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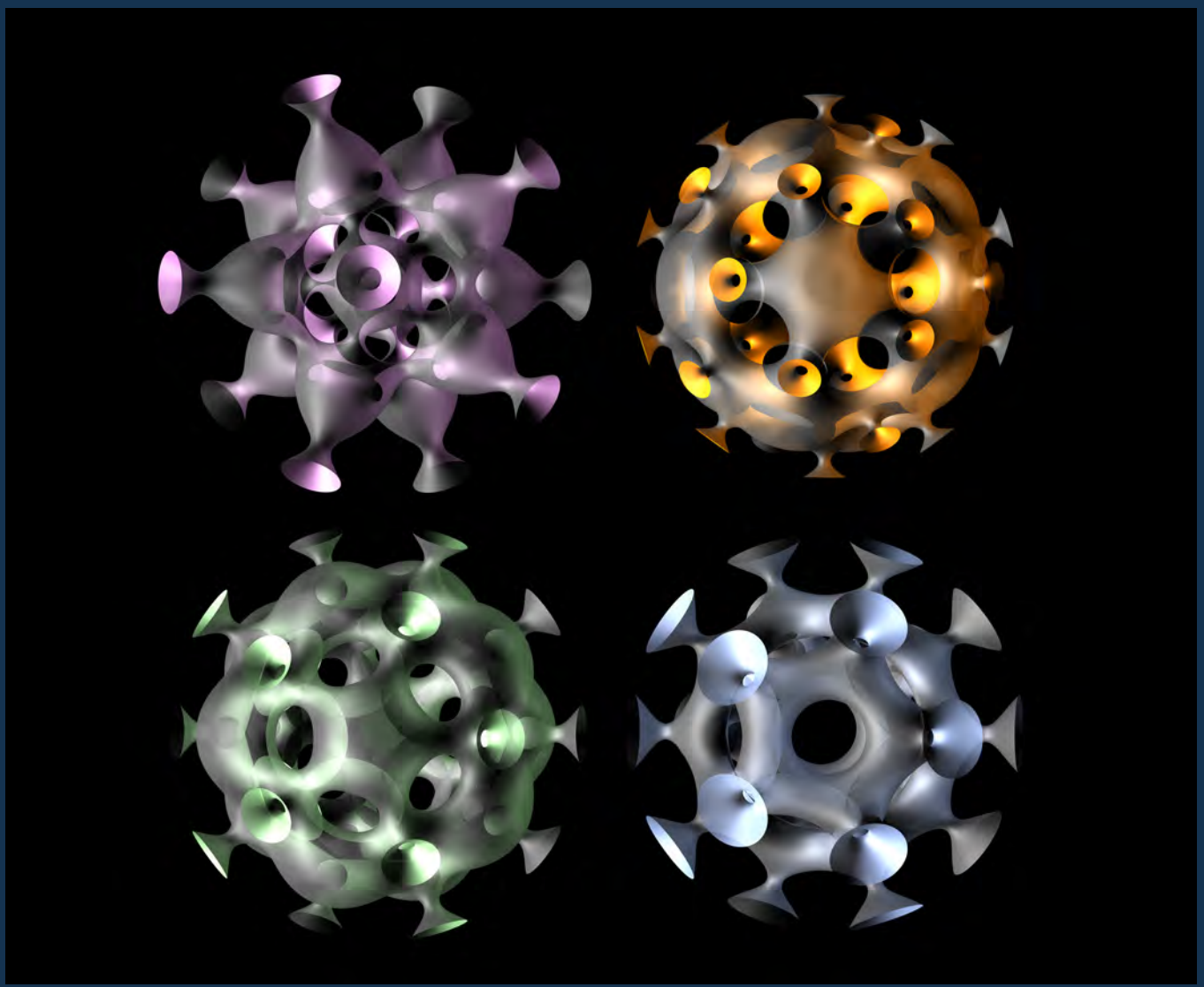

JORDAN ALGEBRAS AND SYMMETRIC MANIFOLDS

MANIN MATRICES QUADRATIC ALGEBRAS & OPERADS

NOTES OF A NUMERICAL ANALYST

EDITORIAL OFFICE

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CONTENTS

LMS Council 2021–22

The results of the 2021 LMS Elections to Council and Nominating Committee were announced at the LMS Annual General Meeting on 12 November 2021. Council membership for 2021–22 is as follows:

President:
Professor Ulrike Tillmann FRS (University of Oxford)

Vice-Presidents:
Professor Iain Gordon (University of Edinburgh)
Professor Catherine Hobbs (University of the West of England)

Treasurer:
Professor Simon Salamon (King's College London)

General Secretary:
Professor Robb McDonald (University College London)

Programme Secretary:
Professor Chris Parker (University of Birmingham)

Publications Secretary:
Professor Niall MacKay (University of York)

Education Secretary:
Dr Kevin Houston (University of Leeds)

Member-at-Large (Women and Diversity):
Professor Sara Lombardo (Loughborough University)

Members-at-Large of Council:
*Professor Peter Ashwin (University of Exeter)
*Professor Anne-Christine Davis (Cambridge University)
Professor Elaine Crooks (Swansea University) (re-elected to Council)
Professor Andrew Dancer (University of Oxford) (re-elected to Council)
Dr Jessica Enright (University of Glasgow)
*Professor Minhyong Kim (International Centre for Mathematical Sciences)
Professor Frank Neumann (University of Leicester) (re-elected to Council)
Dr Rachel Newton (King's College London)
Professor Brita Nucinkis (Royal Holloway, University of London) (re-elected to Council)
*Professor Anne Taormina (University of Durham)
*Dr Amanda Turner (Lancaster University)

*Members elected in 2020 who are continuing with the second year of their two-year term.

The AGM also saw Professor John Hunton step down as Publications Secretary and Dr Tony Gardiner and Dr Mark McCartney step down as Members-at-Large on Council. The LMS is grateful for their contributions, details of which can be found on page .

LMS Nominating Committee:

Also at the AGM, Dr Nira Chamberlain (Babcock International Group) and Professor Philip Maini (University of Oxford) were elected to the Nominating Committee for three-year terms of office. Continuing members of the Nominating Committee are Tara Brendle (Chair from 2022), Chris Budd, Beatrice Pelloni, Mary Rees and Gwyneth Stallard. Council will also appoint a representative to the committee.

Members of Council 2021–2022

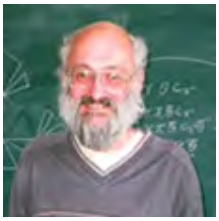
Ulrike Tillmann
President

Cathy Hobbs
Vice-President

Iain Gordon
Vice-President

Simon Salamon
Treasurer

Robb McDonald
General Secretary



Chris Parker
Programme Secretary

Niall MacKay
Publications Secretary

Kevin Houston
Education Secretary

Sara Lombardo
Member-at-Large (Women and Diversity)

Peter Ashwin
Member-at-Large

Anne-Christine Davis
Member-at-Large

Elaine Crooks
Member-at-Large

Andrew Dancer
Member-at-Large

Jessica Enright
Member-at-Large

Minhyong Kim
Member-at-Large

Frank Neumann
Member-at-Large

Rachel Newton
Member-at-Large

Brita Nucinkis
Member-at-Large

Anne Taormina
Member-at-Large

Amanda Turner
Member-at-Large

Incoming Officers and Members of Council

Publications Secretary:
Niall MacKay is Professor of Mathematics in the Department of Mathematics at the University of York. He gained his PhD at the University of Durham in 1992; previous appointments have been as a Japan Society for the Promotion of Science Fellow, RIMS, Kyoto University (1992–1993), PPARC Research Fellow and fellow of Queens' College, Cambridge (1993–1995), Stokes Fellow, Pembroke College, Cambridge (1995–1998), and Lecturer, University of Sheffield (1998–1999). He joined the University of York in 2000, where he has been a Lecturer (2000), Senior Lecturer (2005), Reader (2009), Professor (2014) and Head of Department (2015–2021). Professor MacKay's research interests are integrable systems and quantum groups; operations research and history. His service to the LMS has been as a member of the Education Committee from 2004–2009 and 2011–2014, as an Editorial Adviser from 2005–14, as a Council Member-at-Large and a member of the Newsletter Editorial Board, Personnel Committee and Publications Committee during the last year.

Member-at-Large (Women and Diversity):
Sara Lombardo is a Professor of Mathematics at Loughborough University. She gained her PhD at the University of Leeds in 2004 and has held various post-doctoral positions in Leeds, Manchester, Rome and Amsterdam, before returning to the UK in 2010, accepting a Senior Lectureship at Northumbria University in Newcastle-upon-Tyne. Promoted to Reader in 2014 and serving as Head of Mathematics between 2014 and 2017, Professor Lombardo obtained a personal Chair in 2017 and is currently an Associate Dean at Loughborough. Her research interests are integrable systems, bringing together algebra, analysis and geometry to tackle problems in mathematics and mathematical physics. Professor Lombardo is currently on the EPSRC Mathematical Sciences Strategic Advisory Team (SAT) and will be SAT Chair from January 2022. She is also EDI Champion for EPSRC. Professor Lombardo has been a member of the LMS Women in Mathematics Committee (now the Committee for Women and Diversity in Mathematics) since 2017 and is also a member of the Good Practice Steering Group.

Members-at-Large:
Jessica Enright is a Senior Lecturer in the School of Computing Science at the University of Glasgow. She received her PhD from the University of Alberta in 2011. Previous appointments include postdoctoral research associate modelling animal disease outbreaks at the University of Glasgow from 2012–2015, a Lecturer in Mathematical Biology at the University of Stirling from 2015–2018 and a Lecturer in Global Academy of Agriculture and Food Security at the University of Edinburgh from 2018-2019. Dr Enright's research interests include graph theory and complex networks, particularly graphs and networks with temporal, spatial, or geometric structure; combinatorial games on graphs; and application and modelling involving infectious disease. A member of the EPSRC early-career forum for mathematics, she has been General Secretary of the Edinburgh Mathematical Society since 2017 and a member of the Young Academy of Scotland since 2018. She has served as a member of the Society's Newsletter Editorial Board since 2020.

Rachel Newton is a Reader in Number Theory at King's College London. She received her PhD from the University of Cambridge in 2012. Her previous appointments were as a postdoc at Leiden University (2012–2013 and 2014) and the Max Planck Institute for Mathematics (MPIM) (2013–2014), a European Post-Doctoral Institute Fellow at MPIM and IHES (2014–2015), and as a Lecturer/Associate Professor at the University of Reading (2016–2021). Dr Newton's research interests are in number theory, especially rational points on algebraic varieties, local-global principles and Brauer–Manin obstructions. For the LMS, she has been a member of the Early Career Research Committee since 2017, Society representative and chair of the 2021 Cecil King Travel Scholarship assessment panel and LMS Departmental Representative for the University of Reading 2016-2017. Dr Newton is a co-organiser of the Egham–Reading–London Arithmetic Statistics Seminar, an LMS Scheme 3 research network running since 2019.

Retiring Officers and Members of Council

Jonathan Keating
LMS President 2019–2021

After serving as LMS President for two years, Professor Jonathan Keating FRS handed over the badge of office at the AGM on 12 November 2021.

