knowledge about numbers. In her pursuit of elegance as an algebraist, Lynne sought explicit formulas to express certain fundamental quantities discovered in particular by Carl Ludwig Siegel in the 1930s. Lately, she was particularly proud of her 2018 paper *Explicitly realizing Average Siegel theta series as linear combinations of Eisenstein series* in the *Ramanujan Journal*, where she said she succeeded in doing something that had defeated Siegel.

She would stress that mathematics constituted largely her personal aesthetic quest, rather than the pursuit of fame and honours. Nonetheless, Lynne's career since its early days has been marked by an unambiguous pattern of strong leadership. Her dedication to teaching and scholarship inspired generations of students. Her commitment to equality and diversity was selfless, unparalleled and contagious. In the early 2000s, serving in a position of honour as Program Officer for the US National Science Foundation, she was an active member of the managing team of the Vertical Integration of Research and Education (VIGRE) program, aiming at broadening the backgrounds of those who seek careers in the mathematical sciences. There, she was also a lead member of the Early Career Development Program management team. Since 1988 she gave some 20 invited presentations on Women, Diversity, and Education becoming a role model for a whole generation of female scholars.

Professor Lynne Walling was Head of Department of Mathematics in Boulder in 2004–06. In Bristol she was Head of Pure Mathematics in 2011–2015 and Director of the Institute of Pure Mathematics from 2018. In these leadership positions, she regarded ensuring the well-being of her colleagues, supporting and recognizing their achievements and promoting their success as perhaps her greatest responsibility. She was also heavily involved in matters regarding teaching, introducing regular teaching panels for staff, developing the curriculum and introducing social events with staff and students. She was also always an extremely popular lecturer with the students.

She knew how to hold her head high and to be unapologetic in advocating for what she believed. She was a true artist in everything she did — her mathematics, her artwork and craftsmanship, which preceded her mathematics and which she carried with her all her life, and in her general quest for self-realisation. She was a remarkably courageous and generous person, colleague and friend, and she taught this to people who found themselves within the sphere of her magnetism. The last lesson she taught those surrounding her is that it is possible to face death on your own terms and undefeated.

Lynne was one of the organisers of Building Bridges: a series of joint EU/US workshops and summer schools in automorphic forms and related topics. The second event in the series took place in Bristol in 2014 and was an LMS–CMI research school. The fifth one had been planned to take place in 2020 in Sarajevo but was postponed due to because of the covid-19 pandemic. Unfortunately, Lynne will not be there when it takes place. She will be remembered as the fountainhead of the events' mathematics, a strong and determined voice to ensure the diversity of their speakers and participants' backgrounds and origins, and its overall heart and soul.

Lynne was also an organiser for the LMS-supported 2016 Bristol Mathematics Colloquium. Once again, in line with her dedication and ethics, Lynne worked tirelessly to ensure the highest standard for this conference.

We thank Lynne's father, Stuart Walling, and many of her friends and colleagues, in particular Tom Shemanske, Sol Friedberg, Jonathan Robbins, as well as the members of the Celebration of LW WhatsApp group, for providing material and help in the preparation of this obituary.

## David Laurence Wilkens: 1944 – 2021

David Wilkens, who was elected a member of the London Mathematical Society on the 19 October 1972, died on Sunday 13 June 2021.

*Rob Curtis writes:* Dave was born to Laurence and Ethel Wilkens on 4 August 1944 and spent his early life living in the family home in Little Lever, near Bolton. He attended the Canon Slade Grammar School, Bolton, and in 1962 proceeded to study mathematics at Liverpool University. Having distinguished himself there he was taken on as a PhD student by C.T.C. Wall and in 1971 submitted a thesis entitled *Closed (s-1)-connected (2s+1)-Manifolds, s = 3,7*. Murray MacBeath, at that time Head of Mathematics at Birmingham University, was keen to attract a strong topologist and Dave was appointed to a lectureship in 1969, before the submission of his dissertation. In 1966 he had married Patricia, who for many years was known by her second name Adele; they had met at a Valentine's Ball at Liverpool University and married after she returned from a spell teaching English in France. They lived near the university for the whole of their long married life together.

Dave thrived at Birmingham and became a highly regarded and professional lecturer; his notes were invariably immaculately produced and much appreciated by our students. In those days we also had responsibility for teaching mathematics to engineers and scientists and Dave made a valuable contribution outside our department. He played an active part in the running of the department, successfully and conscientiously taking on important roles such as that of Admissions Tutor. His main research interest was in a topological approach to groups acting on infinite trees, and he often collaborated in this work with another colleague, Howard Hoare. Dave was an enthusiastic participant in the Midlands University Pure Mathematics Seminar (MUPMS) which was held once a term, moving around the region from department to department. He also attended several ICMs including those in Nice, Zürich and Berlin.