The Society has benefited greatly from the superb leadership of Professor Keating over the past two years. He has demonstrated unwavering energy, enthusiasm and ambition for the Society and its objectives while guiding it securely through a very turbulent time national and globally.

During his time as President, Professor Keating was met with unprecedented challenges arising from the Covid-19 pandemic. The Society had to change the way in which it conducted its core activities, with De Morgan House closed, Society meetings and events moved online and profound limitations on face-to-face collaboration within the mathematical community. Professor Keating ensured that the Society rose to these challenges. At the start of the pandemic, a special 'Covid' reserve fund was set up to provide extra support for early career researchers, and to cover the costs of adapting some of the Society's existing grant schemes to make them more relevant and effective during the pandemic. For example, extra funds were provided for Joint Research Groups to produce online lecture series and, when the pandemic restrictions began to be lifted, the Society launched its Research Reboot scheme allowing researchers a period of time away from their usual environments and responsibilities to concentrate fully on their research. The Society also continued to co-sponsor the highly successful *Teaching and Learning Mathematics Online* (TALMO) events, helping colleagues who have been required to carry out their teaching online.

The Society has learned valuable lessons from the pandemic, including that online events can be as effective as in-person events, as well as being cheaper, more inclusive and more environmentally friendly. Building on this insight, and on a Carbon Trust Energy Efficiency Audit of De Morgan House, Council under Professor Keating's leadership has adopted an Environmental Policy Statement reinforcing the Society's ongoing commitment to sustainability.

Professor Keating has been a strong supporter of diversity and inclusion in mathematics. He has built bridges across the mathematics community through his wide network of contacts, his skill in developing relationships both within academia and beyond, and his ability to interest generous benefactors in the work of the Society. Together with Dr Nira Chamberlain, President, Institute of Mathematics and its Applications, Professor Keating championed the development of the Black Heroes of Mathematics Conference. There have now been two conferences attracting up to 700 participants online with support from across the mathematical community, including the International Centre for the Mathematical Sciences (ICMS), Isaac Newton Institute (INI), British Society for the History of Mathematics (BSHM) and the Mathematical Association (MA). With the generous support of Dr Tony Hill, the Society developed and launched the Levelling Up online tutoring programme for A-level mathematics students from under-represented backgrounds. Following a pilot with Durham University and the University of Leicester, this programme is now expanding across England. Ongoing donations from the Liber Foundation have meant that Society has been able to continue to award Emmy Noether Fellowships to support mathematical researchers with caring responsibilities. The Society has also developed and published its Caring Costs Policy which outlines the funding available to allow those with caring responsibilities to attend events that the Society supports.

Professor Keating has worked extensively to increase the Society's public profile and he has acted as an advocate and spokesperson for the Society across a wide range of issues. He oversaw the publication by the Society of several statements on matters of concern to the mathematical community, including regarding mathematics departments at threat of closure and risks to the human rights of mathematicians. A major initiative on which the Society has taken a lead under Professor Keating's guidance is the Protect Pure Maths campaign, which is helping to ensure the long-term health of all areas of the mathematical sciences. The goals of this campaign include ensuring that the mathematical sciences are funded properly, that they are represented and understood in Parliament and that further cuts to mathematics research, including pure mathematics research, in universities are prevented.

The impact of changes in the academic publishing market, including the drive towards open access, became apparent earlier this year when the Society

renewed its contract for publication of its core journals. As a result, the Society must now adapt to significant reductions in its income from its publishing activities. Professor Keating and the Council acted decisively in reviewing the Society's budgets and adjusting the Society's expenditure so that the Society's finances have now been placed on a solid foundation for the future.

Professor Keating's final duty as President was to welcome back LMS Members face-to-face at the 2021 LMS AGM, which was the first in person Society meeting to take place since March 2020. This allowed LMS members and those from the wider mathematics community to once again meet and to network. It also marked the first time the Society had run a hybrid meeting where those unable to attend in person a could join a live stream.

The Society would like to thank Professor Keating for his dedication and for his passion and leadership during his two years in office. The Society wishes him well for the future. At the AGM, Professor Keating handed over the badge of Presidential Office to Professor Ulrike Tillmann FRS.

John Hunton
LMS Publications Secretary 2013–2021

After eight years, Professor John Hunton, Deputy Head of Mathematics, Durham University, has stepped down as LMS Publications Secretary.

Professor Hunton chaired the LMS Publications Committee, driving the Society's Publications Strategy, leading its management of contracts, appointment of editorial boards and maintaining positive publishing partnerships, including those with Cambridge University Press (CUP), Wiley and Institute of Physics Publishing (IOPP).

During Professor Hunton's tenure as Publications Secretary significant shifts have taken place within scholarly publishing. He led on several major initiatives to develop the LMS publications offering to the mathematical research community, with the continued production of high-quality research journals and ensured a robust income to the Society, which is critical to funding the Society's charitable activities.

In 2014 Professor Hunton oversaw the offering of free online access to the *Bulletin, Journal* and *Proceedings of the LMS* to individual members and the adoption of a Publishing Ethical Policy for the Society's journals, including guidance on the expected behaviour of authors, referees and editors. Starting in 2015 LMS individual members were also offered free online access to *Nonlinearity*.

He led the Society through two publishing partnership tenders for the LMS-owned journals in 2015 and in 2020 and successfully oversaw the renewal of contractual arrangements for several other key partnerships. Such contract negotiations are lengthy and complex, and the Society is indebted to Professor Hunton for his knowledge and experience in helping to obtaining favourable agreements for the Society.

In 2017 Professor Hunton convened a two-day Publications Retreat, to stimulate discussions on all aspects of LMS publishing operations and to identify priorities for the work of the LMS Publications Committee. Following discussions at this Retreat and during 2017–18 LMS Council agreed new Aims and Objectives for its publications, which together formed a three-year Publications Strategic Plan. The new Aims focused on the provision of services for authors and readers, leadership and representation within UK mathematics publishing, and sustainability of the Society's income from publishing.

In the past two years a significant development for LMS publications has been the change to the identities of the LMS journals with a relaunch of *Proceedings of the London Mathematical Society* in 2019 and *Transactions of the London Mathematical Society* in 2021, both with new, separate Editorial Boards.

Building relationships with mathematicians around the world is incredibly important to the Society. Professor Hunton chaired a publications panel at the 7ECM held in Berlin in 2016 and in 2019 he was part of an LMS delegation, led by the LMS President, to Mainland China and Hong Kong that explored new opportunities for joint publications. The delegation met the leadership of the Chinese and Hong Kong Mathematical Societies, and several other key mathematical institutes.

Professor Hunton also played a prominent role in communicating the Society's views on a range of issues affecting academic publishing. In 2019 the Society responded to the Guidance on the Implementation of Plan S (for full and immediate Open Access) and, as part of the UKRI Open Access Review, provided evidence on the value of the Society to the research community.

The Society had previously provided evidence to the RCUK Open Access Review in 2014.

Professor Hunton's wide experience has benefited the Society and the mathematics community during his tenure as LMS Publications Secretary. The LMS is extremely grateful to him for the support he has given to the Society and the wider Mathematical Sciences community and for the time and energy he has devoted to the Society's publications activities and wishes him well for the future.

Tony Gardiner
Member-at-Large 2015–2021

Dr Gardiner has served as a Member-at-Large on Council from 1992–1997 and 2015–2021, as well as being Education Secretary from 2011–2012. He has served on many committees and groups: Education Committee (1996–2000 and 2011–2018), playing a major role in the committee that produced the influential report *Tackling the mathematics problem* in 1995 and helping to set up the Cecil King Travel Scholarships in 2000. He was also the link for Undergraduate Summer Schools from 2015–2017 and is currently a Holgate Lecturer. The LMS thanks Dr Gardiner for his contributions to Council meetings, committee membership and continuing involvement with the work of the Society.

Mark McCartney
Member-at-Large (Librarian) 2018–2021

Dr McCartney has been Member-at-Large (Librarian) on Council since 2018. He was a member of the LMS Newsletter Editorial Board from 2017–2020, acting as the Reviews Editor and has been a member of Education Committee since 2020. Dr McCartney was President of the British Society for the History of Mathematics from 2018–2020 and is Associate Editor of the *International Journal of Mathematical Education in Science and Technology*. The Librarian no longer has to be a Council Member-at-Large, but Dr McCartney will continue as Librarian following Council's decision to reappoint him for 2021–2022. The LMS is grateful to Dr McCartney for his ongoing commitment to achieving the aims of the Society over several years.

Speeches at the 2021 LMS AGM Reception and Dinner

Jon Keating, retiring LMS President:

It is a tradition that before our post-AGM dinner, every second year, we have two speeches, the first by the immediate past-President, which is backward looking, and the second by the new President, which is forward looking, and which tries to anticipate the opportunities and challenges to come. In preparing this speech I looked back at the one I gave two years ago and I fear my crystal ball then may not have been functioning quite as I had hoped, because the period since has certainly not turned out as I had anticipated. So I am pleased now to be talking about the past, not trying to foresee the future again.

In the context of what we have all experienced recently, it feels particularly special to be meeting in person again. One of the main reasons for having a Society like ours is to support and enrich the whole mathematical community and there really is no better way of achieving that than in seeing each other and in talking face-to-face, especially at an evening event like this, over dinner.

Being LMS President is a great privilege. Usually because one gets to travel the country, and occasionally further afield, meeting and talking to mathematicians, and I had greatly looked forward to doing this. But for obvious reasons that has not been my experience. My one international trip was just before I took over the Presidency, in early November 2019, when I was asked by my predecessor to join a delegation to China, to participate in the 13th National Congress of the Chinese Mathematical Society and to promote our journals there – a trip that, in retrospect, might have been more risky than I at least understood at the time.

Instead I have been privileged in different ways: over the past two years, I have got to work closely with the Society's Officers, Council Members, other Society Members who volunteer their time and energy, and with the wonderful LMS staff, to redirect our support for the community at a time when it has faced unique and serious challenges, and so when that support has been particularly important.

I really cannot praise the colleagues with whom I have worked highly enough. They have all dedicated huge time and thought to considering how the Society should position itself and use its resources to best effect during this period of challenge and uncertainty, as well as to anticipating how we might need to do this in the future; a future that is not at all easy to predict. I believe that the Society is extremely fortunate to have: a quite exceptionally capable set of officers; a Council that is fully engaged, which has the community's interests at the front of its collective mind and which has combined in a most impressive way the multiple requirements of providing challenge and scrutiny, constructive, collegial debate, and strategic thinking; we are also fortunate to have Members who unselfishly volunteer to support the Society's work; and a highly talented and committed staff.

I would like to thank them all. It is invidious to pick out individuals, but had we been able to have our annual dinner last year I would have sung the praises of long-serving Officers who stepped down then — Rob Curtis and Stephen Huggett — both of whom have made outstanding contributions to the Society over many years. We are very fortunate indeed that Simon Salamon and Robb MacDonald have stepped so ably into their roles. I would also have lauded other members of Council who stood down last year: Sasha Borovik, Tara Brendle, Dai Evans, Richard Pinch, and Mariya Ptashnyk, who collectively made significant contributions to Council's work.

This year John Hunton will finish as our Publications Secretary — a role that is hugely important to the Society — and we are grateful to him for overseeing the renewal of our publishing contract. And Mark McCartney, and Tony Gardiner will step down from Council, again after many years of excellent contributions to its work.

The Society's staff have been simply outstanding, despite having to work remotely until recently. They have kept the Society's business running most effectively. This has been a wonderful team effort, but I would like to pay particular tribute to Caroline Wallace for her leadership during this challenging period. I cannot begin to do adequate justice to her.

I should also like to express the Society's deep gratitude to all of our donors, whose support is vital to the Society's function: a significant number of people make donations and bequests and these feed directly in to our charitable work. I would particularly like to thank all of those who have become De Morgan Donors, making substantial gifts that added considerably to our ability to support mathematicians and mathematical events over the past two years, the Liber Foundation, for supporting the LMS Emmy Noether Fellowship scheme for mathematicians with caring responsibilities, Tony Hill, for his support of the Levelling Up: Maths initiative, providing tutoring for A-level students from disadvantaged backgrounds and underrepresented minorities, and XTX Markets, for their support of our activities in promoting mathematics with key opinion formers and more widely. These have been some of our major initiatives over the past two years and they would not have been possible without this generous support.

One thing I would like to highlight is that over the past two years we have enjoyed excellent and highly cooperative relations with our sister learned societies in the mathematical sciences. I should like to mention in particular our collaboration with the IMA and the BSHM to establish meetings showcasing the work of black mathematicians, we hope inspiring black students to become mathematicians. It was wonderful to see over 700 participants registering for this year's event from across the world, and it is a pleasure to be able to thank Nira Chamberlain for his leadership of what I believe to be an important collaborative venture.

What have I learned over the past two years? Well, I have certainly learned to use Zoom, and my speed of reaction now when reminded to unmute is, I think, reasonably competitive. I have also learned that those parts of Meetings where one discusses a risk register are perhaps less prosaic than I had previously thought them to be — the Society has benefitted enormously from the care taken in the past in the unglamorous work of preparing us for eventualities that at the time must have seemed highly remote. But more seriously, I have learned that our Society benefits from a robustness, a resilience, a self-confidence that comes from our long history and which has been laid down over many generations. I have also been reminded of how important our sense of community is to us all – it has been most impressive to see how people have donated money and volunteered their precious time to help others, for example students, early career researchers, and members of the community who have had caring responsibilities, and also how the community has rallied round to support its members who have found themselves in difficulties. Again, the foundations for this sense of community have been laid down over generations, through meetings and evening events like this one tonight. It is a precious resource, and one we should continue to foster.

I do believe that the Society is entering an important phase of its history, with the move of mathematics online, fundamental changes to our working practices, increasing pressure on our resources, and with the challenges we face relating to climate change. I am therefore delighted that we have the ideal person in Ulrike Tillmann to take us forward.

With that in mind, let me offer a toast: "To the continued health of mathematics and the London Mathematical Society"

Ulrike Tillmann, Incoming LMS President:

I am very honoured to have been elected President of the LMS. The LMS has been a mathematical home for me for many years.

It is 30 years ago when as a young postdoc, recently arrived from the US, I gathered courage to ask Graeme Segal and Ray Lickorish to sign my application for membership. Receiving the news that it had been approved, I had a strong feeling of belonging, of arrival.

Later, the first funding I applied for was an LMS Scheme 1 grant for a British Topology Meeting. It was a relatively small pot of money but it both gave me encouragement and enabled me, in this case to organise the conference. For both I am very grateful!

This is really what I think the LMS does best: building community, supporting and enabling budding mathematicians. The latter in particular must be one of the most valuable things that we do. And we do it in increasingly many different ways. Whether it is the Undergraduate Research Bursaries, the Early Career Fellowships, or now the Levelling Up Scheme.

Looking forward, in the coming years we will need to discuss ideas for a National Academy. There is no doubt in my mind that as a community we will have to grow together, recognise and build on each other's strength. Jon (Keating) and Nira (Chamberlain) led the way; Paul (Glendinning) and I are already talking. And with Alison (Etheridge) at the head of CMS the conversations across the whole of the mathematical sciences will be effective. Whatever the outcome, a United Nations or a nation state, we will need to make sure that there is a net positive for mathematics!

My intention is to keep this short as dinner is waiting. But I want to thank Jon for the thorough induction he has given me. I have learnt a lot watching him in the past year. Not only did he have to navigate the Covid months, he also had to steer Council through tough decisions that saw significant budget cuts. He will be a difficult act to follow! Let me also say that I am looking forward working with Caroline (Wallace), Cathy (Hobbs) and Iain (Gordon), and the whole of Council.

There are two anniversaries that we are marking today, and it gives me great pleasure to invite you to toast our two celebrants.

10 Years: Elizabeth Fisher, Membership and Grants Manager

I remember well when Elizabeth started her job in January 2010. She seemed so young but tremendously dependable and capable, managing a whole range of different activities: from taking minutes at the Prizes Committee, and Research Meetings Committee, corresponding with applicants and referees, and preparing for the AGM or one of the many other Society meetings.

There is roughly one meeting a month, that's over 100 meetings over the time Elizabeth has been with us (Elizabeth nods, confirming her attendance at most of these) in addition to similar number of committee meetings.

It is an enormous amount of work Elizabeth has done for the Society over the last 10 years in her calm and steady way. Quite literally, the Society would not have functioned without her. We owe her a great deal!

40 Years: Susan Oakes, Administrative Editor for the LMS Newsletter

Susan took her position with the LMS in January 1981 as the first full time employee. By the time I got involved with the Society, Susan was already a legend. For many she was the Society, or at least the one who kept everything together.

Susan officially retired from her job as LMS Administrator in 2008 but she remains actively involved in the Society's business and not only with the LMS Newsletter. For one example, Caroline (Series) convened a committee to plan for celebrating our 21 years in De Morgan House. As you can imagine, Susan — and I quote Caroline here — "became a central and indispensable member".

To finish, let me quote from the 2001 Treasurer's report by Alun Owen Morris: "... her incomparable knowledge of the Society's affairs and her constant dedication to the Society's needs are a source of inspiration to us all." 20 years later, these words that remain as true today as then!

Susan Oakes, Administrative Editor for the LMS Newsletter:

First I would like to thank the LMS for inviting me to this reception and the dinner. But even more important to thank the LMS for employing me 40 years ago. I was the first full time member of staff. Prior to me there had twice been part-timers in succession. And that's since 1865. It must have been incredible how much work the Council and especially all the Officers were dealing with prior to employing a member of staff. It was thanks to an Officer back in 1981 that I became aware of the job vacancy. At the time I was the mathematics departmental secretary at Queen Elizabeth College London. David Brannan, also at QEC, was at the time LMS Council & General Secretary, who happened to place on my desk the LMS job advertisement. As they say, the rest is history. I am so fortunate to have over the years met so many mathematicians, including Chern, Conway and Hawking to name but a few, and been all over the world due to having LMS stands at meetings such as the ICM in Beijing.

As most of you know the LMS used to be at Burlington House. We rented half an office from the Royal Astronomical Society on the top floor with no lift. People such as the dear Treasurer the late Rolph Schwarzenberger, the late Alan Pears the Meetings and Membership Secretary and Nick Bingham the Bulletin Book Reviews Editor, who is here today, often managed to clamber up the stairs to meet with me. Fortunately for the Council they met at a meeting room on the ground floor. All the technology I had to run the office was a golf ball typewriter, a rolodex membership form holder and a filing cabinet. The LMS then employed two part-timers so I managed to move us to another room, and then the LMS had its own office. 22 years ago it was decided we should have our own property. A small group of us endeavoured to find somewhere. Needless to say we did look in the Spitalfields area. Being at Russell Square not too far from UCL is great because our Library is there and of course our first President Augustus de Morgan was there.

My first President was Barry Johnson. Since then I have had the pleasure of working with twenty one Presidents of which two were female Presidents, Frances Kirwan and Caroline Series and now it will be my third, Ulrike. The only one I did not work for was Mary Cartwright as she was President back in 1961.

It's now 21 years that we are at De Morgan House. It was when we moved to DMH that it was decided to have an Executive Secretary. Ben Garling was the first one, having been LMS Meetings & Membership Secretary, and now it's Caroline [Wallace] the fourth one, who began during lockdown and has risen so well to the incredible challenge of the current Covid era. She motivates the staff, who are working so efficiently. I know this because these days I am just involved in the production of the Newsletter and they are all great to work with. Once again, thank you LMS for employing me for over forty years. Thank you.

Update on LMS Publications

Professor Niall MacKay was elected to the role of Publications Secretary at the 2021 AGM. He takes over the role from Professor John Hunton who stepped down from the role after eight years of excellent service against a background of significant changes within the scholarly publications landscape.

In 2022 the Society's second contract term with Wiley, for the production, sales and marketing of the *Bulletin, Journal, Proceedings, Transactions, Journal of Topology* and *Mathematika*, begins. This contract renewal was agreed by the Society following the conclusion of a tendering process and the two parties look forward to continuing their partnership.

In 2022 no printed edition of these six journals will be produced. This decision was taken in response to the increased prevalence of institutions with online-only access (over 6000 in 2020) and the associated sharp decline in institutional print subscriptions (at most 16 per journal in 2020).

LMS Members will continue to be able to obtain free online subscriptions to the *Bulletin, Journal, Proceedings* and *Mathematika*.

Readers will also notice that changes to the style and layout of articles published in the journals from the beginning of 2022. The journals have also adopted the AMS referencing style for published articles. Authors remain welcome to submit in any style they wish.

The Society's journals have seen a 5% growth in submissions compared with the previous 12 months and 20% more articles were published in 2021 volumes compared to 2020. Further growth in published article output in 2022 is predicted.

A significant contribution to the success of the Society's journals can be attributed to the work of the volunteer Editors, Editorial Board members and reviewers. Thanks go to the 90+ Editors and Board members working across the five LMS-owned journals and 900+ mathematicians who provided a referee report or quick advice report for the journals.

The Society would like to thank those who finished their terms on the joint Board, serving the *Bulletin* and *Journal*, in 2021: Karin Baur, Raf Bocklandt, David Craven, Adam Harper, Alan Haynes, Tuomas Hytönen, Peter Linnell, Darren Long and Nikolay Nikolov. For their time on the Board of the *Proceedings*, we thank Nalini Anantharaman and Anna Wienhard.

Several long-standing members of the *Journal of Topology* Board stepped down in 2021. The Society thanks Arthur Bartels, Simon Donaldson and Frances Kirwan for their long service to the journal. Thanks also go to Marc Lackenby who stepped down as Managing Editor in November 2022. He is succeeded in this role by Ivan Smith.

Suzanne Abbott
Publications Development Manager

The Magical Ewens Sampling Formula: 2017 Presidential Address

An article by Professor Simon Tavaré which is based on his Presidential Address given at the LMS AGM on 10 November 2017 and entitled 'The Magical Ewens Sampling Formula' (doi.org/10.1112/blms.12537) has been published in the Bulletin of the London Mathematical Society. In this article Professor Tavaré shares his view on developments around the theme of the Ewens sampling formula (ESF), which was first published 50 years ago (W.J. Ewens, 'The Sampling Theory of Selectively Neutral Alleles', Theoret. Popul. Biol. 3 1972), illustrating the contribution of several of the Society's past presidents to the theory around the ESF.