He was always keen on foreign travel, starting with a road and rail trip to Greece in 1963 which he made with his friend Dave Moss, who had been at been at school with him. He and Pat made many trips to the Greek Islands, North Africa and the Caribbean.

Dave was an enthusiastic sportsman, playing rugby well beyond the recommended age limit. He and I played squash together for many years and, when we became too old for that, we played golf. On occasions, in the annual Birmingham versus Liverpool Staff golf match, if Liverpool failed to produce a full team, Dave played for them as he had such strong Liverpudlian credentials. He was also famous for winning a bottle of whisky from Johnnie Walker when he scored a hole-in-one during a medal match.

In 2015 Dave was diagnosed as suffering from secondary kidney cancer, but happily he was able to lead a full life for most of the intervening years. Eventually though he ran out of treatments and succumbed to the inevitable. He will be remembered as a superb colleague, a loyal and generous friend, and a man of few words but immense integrity; he will be sorely missed.

## Frank R. Drake: 1936 – 2021

Dr Frank Drake, who was a member of the London Mathematical Society from 17 November 1966 to October 1996, died on 16 May 2021 aged 84.

*Dugald MacPherson writes:* Frank played an important role in UK Logic, both in its development at the University of Leeds, and through his work for the British Logic Colloquium. After growing up in Sheffield, he obtained his PhD with Tim Smiley in Cambridge in 1963, and was a co-student in Cambridge of the leading logician Alastair Lachlan. He was appointed as a Lecturer at Leeds in 1962, on the same day that the logician John Derrick was also appointed in Leeds; they were both lifelong members of the logic group in Leeds, developed initially by Martin Loeb. Though his early training and first publication was in modal logic, Frank switched into the rapidly developing area of set theory. In the summer of 1967 he attended the NSF Summer Institute at UCLA on Axiomatic Set Theory, where his interest in large cardinals was stimulated. This was shortly after Cohen's proof of the relative consistency of the negation of the Continuum Hypothesis; proved via Cohen's invention of set-theoretic forcing, this led to huge advances in set theory in the late 1960s.

Back in Leeds, Frank gave graduate lectures on constructible sets and forcing, and in 1974 published a now-classic text: *Set Theory — An Introduction to Large Cardinals* (North Holland). A later textbook with D. Singh, *Intermediate Set Theory* (Wiley 1996), was pitched at a more elementary level but introduces the constructible universe and forcing.

Frank wrote a number of articles around 1967–75 on set-theoretic independence results, forcing, and large cardinals. In the mid-1980s he wrote on mathematical foundations and reverse mathematics, a still-thriving field (then new) which explores the logical strength of axioms systems needed to prove key mathematical results. Frank had several PhD students, including John Truss (Leeds), who has published on many topics around set theory, model theory, permutation group theory, and combinatorics.

In 1977 Frank was invited, by Robin Gandy and John Shepherdson, to become the first Honorary Secretary of the newly-formed British Logic Colloquium, the society representing Logic (in mathematics, philosophy, and computer science) in the UK. He undertook this with huge energy and efficiency, carrying out key initial work setting up the BLC and continuing to work for it until the 1990s. Among further activities, he was a lead organiser of the 1986 Logic Colloquium in Hull, editing the *Proceedings* with John Truss.

Frank became a Senior Lecturer in Leeds, and held a series of senior administrative roles. He was Head of the Department of Pure Mathematics 1978–82, agreeing to serve the additional fourth year to enable his successor to settle into Leeds. He then held roles as School of Mathematics Admissions Tutor and Director of Undergraduate Studies, and is remembered as a kind, imperturbable and supportive colleague. He retired in 1996, but took on many other activities, such as, for many years, the role of volunteer guide at Harewood House just outside Leeds.

## Death Notices

We regret to announce the following deaths:

- Brian Sleeman, formerly of the Dundee and Leeds Universities, died 19 July 2021.
- Victor Snaith, formerly of Sheffield University, who died 3 July 2021.

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## British Science Festival

Location: Anglia Ruskin University
Date: 7–11 September 2021
Website: tinyurl.com/yr2bkw86

The main mathematics speakers are Chris Budd OBE and Nira Chamberlain, who is President of the Mathematical Sciences Section. Nira's talk is *Stopping the AI-pocalypse* on Tuesday 7 September, and Chris, assisted by Tom Crawford, Tina Zhou and Tosin Babasola will ask *Can YOU predict the climate?* on Saturday 11 September.