LMS Supports Namibian Olympiad in Mathematics

The Society will be providing financial support for the inaugural Namibian Olympiad in Mathematics. A current recipient of an LMS Mentoring African Research in Mathematics (MARM) grant, Professor Letterio Gatto (Politecnico di Torino) has been collaborating with colleagues at the University of Namibia (UNAM) with the goal of obtaining support for the Olympiad. Financial support was secured from the Namibian National Council for Research Science and Technology (NCRST) to host the Olympiad and the LMS MARM affiliation at UNAM was crucial in providing the necessary expertise and guidance to make this possible.

MARM links African academics with their UK and European counterparts via professional mentoring partnerships. This provides the means and opportunities for African mathematicians to develop international working relationships. The Society believes that enhancing and developing relationships with African academics and students will help ensure that pursuing first rank mathematical careers within Africa will be both an achievable and attractive option. Events such as the Olympiad in Mathematics are one

way of fostering the next generation of mathematicians and promoting the benefits of mathematics to young people, one of the core aims of the Society.

More information about MARM is available at tinyurl.com/3fxwyfft. Information about the LMS MARM UNAM affiliation is available at tinyurl.com/2sht86ej.

Forthcoming LMS Events

The following events will take place in forthcoming months:

LMS South Wales and South West Regional Meeting and Workshop: 4–6 January 2022, Braces in Bracelet Bay, Swansea.

LMS Invited Lecture Series 2022 — The Mathematics of Deep Learning: 28 Feb–4 March 2022, Centre for Mathematical Sciences, University of Cambridge.

Diverse Perspectives on Alan Turing and Society Meeting: 4 March 2022, De Morgan House, London.

LMS Midlands Regional Meeting and Workshop — Mirrors, Moduli and M-theory in the Midlands: 4–7 April 2022, Birmingham.

A full listing of upcoming LMS events can be found on page 60.

OTHER NEWS

News from the James Clerk Maxwell Foundation

The James Clerk Maxwell Foundation was set up in 1977, with the aim of providing grants and donations to academics. In 1993 it bought the house that was the birthplace of James Clerk Maxwell at 14 India Street. Elmer Rees, on behalf of the newly formed ICMS, and David Ritchie of the Foundation approached the Scottish Government to award a loan of £100,000 to complete the purchase; the intention being that the Foundation would rent out a part of the building to the International Centre for Mathematical Sciences (ICMS). The purchase of the building transformed the activities of the Foundation and we now are open for visitors and scholars of Maxwell.

The Foundation has essentially three functions, one charitable and the other two commemorative. Its general purposes are:

- to promote, encourage and advance the study of, the research into and the dissemination of knowledge relating to STEM;
- to commemorate, with no view to profit, the said James Clerk Maxwell;
- create a small museum at India Street to commemorate the scientific achievements of James Clerk Maxwell.

Our assets are the house and our excellent Trustees and Fellows who are involved in various outreach activities and are enthusiastic in giving their time to promote Maxwell and his works at all levels, from school to distinguished scientists. Our small museum at India Street is arranged over three rooms: the first contains family portraits and displays relating to Maxwell's major advances on colour and electromagnetics; the library has wall panels on his many other scientific achievements; and upstairs we have panels on his life and we display watercolours by his cousin and aunt. We welcome individual visitors to India Street for guided tours by arrangement and we publish a Newsletter twice a year that is distributed widely. We also plan to open up for more events next year.

We are pleased to welcome back the ICMS to the Foundation's home at 14 India Street; they will take up the second-floor offices that will be used by visiting scholars and will be hosting some receptions in the first-floor rooms.

We are also pleased to announce the appointment of a new Trustee and three new Fellows to the Foundation:

Martin Hendry, Professor of Gravitational Astrophysics and Cosmology at Glasgow, is a new Trustee. Martin has longstanding involvement with Maxwell. He was instrumental in obtaining the funding for the Maxwell Torch and he is currently Programme Convenor at the RSE.

Catherine Heymans, the Astronomer Royal for Scotland, Professor Michael Kosterlitz, Nobel Laureate 2016, and Sir Roger Penrose OM, Nobel Laureate 2020, have agreed to be Fellows of the Foundation. All are well known for their outreach activities and commemorating Maxwell.

If you are interested in visiting the house, visit the webpage clerkmaxwellfoundation.org/html/visit_booking.html for further information.

Peter Grant, Interim Chair
Trustees of the JCMF

PROMYS Europe 2022 — Call for Applications

PROMYS Europe, a challenging mathematics summer programme based at the University of Oxford, is seeking applications from pre-university students from across Europe who show unusual readiness to think deeply about mathematics, as well as from undergraduate students who would like to work with them as counsellors.

PROMYS Europe is designed to encourage mathematically ambitious students who are at least 16 to explore the creative world of mathematics. Participants tackle fundamental mathematical questions within a richly stimulating and supportive community.

The programme is dedicated to the principle that no one should be unable to attend for financial reasons. Most of the cost is covered by the PROMYS Europe partnership and by generous donations from supporters. In addition, full and partial financial aid is available, for those who need it.

For more information and details of how to apply visit the PROMYS Europe website promys-europe.org.

MATHEMATICS POLICY DIGEST

Mathematical Sciences Funding

Following the government's announcement in January 2020 to invest additional funding into Mathematical Sciences, UK Research and Innovation (UKRI) has awarded around £104 million of additional funding to the discipline, over and above EPSRC's core Mathematical Sciences Theme budget. The additional funding has covered institutes, small and large research grants, fellowships, doctoral studentships and postdoctoral awards.

Following the Comprehensive Spending Review on 27 October 2021, the Department for Business, Energy and Industrial Strategy (BEIS) and UKRI will now set out how to meet the commitment to invest additional funding into Mathematical Sciences in the coming years.

A recent blog written by Isobel Stephen (UKRI Executive Director, Strategy, Performance and Engagement) outlines the process for allocations within UKRI: tinyurl.com/yzbxbcc7.

Digest prepared by Dr John Johnston
Society Communications Officer

Note: items included in the Mathematics Policy Digest are not necessarily endorsed by the Editorial Board or the LMS.

Reminders: Deadlines for Upcoming LMS Calls

- Call for Applications for Early Career Fellowships 2022; deadline 14 January 2022. Full details in the September 2021 issue, page 19 and on the LMS website at bit.ly/3Ek7uWD.
- Call for Proposals for LMS-Bath Symposia 2023; extended deadline 31 January 2022. Full details in the November 2021 issue, page 10 and on the LMS website at bit.ly/3ogVNui.
- Call for Applications for Atiyah Fellowships 2022-23; deadline 31 January 2022. Full details in the November 2021 issue, page 13 and on the LMS website at bit.ly/3oljqlw.
- Call for Nominations for the Hardy Lecture Tour 2023; deadline 31 January 2022. Full details in the November 2021 issue, page 11, and on the LMS website at bit.ly/3oiYoE1.
- Call for Nominations for the Invited Lecture Series 2023; deadline 1 February 2022. Full details in the November 2021 issue, page 12, and on the LMS website at bit.ly/31tkQ4o.
- Call for Expressions of Interest for the LMS Undergraduate Summer School 2023; deadline 22 February 2022. Full details in the November 2021 issue, page 11, and on the LMS website at bit.ly/3Ek85Yn.

LMS Grant Schemes

LMS Grant Schemes
The next closing date for research grant applications (Schemes 1,2,4,5,6 and AMMSI) is 22 January 2022. Applications are invited for the following grants to be considered by the Research Grants Committee at its February 2022 meeting. Applicants for LMS Grants should be mathematicians based in the UK, the Isle of Man or the Channel Islands. For grants to support conferences/workshops, the event must be held in the UK, the Isle of Man or the Channel Islands:

Conferences (Scheme 1)
Grants of up to £5,500 are available to provide partial support for conferences. This includes travel, accommodation and subsistence expenses for principal speakers, UK-based research students, participants from Scheme 5 countries and Caring Costs for attendees who have dependents.

Visits to the UK (Scheme 2)
Grants of up to £1,500 are available to provide partial support for a visitor who will give lectures in at least three separate institutions. Awards are made to the host towards the travel, accommodation and subsistence costs of the visitor. Potential applicants should note that it is expected the host institutions will contribute to the costs of the visitor. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents.

Research in Pairs (Scheme 4)
For those mathematicians inviting a collaborator, grants of up to £1,200 are available to support a visit for collaborative research either by the grant holder to another institution abroad, or by a named mathematician from abroad to the home base of the grant holder. For those mathematicians collaborating with another UK-based mathematician, grants of up to £600 are available to support a visit for collaborative research either by the grant holder to another institution or by a named mathematician to the home base of the grant holder. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents.

Collaborations with Developing Countries (Scheme 5)
For those mathematicians inviting a collaborator to the UK, grants of up to £3,000 are available to support a visit for collaborative research, by a named mathematician from a country in which mathematics could be considered to be in a disadvantaged position, to the home base of the grant holder. For those mathematicians going to their collaborator's institution, grants of up to £2,000 are available to support a visit for collaborative research by the grant holder to a country in which mathematics could be considered to be in a disadvantaged position. Applicants will be expected to explain in their application why the proposed country fits the circumstances considered eligible for Scheme 5 funding. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents. Contact the Grants

team if you are unsure whether the proposed country is eligible, or check the IMU's Commission for Developing Countries definition of developing countries (tinyurl.com/y9dw364o).

Research Workshop Grants (Scheme 6)
Grants of up to £10,000 are available to provide support for Research Workshops. Research Workshops should be an opportunity for a small group of active researchers to work together for a concentrated period on a specialised topic. Applications for Research Workshop Grants can be made at any time but should normally be submitted at least six months before the proposed workshop.

Computer Science Small Grants (Scheme 7)
Grants of up to £750 are available to support visits for collaborative research at the interface of mathematics and computer science. Applications from early career researchers are particularly encouraged.

African Mathematics Millennium Science Initiative (AMMSI)
Grants of up to £2,000 are available to support the attendance of postgraduate students at conferences in Africa organised or supported by AMMSI. Application forms for LMS-AMMSI grants are available at ammsi.or.ke.

The next closing date for early career research grant applications (Schemes 8-9 and ECR Travel Grants) is 22 February 2022. Applications are invited for the following grants to be considered by the Early Career Research Committee at its March 2022 meeting:

Postgraduate Research Conferences (Scheme 8)
Grants of up to £2,500 are available to provide partial support for conferences, which are organised by and are for postgraduate research students. The grant award will be used to cover the costs of participants. In addition, the Society allows the use of the grant to cover Caring Costs for those who have dependents.

Celebrating new appointments (Scheme 9)
Grants of up to £400-£500 are available to provide partial support for meetings to celebrate the new appointment of a lecturer at a university. Potential applicants should note that it is expected that the grant holder will be one of the speakers at the conference. In addition, the Society allows the use of the grant to cover Caring Costs for those who have dependents.

ECR Travel Grants
Grants of up £500 are available to provide partial travel and/or accommodation support for UK-based Early Career Researchers to attend conferences or undertake research visits either in the UK or overseas.

For full details of these grant schemes, and to find information on how to submit application forms, visit the LMS website: lms.ac.uk/content/research-grants. Queries regarding applications can be addressed to the Grants Administrator Lucy Covington (020 7927 0807, grants@lms.ac.uk), who will be pleased to discuss proposals informally with potential applicants and give advice on the submission of an application.

Research Reboot Scheme

An addition to the Research in Pairs (Scheme 4) grant. Application deadline: 22 January 2022

In response to a proposal from its Covid working group, the Society's Council has approved a new 'Research Reboot' grant scheme. This scheme aims to help mathematicians restart their research activities following the intense disruption and upheaval of the pandemic.

Researchers may have found themselves with very little time for research due to illness, caring responsibilities, increased teaching or administrative loads, or other factors. This scheme offers funding for travel, accommodation, subsistence and caring costs for applicants so that they can leave their usual environment to focus entirely on research for a period from two days to a week. For applications submitted by the next deadline (22 January 2022), the Reboot Retreats should take place between 15 March–30 June 2022. Applicants should be mathematicians based in the UK and may be at any career stage.

The value of the award will be a maximum of £100 per day for accommodation, subsistence, travel and other necessary expenses to enable the research project. An additional £100 per day may be applied for to cover Caring Costs for those who have dependents. For more information and the application form visit: lms.ac.uk/grants/research-pairs-scheme-4. Prospective applicants are advised to consult the guidance available at bit.ly/3y3ta7f. If you have any queries contact grants@lms.ac.uk.

Online Lecture Series: An addition to the Joint Research Groups (Scheme 3) grant. Application deadline: 22 January 2022

The value of the award will be £1,000 per financial year for those who would like to organise an Online Graduate Lecture Series. The Society built on its

Scheme 3 research networks and expertise to enhance the education of beginning postgraduate students and to support early career researchers. The introduction of the online lecture series element to the Joint Research Groups follows the successful Online Lecturer Series grant scheme, which was run in 2020 in response to the impact of the Covid-19 pandemic on the mathematical community, However, applications for this element of the Scheme 3 grants is open both to Joint Research Groups (new and current) and to mathematicians who are not part of a Joint Research Group. For more information and the application form, please visit: lms.ac.uk/grants/joint-research-groups-uk-scheme-3. Prospective applicants are advised to consult the guidance available here: bit.ly/3lx7zyF. If you have any queries contact grants@lms.ac.uk.

LMS Undergraduate Research Bursaries in Mathematics 2022. Application deadline: Tuesday 1 February 2022

The Undergraduate Research Bursary scheme provides an opportunity for students in their intermediate years to explore the potential of becoming a researcher. The award provides support to a student undertaking a 6–8 week research project over Summer 2022, under the direction of a project supervisor.

Students must be registered at a UK institution for the majority of their undergraduate degree and may only take up the award during the summer vacation between the intermediate years of their course. Students in the final year of their degree intending to undertake a taught Masters degree immediately following their undergraduate degree may also apply. Applications must be made by the project supervisor on behalf of the student. For further information contact Lucy Covington (urb@lms.ac.uk).

LMS Research Schools and Research Schools in Knowledge Exchange 2023

Grants of up to £15,000 are available for LMS Research Schools, one of which will be focused on Knowledge Exchange. The LMS Research Schools provide training for research students in contemporary areas of mathematics. The Knowledge Exchange Research Schools will primarily focus on Knowledge Exchange and can be in any are of mathematics.

The LMS Research Schools take place in the UK and support participation of research students from both the UK and abroad. The lecturers are expected to be international leaders in their field. The LMS Research Schools are often partially funded by the Heilbronn Institute for Mathematical Research (heilbronn.ac.uk). Information about the submission of proposals can be found at lms.ac.uk/events/lms-research-schools along with a list of previously support Research Schools. Applicants are strongly encouraged to discuss their ideas for Research Schools with the Chair of the Early Career Research Committee. Professor Chris Parker (research.schools@lms.ac.uk) before submitting proposals. Proposals should be submitted to Lucy Covington (research.schools@lms.ac.uk) by 22 February 2022.

Clay Mathematics Institute Enhancement and Partnership Program

To extend the international reach of the Research School, prospective organisers may also wish to consider applying to the Clay Mathematics Institute (CMI) for additional funding under the CMI's Enhancement and Partnership Program. Further information about this program can be found at tinyurl.com/y72byonb. Prospective organisers are advised to discuss applications to this program as early as possible by contacting the CMI President, Martin Bridson (president@claymath.org). There is no need to wait for a decision from the LMS on your Research School application before contacting the CMI about funding through this program.

LMS Early Career Fellowships 2021–22

The LMS Early Career Fellowships are supported by the Heilbronn Institute of Mathematical Research (HIMR) with funding from EPSRC–UKRI. The 2021–22 round is now open to applications. The deadline is 14 January 2022.

To support early career mathematicians in the transition between PhD and a postdoctoral position, the London Mathematical Society offers up to 8 Fellowships of between three and six months to mathematicians who have recently or will shortly have received their PhD. The award will be calculated at £1,300 per month plus a travel allowance. The fellowships may be held at one or more institutions but not normally at the institution where the Fellow received their PhD.

For further details, including how to apply, visit the Society's website at tinyurl.com/52zjp4h8. Contact fellowships@lms.ac.uk with any queries.

LMS–Bath Mathematical 2023

Call for proposals: extended deadline to 31 January 2022

The London Mathematical Society is pleased to announce its Call for Proposals for the LMS–Bath Mathematical Symposia to be held at the University of Bath in 2023. One of the LMS–Bath Symposia will be funded by the Isaac Newton Institute.

Further information, in particular regarding available funding, will be published on the Symposium website bathsymposium.ac.uk.

Formerly known as the LMS–Durham Symposia, the LMS–Bath Mathematical Symposia will be held at the University of Bath between 2020 and 2025. The Symposia are an established and recognised series of international research meetings, since their foundation in 1974, that provide an excellent opportunity to explore an area of research in depth, to learn of new developments, and to instigate links between different branches.

The format is designed to allow substantial time for interaction and research. The meetings are by invitation only and will be held in July/August, with up to 50 participants, roughly half of whom will come from the UK. A novel element of the symposia is that they will be complemented by a summer school, to prepare young researchers such as PhD students, or a 'research incubator', where problem(s) related to the topic of the conference is studied in groups. These entire event, summer school/incubator and workshop, will typically last around two weeks.

Prospective organisers should send a formal proposal to the Grants Team (grants@lms.ac.uk) by 31 January 2022. Proposals are approved by the Society's Research Grants Committee after consideration of referees' reports.

Proposals should include:

- A full list of proposed participants, divided into specific categories:
  - Category A – Scientific Organisers
  - Category B – Key Overseas Participants
  - Category C – Key UK-based Participants
  - Category D – Important Overseas Participants
  - Category E – Important UK-based Participants
- Proposers are encouraged to actively seek to include women speakers and speakers from ethnic minorities, or explain why this is not possible or appropriate.
- A detailed scientific case for the symposium, which shows the topic is active and gives reasons why UK mathematics would benefit from a symposium on the proposed dates.
- Details of additional support from other funding bodies, or proposed avenues of available funding.
- Indicative plans for the summer school or research incubator.
- Where appropriate, prospective organisers should consider the possibility of an 'industry day'.

For further details about the LMS Mathematical Symposia, please visit the Society's website: lms.ac.uk/events/mathematical-symposia or the LMS-Bath symposia's website: bathsymposium.ac.uk

Before submitting: Organisers are welcome to discuss informally their ideas with the Chair of the Research Grants Committee, Professor Andrew Dancer (grants@lms.ac.uk).

Maximising your LMS Membership: Free Online Journal Access

Since the London Mathematical Society was founded in 1865 it has published high-quality peer-reviewed papers in a growing collection of esteemed journals. Beginning with the Proceedings of the London Mathematical Society in 1865, the portfolio of journals published by the Society now comprises 12 well-regarded titles.

LMS Members can benefit from free online access to the following journals from the Society's collection:

- *Bulletin of the London Mathematical Society:* Publishing leading research in a broad range of mathematical subject areas since 1969, the *Bulletin* features high-quality and well-written research articles, authoritative survey articles and obituaries of distinguished mathematicians.
- *Journal of the London Mathematical Society:* Since 1926, the *Journal* has welcomed papers on subjects of general interest that represent a significant advance in mathematical knowledge, as well as submissions that are deemed to stimulate new interest and research activity.
- *Proceedings of the London Mathematical Society:* The flagship journal of the LMS, the *Proceedings* publishes articles of the highest quality and significance across a broad range of mathematics.
- **New for 2022** *Mathematika:* Published by the LMS on behalf of its owner UCL, *Mathematika* features both pure and applied mathematical articles, and has done so continuously since its founding by Harold Davenport in the 1950s.
- *Nonlinearity:* Owned and published jointly with the Institute of Physics, *Nonlinearity* covers the interdisciplinary nature of nonlinear science, featuring topics which range from physics, mathematics and engineering through to biological sciences.

Members can sign up to free online access to any or all of these titles either by logging into their LMS user record here: lms.ac.uk/user and going to the 'My LMS Membership' tab, or by returning a completed subscription form for the current year (which is available to download from the LMS website at: lms.ac.uk/membership/paying-your-subscription) by email (to: membership@lms.ac.uk) or by post (to: LMS Membership, De Morgan House, Russell Square, London WC1B 4HS, UK).

The Society's fully open access journal *Transactions of the London Mathematical Society* is available to both members and non-members. It welcomes papers of general or specialised nature that represent a significant advance in mathematical knowledge. The papers can be read online at tinyurl.com/ptj2pu5s. Visit our website lms.ac.uk/publications for more information about our publications or contact us at membership@lms.ac.uk if you have any queries.

Elizabeth Fisher
Membership & Grants Manager

LMS Council Diary — A Personal View

Council met via video conference on Friday 15 October 2021. The meeting began with the President's business, including reports on the successful Atiyah Conference at the Isaac Newton Institute, the second online Black Heroes of Mathematics conference, and ongoing involvement in the Protect Pure Maths campaign, which is going well and has been reported in several newspapers.

The next major item of business was a report from the Publications Secretary on the signing of a new publications contract between the Society and Wiley. The Treasurer then presented the draft Trustees' Report, including the Annual Accounts for the year 1 August 2020–31 July 2021, which Council agreed to recommend to the upcoming Annual General Meeting on 12 November, and a report on a recent meeting of the Investment Sub-committee. Taking into account recommendations from Investment Sub-committee, and the Council's commitment to sustainability through its Environmental Policy statement, Council agreed that a significant fraction of the Society's investments should be moved to an (even more) socially responsible investment fund, and that a plan should be developed for moving all of the Society's investments to the new fund.

Other business included an item on Committee Membership, and an update on Society Membership, in which the Treasurer noted that membership numbers seemed to be bouncing back, though gender balance remains a concern.

The meeting concluded with the President thanking members of Council for their contributions.

Elaine Crooks
Member-at-Large

LEVELLING UP

The latest updates about the Levelling Up: Maths scheme being developed by the LMS, made possible by a generous donation from Dr Tony Hill. The scheme seeks to widen participation of those who are under-represented in mathematics. It is part of a broader Levelling Up: STEM project which also covers Physics and Chemistry.