## Spectral Theory and Partial Differential Equations

Location: Durham University
Date: 15–16 September 2021
Website: tinyurl.com/y79e244k

This meeting is to celebrate the recent appointments of Sabine Boegli and Katie Gittins at Durham University. Please see the website for further information including registration and the list of speakers. This meeting is supported by the London Mathematical Society and the Department of Mathematical Sciences at Durham University.

## New Lecturers in Mathematical Sciences Induction Course

Location: Isaac Newton Institute & Hybrid
Date: 8–9 September 2021
Website: tinyurl.com/xu4f3np3

This IMA course has been designed by the mathematics community for anyone new to or with limited experience of teaching mathematics or statistics within UK HE. Course leaders will have significant experience of teaching in the mathematical sciences.

## Twistor Theory and Beyond

Location: Cambridge & online
Date: 27-29 September 2021
Website: indi.to/SnpP6

This workshop will review the achievements of twistor theory from pure mathematics to theoretical physics and assess the most promising current directions for research. There will be talks on the many different facets of twistor theory by leading researchers.This will be a hybrid in-person/online event; online registration is required and applications for in-person attendance will be considered.

### LMS Meeting

# Annual General Meeting

12 November 2021, 3–6 pm

Location: Goodenough College, Mecklenburgh Square, London
Website: tinyurl.com/cu4u62ry

15.30: Supporting lecture: Professor Jens Marklof
16.30: Tea/coffee break
16.55: Announcement of LMS election results
17.00: Presidential address: Professor Jon Keating
18.00: Close of meeting

The meeting will open with Society business, including the presentation of certificates to all the 2021 LMS prize winners. These lectures are aimed at a general mathematical audience. All interested, whether LMS members or not, are most welcome to attend this meeting.

The meeting will be followed by a reception at Goodenough College. For further details about the AGM, contact Dr John Johnston (lmsmeetings@lms.ac.uk).

The Society's Annual Dinner will also be held on 12 November at 7.30 pm at Goodenough College. The cost of the dinner is £60.00, including drinks. To reserve a place at the dinner, email Clare Ralphs at AnnualDinner_RSVP@lms.ac.uk by Friday 29 October.

# Society Meetings and Events

## September 2021

1-10 Northern Regional Meeting, Conference in Celebration of the 60th Birthday of Bill Crawley-Boevey, University of Manchester
9-10 LMS Prospects in Mathematics Meeting (online)
30-1 Oct LMS/IMA Joint Meeting, Maths in Human Society (online)

## October 2021

5-6 Joint Meeting Black Heroes of Mathematics Conference (online)

## November 2021

12 Annual General Meeting, London

## January 2022

4-6 South West & South Wales Regional Meeting, Swansea

# Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website (www.lms.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

## September 2021

1-3 Scaling Limits: From Statistical Mechanics to Manifolds, Cambridge (493)
7-11 British Science Festival, Anglia Ruskin University (496)
8-9 New Lecturers in Mathematical Sciences Induction Course, Isaac Newton Institute (496)
8-10 Mathematics of Robotics Conference (online) (494)
9-10 Heilbronn Annual Conference 2021, Heilbronn Institute (493)
15-16 Spectral Theory and Partial Differential Equations, Durham (496)
16-17 Statistics at Bristol: Future Results and You 2021, Heilbronn Institute (494)
19-24 8th Heidelberg Laureate Forum, Heidelberg, Germany
21-23 Conference in Honour of Sir Michael Atiyah, Isaac Newton Institute, Cambridge (493)
27-29 Twistor Theory and Beyond (online) (496)

## November 2021

18-20 Mathematics in Times of Crisis, online (494)

## December 2021

14-15 Cryptography and Coding Conference, online (495)

## April 2022

25-29 Rational Points on Higher-Dimensional Varieties, ICMS, Edinburgh (495)

## May 2022

18-20 Mathematics in Signal Processing, Aston, Birmingham (495)

## July 2022

24-26 7th IMA Conference on Numerical Linear Algebra and Optimization, Birmingham (487)

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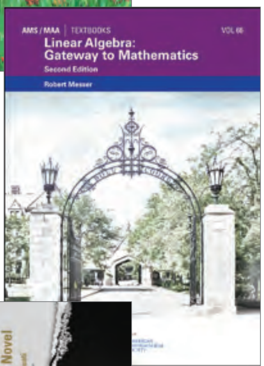


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*Craig A. Stephenson*

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*History of Mathematics, Vol. 45*

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