With a model for success now established from the pilot phase, the Levelling Up Scheme is entering a phase of expansion. As mentioned in the November issue of the LMS Newsletter the Institute of Mathematics and its Applications (IMA) had expressed interest in joining the LMS in administering the scheme. We are delighted that the LMS and IMA are now collaborating fully on the scheme. The LMS and IMA are running separate hub and spoke models in parallel, with each organisation working with different groups of universities, rather than working together on the same universities. A total of nine universities, including the pilot universities, Durham and Leicester, are set to embark on the next phase of the scheme.

As part of the continuing development of the next phase, a workshop is being held in December 2021 where universities new to the scheme will have the opportunity to learn from the experiences of the pilot phase universities, in areas including tutorial development and student recruitment, and to share best practice.

Further support for the scheme has come from the Council for the Mathematical Sciences (CMS). At its board meeting in October 2021 the CMS agreed to endorse the scheme and support the LMS/IMA model. The CMS endorsement is an affirmation of the importance of the scheme to broadening the opportunities for young mathematicians from all backgrounds.

The Society will continue to work closely with the IMA on the shared aim of widening participation of those who are under-represented in mathematics.

More information about the Levelling Up Scheme is available at levellingupscheme.co.uk

John Johnston
Society Communications Officer

Report: LMS Annual General Meeting 2021

The 2021 LMS Annual General Meeting was held on Friday 12 November, with the President, Professor Jon Keating FRS, in the Chair. Last year's AGM had been virtual, using Zoom; this year's was hybrid with some members present in the Great Hall of Goodenough College and others online.

After the confirmation of the minutes of the July general meeting, a Vice-President, Professor Cathy Hobbs, described some of the society's activities over the past year. This was followed by the report of the Treasurer, Professor Simon Salamon. He assured us that the Society is still in a strong position financially, despite the effects of both the pandemic and the continuing issue of the publication model. These had led to some changes in the Society's activities, and more would be needed in the future. A theme that ran through both reports is that the Society intends to operate in ways that are more cost effective, more inclusive and more environmentally friendly. The Society continues to receive gifts and bequests, including in the past year a copy of the rare book *Urania Propitia* and a £50 banknote with the serial number AA01001936, the last four digits being the date of Alan Turing's paper 'On Computable Numbers'.

The resolutions to adopt the Treasurer's and Trustees' Reports and to appoint the auditors for the next year were carried. This was followed by the presentation of this year's prizes. Some of the recipients were able to attend in person, others were online. The meeting congratulated all of them, and also (collectively!) the 322 mathematicians newly elected to various kinds of membership. Ten new members who had been elected at previous meetings came forward to sign the Members' Book.

After the tea break, the results of the elections for vacancies on Council and Nominating Committee were announced. The President thanked those who were retiring for their contributions to the Society. He then handed over the badge of office to the incoming President, Professor Ulrike Tillmann FRS, the Director of the Isaac Newton Institute. Professor Tillmann thanked Professor Keating for guiding the Society through a very difficult time, and after the vote of thanks from the meeting invited him to deliver his Presidential Address.

As is traditional, the Supporting Lecture, given this year by Professor Jens Marklof of Bristol University, was delivered before the election results. This practice arose because voting used to take place at the meeting and the scrutineers needed time to do the counting. This year's lectures complemented each other especially well.

Both lectures dealt with the interplay between mathematics and physics. As Professor Keating reminded us, while mathematics has always been closely connected with physics, in the 19th century the links had mostly been with differential equations, while in the 20th century geometry became more involved. Now number theory is playing an important role.

Professor Marklof described how the study of random lattices, not just 'on average' but as intrinsically random objects, has applications both in various branches of mathematics and also in some basic models in physics. (You can read about this in his article in Newsletter 493.) Professor Keating spoke about random matrices in the connection between the theory of complex quantum systems and certain properties of the Riemann zeta-function. It may have seemed surprising that a lecture on quantum theory should begin with an account of Gauss' remarkably thorough study of the distribution of primes, but by the end the connection was clear enough.

The general tone of the AGM was reassuringly upbeat. Those of us who were there, whether in person or virtually, could appreciate how the efforts of Council, the staff, and the many others who contribute their time and expertise to the Society have enabled it to come largely unscathed through the challenges of the past two years.

Peter Saunders
King's College London

Photographs from the LMS AGM 2021



(l to r) Outgoing President Jon Keating; Cathy Hobbs gives the Vice-President's report; incoming President Ulrike Tillmann

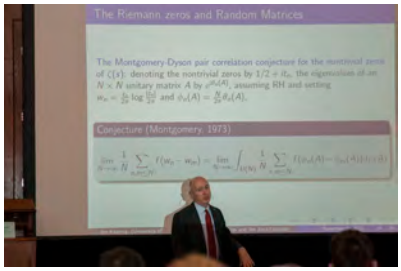


(l to r) LMS Prize Winners Viveka Erlandsson, Endre Süli and Ehud Hrushovski



New LMS members sign the Members' Book





Jens Marklof and Jon Keating give the Supporting Lecture and Presidential Address, respectively



Reception and Annual Dinner

Report: Black Heroes of Mathematics Conference UK

"Mathematics is easy, teaching it is hard." Do you agree or disagree with this statement? This was the title of the opening presentation by Professor Mamokgheti Phakeng for the 2021 edition of the Black Heroes of Mathematics Conference UK held from 5–6 October 2021. The live talk emphasised how top educators inspire and motivate students, and the importance of community engagement.

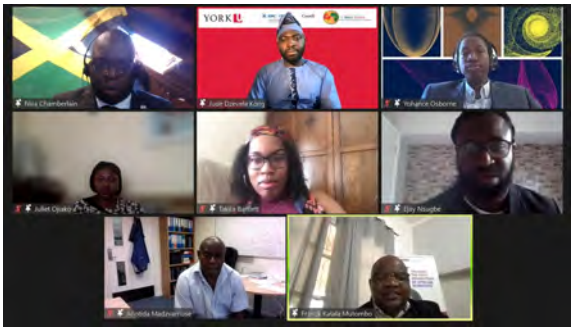
Talks were a blend of technical and nontechnical talks about research and initiatives speakers were involved with. Speakers consisted of both early career and established researchers with connections made for future collaborations between the young and old. Talks on physics and artificial intelligence added a new flavour to the talks for Day 1. The theme for Day 2 was on youth and mathematics education.

Prism Exeter and some universities hosted watch parties which brought a wide audience to this year's conference. Partners and supporters for this year's conference were: The British Society for the History of Mathematics, the International Centre for Mathematical Sciences, the Institute of Mathematics and its Applications, the Isaac Newton Institute, the London Mathematical Society, ICTP–East African Institute for Fundamental Research, African Institute for Mathematical Sciences and the Mathematical Association.

Yohance Osborne gave a talk entitled *Curiously Riding the Waves of Life Towards Success in Mathematical Sciences*. One highlight was the passionate teacher life-changing experience of Yohance. Talks for Day 1 ended with a panel discussion on the theme of mathematics.

The Mathematics Education talk by Susan Okereke highlighted the falling standard of numeracy in the UK and the need for society to value numeracy and celebrate mathematics. Dr Juliet Ojiako pulled a surprise for the audience when she played the video of a special animation she made for her talk entitled *I Can Be a Hero Too!* The talk emphasised the importance of role models to young people and how science and mathematics is for everyone.

The panel discussion for Day 2 got interesting with panelists giving their versions of the definition of who a hero of mathematics is. Panelists included Prof. Clifford V. Johnson, a theoretical physicist and consultant for science fiction films and television shows including Avengers: Endgame and Star Trek: Discovery.



Attendees at the panel discussion

There were also discussions about the lack of resources for young children on Black Heroes of Mathematics. Maurine Atieno Songa, one of the speakers, is the author of two books which are great resources: *In Dana's Honor* and *Beyond the Crevices*.

The full playlist of talks is available at tinyurl.com/ybpjv4xr.

Angela Tabiri
African Institute for Mathematical Sciences Ghana

Report: LMS Graduate Student Meeting

The LMS Graduate Student Meeting was held on 8 November. As with many meetings over the last couple of years it took place on Zoom, but this in no way prevented it from being a highly stimulating event. The meeting served as a 'warm-up' to the AGM held four days later, the highlight of which was to be Jonathan Keating's presidential address on *Random Matrices and the Riemann Zeta-Function*.

To help prepare us for the address, Jon's former PhD student Nina Snaith delivered an incredibly interesting and engaging talk on *Hollywood's Hippest Mathematics: Random Matrices and Riemann Zeros*. We were told the story of Hugh Montgomery and Freeman Dyson's fortuitous meeting over tea at the Institute for Advanced Study. Intending only to fill an awkward silence, Montgomery shared some of his progress on the distribution of Riemann zeros on the critical line. Dyson jumped in with the observation that Montgomery's distribution bore striking resemblance to that of eigenvalues of random unitary matrices he had been using to model heavy atomic nuclei.

This profound and mysterious link led some mathematical physicists (including Jon Keating) to

predict certain number theoretic results with astounding accuracy. Keating and Snaith used random matrices to predict the 'hard part' of the leading coefficient of all the moments of the Riemann zeta-function, where purely number theoretic approaches could only tackle low-order moments. Nina recalled with amusement an encounter at a conference, when eminent analytic number theorist Peter Sarnak challenged the physicists to produce something of real significance for number theory. Their moment calculations certainly rose to that challenge!

To conclude, Nina remarked that random matrix theory even made an appearance in John Madden's 2005 film *Proof*, where it was cited as an example of mathematics too 'hip' to have been used by a lately deceased professor.

After a tea-break, we were invited to join breakout rooms to hear 15-minute graduate student talks. There were three rooms of three speakers, and attendees were free to switch between rooms as they desired. Many English universities were represented, from York to Bristol, and the topics expounded ranged from fluid dynamics to group theory. I stayed in Room 2 for the duration, hearing presentations by Maram Alossaimi (Bristol) and Theodoros Stylianos Papazachariou (Essex), and giving one myself.

Maram gave us a whistle-stop tour of her classification of Poisson algebras and their spectra. She detailed some of the involved calculations that led to the results and showed us several figures to help us visualise them. Theodoros used several beautiful examples to transport us into the world of Fano threefolds, before briefly explaining how he was able to compactify the moduli space of the threefold family 2–25. My talk introduced the concepts of nonnegative curvature and integrality in the context of polyhedral manifolds. I gave an overview of my results on the singular loci of polyhedral 3-manifolds and then outlined possible further directions.

The meeting ended with an overview of the LMS's provision for graduate students, including a call for applications to the LMS Early Career Fellowships. These are 3–6 month grants to help bridge the gap between PhD and post-doc. If you're interested in applying, visit lms.ac.uk/grants/lms-early-career-fellowships for details.

Thomas Sharpe
King's College London

Report: LMS Computer Science Colloquium 2021

The LMS Computer Science Colloquium was held on Wednesday 17 November 2021 online via Zoom with the topic of the colloquium being 'Mathematical Foundations for Machine Learning'. A large number of participants enjoyed a compelling series of talks.

The first talk of the day was by Benjamin Guedj, permanent member of Inria and also principal research fellow at UCL. He has been working on Machine Learning in the "PAC-Bayes framework for a number of years and his talk was structured as an initial "primer" on this framework, followed by some details of recent developments. His personal passion is generalisation, and he discussed the significance of PAC-Bayes to generalisation bounds. After the introduction, he went on to note some recent developments by his research group, giving details of their recent results for heavy-tailed distributions, and for classification by binary activated deep neural networks. At the end of the talk there were questions - he touched on the relevance of derandomization (given we have a distribution over classifiers). He also had a query from a junior researcher interested in getting involved in PAC-Bayes research. The conversation continued in the coffee/breakout sessions. A few attendees were interested in learning more, and some connections were made, especially helpful for PhD students new in their careers. There was also some general conversation about academic life, and in particular the experience of working in a research field that is becoming a hot area.

The second speaker was Peter Tio, who is a Professor of Complex and Adaptive Systems in the School of Computer Science at the University of Birmingham. His talk *From dynamical systems to kernel based feature spaces and back* discussed how parametrized state space models in the form of recurrent neural networks are often used in machine learning to learn from data streams exhibiting temporal dependencies. Such models are usually composed of two parts: an input-driven dynamical system and a (static) readout mapping from the state space of the dynamical system to the outputs. Peter's talk presented a framework for rigorous analysis of such dynamical systems in a class of recurrent neural networks known as Echo State Networks (ESN). This enables one to understand the deep connection between the state space and the notion of feature space

in kernel machines (a well-known class of machine learning methods). The talk went on to discuss several rather surprising results linking the structure of the dynamical coupling in ESN with properties of the associated temporal feature space.

The next speaker was Aretha Teckentrup from University of Edinburgh. She gave a talk titled *Numerical analysis of Gaussian process regression*. Gaussian process regression is often used as a Bayesian inference procedure resulting in a probability distribution on the unknown function conditioned on the observed data, the so-called posterior distribution. In the talk Dr Teckentrup has given an overview of recent results on convergence properties of Gaussian process regression, and shown how tools from applied mathematics can provide insight into the behaviour of this methodology. She further discusses deep Gaussian processes studying the depth of the processes and how they can be used for inference. This recent work is especially remarkable.

The final speaker of the day was Dr. Alexandros Hollender from Oxford University, who presented a fascinating result on the complexity of gradient descent. Gradient descent dates back to the 1840s and is perhaps the most used and studied method in continuous optimisation. As the main contribution, this work established that computing a Karush-Kuhn-Tucker point of a continuously differentiable function over the domain $[0,1]^2$ is complete for the class PPAD ∩ PLS. Here PPAD (parity argument on a directed graph) and PLS (polynomial local search) are successful computational classes introduced by Papadimitriou and Johnson, Papadimitriou, and Yannakakis, respectively. The PPAD class has attracted significant attention because it contains the problem of computing a Nash equilibrium. The PLS class captures problems of finding a local minimum of an objective function and includes a diverse set of problems including finding a local optimum of the TSP and finding pure Nash equilibria in many-player congestion games. The main result of this work also implies that the class CLS (continuous local search), which was defined by Daskalakis and Papadimitriou as a more "natural" counterpart to PPAD ∩ PLS, is in fact equal to PPAD ∩ PLS. This recent work (with Fearnley, Goldberg, and Savani) received one of the best paper awards at the 53rd Annual ACM Symposium on Theory of Computing (STOC 2021).

The colloquium provided the audience with perspectives of many facets of mathematical foundations of machine learning. The discussion after the talks was lively and stimulating.

Videos of the talks are available on the LMS YouTube channel youtube.com/c/LondonMathematicalSociety.

Peter Ashwin (University of Exeter)
Mary Cryan (University of Edinburgh)
James Davenport (University of Bath)
Prudence Wong (University of Liverpool)
Standa Zivny (University of Oxford)

Report: The British Science Festival

The British Science Festival which had originally been planned for September 2020 went ahead in Chelmsford from 7–11 September 2021, hosted by Anglia Ruskin University. The strong focus on safety meant that audiences were reduced, with distancing in place, and audience members asking questions via a website rather than taking off their masks, but it was wonderful that events planned before the pandemic were finally able to take place.

There were three events focused on mathematics: two lectures by British Science Association Section Presidents, and one Saturday all-day drop-in session.

Stopping the AI-Pocalypse

The first of these events was the Presidential Lecture for the Mathematical Sciences Section, delivered by Dr Nira Chamberlain (left; photo © Jacqueline Rhule-Chamberlain) on Tuesday 7 September. Nira gave a wonderful talk, showing the value of mathematical modelling and simulation. Addressing the question of whether AIs would inevitably replace human decision-makers in a field like investment banking, he had created an evolutionary multi-player contest in which one player (representing an AI) initially held a small advantage, and showed that by imposing a small tax on successful players, it was possible to reduce the probability that the AIs would drive out all the other competitors. Nira's use of multimedia contributed to the excitement of the occasion, with dramatic visual and sound effects supporting him.

The Lady with the Maths

This year's President of the British Science Association General Section is Dr Noel-Ann Bradshaw (London Metropolitan University; left; photo © Jonathan Histed). Noel-Ann's Presidential Lecture on Wednesday 8 September was delivered in the person of Florence Nightingale. While many still associate Nightingale primarily with nursing, Noel-Ann (as her title suggest) told us about her pioneering use of data to persuade those in power of the need to change. Nightingale appreciated the value of representing data diagrammatically, and her writings reveal her understanding of how to communicate information visually, both accurately and effectively. Noel-Ann selected particularly telling quotations from Nightingale's writings and correspondence, telling us about her childhood interest in numbers and mathematics and how she overcame the obstacles facing a woman of her class and time. Nightingale's data-driven approach to influencing public policy has never seemed so relevant, and this entertaining talk generated a great deal of interest and discussion.

Can you predict the climate?

The final mathematics event was held on Saturday 11 September, presented by Tosin Babasola (Bath), Prof Chris Budd OBE (Bath), Tina Zhou (Bath) and Dr Tom Crawford (Oxford).

With COP-26 taking place in Glasgow in November 2021, it seemed timely and highly appropriate to present a show related to climate change at the British Science Festival. Everyone has heard about climate change, but very few people understand that mathematics lies at the heart of the predictions both of the way that the climate will change, and also the effects that this change will have on us all.

Wanting to show how mathematics plays such a vital role in predicting climate change, the team decided to run a hands-on show called 'Can you predict the climate?'. The centre-piece of this was a set of climate models which the general public could interact with and allowed them to see the effects on global temperatures of changing the Carbon Dioxide level in the atmosphere. These codes were based on mathematical models of the climate, written in Python (at the University of Bath), and they were compact enough to run on an inexpensive Raspberry Pi system. Whilst in no way as advanced as the sophisticated climate models used to inform the IPCC (and which run on a super computer), these simple climate models were all firmly based on sound physical principles, and are certainly good enough to demonstrate the key principles underlying climate change.

The codes themselves can be downloaded from the website https://tinyurl.com/m794zdt2 . This website also contains much more information about the mathematical basis of climate models, and animated movies showing the predictions of these models. These are all aimed at school students (and their teachers) and the team hope that they will be used in schools across the country to teach both the principles of climate change and also the mathematics underlying the models used to predict this. Alongside the Raspberry Pi demonstrations was a double pendulum, used to show the role played by chaos in mechanical systems, and a Stirling engine to show the basic thermodynamic principles involved in atmospheric dynamics. The audience were also shown the importance of the Coriolis effect by trying to throw bean bags into a target whilst spinning around.

The hands-on show took place in a Geodesic Dome just outside the Marconi Plaza in Chelmsford. It was open to the general public, and there was a steady flow of families throughout the day. A lot of interest was shown in the exhibition, and we were all asked many questions about climate change. Whilst the adults enjoyed looking at the posters and computer simulations, the younger audience enjoyed playing with the mathematical puzzles that we had on display, and trying to throw the bean bags. It was particularly encouraging that some families stayed at the exhibition for half an hour or more.

The organisation of the whole of the British Science Festival was excellent, and the team enjoyed visiting the other exhibitions in the dome. These included two doctors in period dress showing how medicine was practised in the 17th century, and an excellent demonstration of the critical role played by women in the history and practice of engineering.

The 2022 British Science Festival

The British Science Festival will visit Leicester in 2022, hosted by De Montfort University. The dates are 13–17 September. We are confident that once again there will be interesting events particularly showcasing current mathematics research.

Tony Mann and Chris Budd

Report: The Unity of Mathematics: A Conference in Honour of Sir Michael Atiyah

It is a pleasure to write a report for the LMS Newsletter on this meeting in honour of Sir Michael Atiyah. The title is that of his Presidential Address to the London Mathematical Society (M.F. Atiyah, The Unity of Mathematics. Bull. London Math. Soc. 10 (1978), 69–76).

Atiyah died before the world had heard of covid-19, and arrangements for the memorial meeting that his death so clearly called for had to wait until the successful vaccination programme had made it reasonably safe to do so. We wore masks when moving about in the Institute, but not during talks or having lunch outside (we were fortunate in the weather).

There were twelve hour-long talks on mathematics, with opening and closing half-hour talks on the more personal side of Atiyah's life. While a number of the speakers gave their talks in person, a fair number spoke online via Zoom. The fact that academics have had to spend so much time mastering the mechanics of online delivery of lectures, plus the excellent IT resources and helpful staff at the INI, meant that after a while one barely noticed the difference — a triumph of adaptability.

Atiyah had famously broad interests and was famously a wonderful speaker. As his pupil and obituarist Nigel Hitchin put it ("...throughout his whole career he was famous for his inspiring seminars and lectures linking different themes together. Reconstructing the argument later was usually a different matter" (N.J. Hitchin, Sir Michael Atiyah OM. Biographical Memoirs of Fellows of the Royal Society 66 (2020), 27pp). Exactly. I would gladly listen to Atiyah on anything. Even on things well beyond my mathematical hinterland, he would carry me along — at the time. An hour later, I would realise that the spell had faded, and I was little the wiser. No wonder I considered him a magician — and no doubt I was not alone.

Atiyah will be best remembered for the Atiyah–Singer index theorem of 1962, and this was discussed in several talks (those of Minyong Kim and Nigel Higson, for instance). Incidentally, with the passing of Iz Singer (3.5.1924 – 11.2.2021), the last of the three 'grand old men' (with Raoul Bott (24.9.1923 – 20.12.2005)) has died. Singer began in physics. This no doubt had some influence on Atiyah's lifelong interest in physics, spoken of by Robbert Dijkgraaf, Bernd Schoers and Ed Witten. It was fitting that Witten gave the last full-length talk. Its themes — duality (between electricity and magnetism), the Langlands program (number theory was also discussed by Kim and by Peter Sarnak) and Atiyah's influence — brought together so much of Atiyah's work.

This was a wonderful conference: well-conceived, well planned, and well executed. It was a pleasure to hear so many good talks, and to mix with mathematicians again at long last.

N.H. Bingham
Imperial College London

Celebrating Equality, Diversity and Inclusivity in Mathematical Sciences

Unveiling ceremony

Caroline Nokes MP, the Chair of the Women and Equalities Select Committee, visited the School

of Mathematical Sciences at the University of Southampton on 14 October 2021 for the award ceremony of the MATHS=E+D+I art competition that ran last Spring.

Equality, Diversity and Inclusivity are at the very core of mathematics. The validity of a theorem, the uncanny ability of mathematics to describe, measure and model the world around us are independent of the gender, race, social and economic class, etc. of mathematicians. Mathematics can also act as a force for positive change by highlighting unjustifiable differences, from the gender pay gap to race bias in sentencing. And yet, significant inequalities within mathematics exist. For example, only approximately a third of Maths A-level students are female. The purpose of this art competition was to remind us all that mathematics is open to everyone and that nobody should think that mathematics is not for them because of who they are, rather than what they like.

The winning entry (pictured below) highlighted how diversity doesn't make us 'odd'. The author, Raffaella Mulas, wrote "The number 2 is beautiful and it has many unique properties. It is the only even prime number, and being different from the other prime numbers doesn't make it odd. This is true also for people: being different doesn't make you odd. It makes you beautiful and unique." The other posters emphasised the contributions that mathematicians of very different backgrounds have made and can make to this wonderful field. They are now on permanent display in the University of Southampton Maths Student Centre.

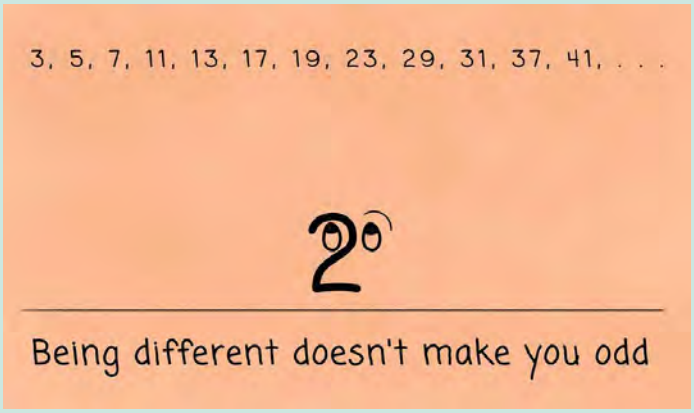
The Head of Mathematical Sciences, Prof Marika Taylor, welcomed Caroline and illustrated some of the actions taken by the School to increase the diversity of students in STEM subjects, including working with schools in India to popularise gravitational waves, to coincide with the planning of the construction of a gravitational wave observatory. Caroline talked about her experience in the House of Commons and how diversity and inclusivity are essential to ensure that legislation is both effective and fair.

Giampaolo D'Alessandro
Chair of the Maths EDI Committee
University of Southampton

UNIVERSITY OF Southampton

MATHS=E+D+I

Being different doesn't make you odd



First Prize

'The number 2 is beautiful and it has many unique properties. It is the only even prime number, and being different from the other prime numbers doesn't make it odd. This is true also for people: being different doesn't make you odd. It makes you beautiful and unique.'

Raffaella Mulas
Post-Doctoral Researcher

Athena SWAN Bronze Award

Records of Proceedings at LMS meetings
Ordinary Meeting, IMA–LMS Joint Meeting, 30 September – 1 October 2021

This meeting was held virtually on Zoom, hosted by the International Centre for Mathematical Sciences (ICMS), as a joint meeting with the Institute of Mathematics and its Applications (IMA) on *Maths in Human Society*. Over 52 members and visitors were present for the first day and over 30 participants for the second day of the meeting.

The meeting began at 1.45 pm on 30 September with Dr Nira Chamberlain, IMA President, in the Chair. Dr Chamberlain welcomed guests, thanked the organising parties, and then introduced LMS President Professor Jonathan Keating FRS, who welcomed guests on behalf of the London Mathematical Society. He then introduced Sarah Black of the ICMS, to cover the housekeeping items for the meeting. The meeting was then handed over to Dr Martine Barons (Vice-President, IMA) who introduced a lecture given by Professor Desmond Higham (University of Edinburgh) titled *Spread of Disease: From Pairwise to Groupwise Modelling and Analysis*. Dr Barons then introduced the second lecture titled *Illustrating Geometry (and Topology)* by Dr Saul Schleimer (University of Warwick).

After the tea break, Dr Barons introduced the final lecture of Day 1 given by Professor Ginestra Bianconi (QMUL) on *Higher Order Networks and Emergent Geometry*.

Dr Chamberlain then thanked the organisers and speakers at the Meeting before handing over to the President of the LMS to close the first day of the meeting before reconvening the next day for the final two talks.

After the close of the first part of the meeting Dr Katrin Leschke (University of Leicester) introduced an arts event highlighting artworks and performances inspired by collaborations with artists and mathematicians.

The following pieces were presented:

Countless Deformations 2.0 by Chloe Aligianni and Lee Allatson, Sounds of Surface by Jenny Hibberd and Andrew Johnston, *The Plastic Number* by Jenny Hibberd and Liam Taylor-West and *Aperiodic Rhythms* by Lee Allatson and Liam Taylor-West

There was a question and answer session after the performances.

The meeting continued at 10.00 am on 1 October with Professor Brita Nucinkis (Chair, LMS Society Lectures and Meetings Committee) in the Chair. Professor Nucinkis welcomed guests and then introduced Dr Martine Barons (IMA) who welcomed guests on behalf of the Institute of Mathematics and its Applications.

Sarah from the ICMS conducted housekeeping items again before Professor Nucinkis introduced the first lecture given by Professor Elsa Arcaute (University College London) on *Nature vs Nurture: Fractals and Hierarchies*.

After a short break Professor Nucinkis introduced the final lecture by Professor Christian Muller (TU Wein) on *Geometric Structures Motivated by Architecture*.

Dr Barons thanked the speakers, guests and organisers and handed over to Professor Nucinkis who concluded the meeting by thanking the speakers and all of the organisers and conference teams from the LMS, IMA and ICMS.

Records of Proceedings at LMS meetings
Annual General Meeting and Society Meeting of the London Mathematical Society, Friday 12 November 2021

The meeting was held as a hybrid meeting in person in the Great Hall at Goodenough College, London and via a live stream over Zoom. Over 110 members and visitors were present for all or part of the meeting, in person and online. The meeting began at 3:00pm, with the President, Professor Jonathan Keating FRS, in the Chair.

The Minutes of the General Meeting, which was held on 2 July 2021, were circulated to members 21 days in advance of this meeting. Copies of those Minutes were also available at the meeting. The President asked those members present and online if there were any clarifications or corrections to those minutes. There were none and the Minutes were confirmed.

The Vice-President, Professor Cathy Hobbs, presented a report on the Society's activities in 2020-2021 and the President invited questions. The Treasurer, Professor Simon Salamon, presented his report online on the Society's finances during the 2020-21 financial year and the President invited questions. Copies of the Trustees' Report for 2020-21 were made available on the day and the President invited members to adopt the Trustees' Report for 2020-21 by a show of hands for those in person and via a poll for those joining online. The Trustees' Report for 2020-21 was adopted.

The President proposed Messrs Moore Kingston Smith be re-appointed as auditors for 2020-2021 and invited members to approve the re-appointment by a show of hands for those in person and via a poll for those online. Messrs Moore Kingston Smith were re-appointed as auditors for 2021-2022.

The President, on Council's behalf, presented certificates to the 2021 Society Prize-winners:

Pólya Prize: Professor Ehud Hrushovski FRS (University of Oxford)
Naylor Prize and Lectureship: Professor Endre Süli (University of Oxford)
Anne Bennett Prize: Dr Viveka Erlandsson, (University of Bristol)
Whitehead Prize: Dr Michael Magee (Durham University)

For the following awards the prize winners were unable to attend in person:

The **Senior Whitehead Prize** was awarded to Professor Tara Brendle (University of Glasgow)
The **Berwick Prize** was awarded to Dr Ailsa Keating (University of Cambridge)
Whitehead Prizes were awarded to Dr Jonathan Evans (Lancaster University), Professor Patrick Farrell (University of Oxford) Dr Agelos Georgakopoulos (University of Warwick) Dr Aretha Teckentrup (University of Edinburgh) and Professor Stuart White (University of Oxford).

30 people were elected to Ordinary Membership: Dr Zahraa Abdallah, Dr Tahani Al-karkhi, Dr Ali Al Khabyah, Dr Lashi Bandara, Dr Ruth Bowness, Dr Ihechukwu Chinyere, Dr Marine Fontaine, Dr Mohammud Foondun, Dr Richard Gratwick, Dr Andrew Harrington, Dr Paul Johnson, Dr Nick Jones, Dr Nikolaos Kallinikos, Dr Wasiur KhudaBukhsh, Dr Henna Koivusalo, Dr Celine Maistret, Dr Armando Martino, Professor Victoria Mifflin, Dr Sofiat Olaosebikan, Dr Jasmina Panovska-Griffiths, Dr Arno Pauly, Mohammod Rahman, Dr Rohini Ramadas, Dr Daniel Ratliff, Dr Pierre-Francois Rodriguez, Dr Rob Silversmith, Dr Lewis Topley, Dr Daniele Turchetti, Dr Shengwen Wang, Dr Yinghui Wei.

49 people were elected to Associate Membership: Lambert A'Campo, Izar Alonso Lorenzo, Maram Alossaimi, Dr Naomi Andrew, Dario Ascari, Francesco Ballini, Filippo Baroni, Katherine Benjamin, Chenjing Bu, Cameron Bunney, Dr Mehmet Siddik Cadirci, Dr Dimitris Chiotis, Xenia Dimitrakopoulou, Dr Joanne Ellison, Alejandro Fernandez, Marc Fersztand, Alvaro Gonzalez Hernandez, Akshay Sateesh Hegde, Alfred Holmes, Simone Hu, Andrés Ibáñez Núñez, Ruhong Jin, Thibault Langlais, Dr Pak-Hin Lee, Dr Isabella Marinelli, Sam Mutter, Nikkita Ngalande, Dr Cian O'Brien, Professor Leonard Paleta, Emma Palmer, Theodoros Stylianos Papazachariou, Alexandru Pascadi, Paweł Piwek, Dr Jack Saunders, Rhiannon Savage, Rehan Shah, Robert Spencer, Margaret Stanier, Dr Yixuan Sun, Filippos Ilarion Sytilidis, Michal Szachniewicz, Matteo Tabaro, Youri Tamitegama, Ognjen Tosic, Milena Vuletic, Jakub Wiaterek, Wojciech Woloszyn, Dr Zoe Wyatt, Dr Beihui Yuan.

6 people were elected to Associate (undergraduate) Membership: Yanpei Cai, Oliver Christensen, Alastair Crossley, Light Ediand, Mohamed Iyaz Shakir, Ben Smith.

1 person was elected to Reciprocity Membership: John Goodwin.

236 people were elected to Associate Membership for Teacher Training Scholars: Mariam Ahmad, Pouyan Ahsani, Rabia Akther, Idiris Ali, Alifiya Alibhai, Harry Allen, Bradley Allison, Alastair Amos, Jack Anderson, Shabana Anjum, Mohammad Ashraf, Sophie Asquith, Christoph Atkins, Elizabeth Atkinson, Martha Austin, Aleena Awais, Ibrahim Aziz, Marta Bacmaga, James Bailey, Kirsty Baker, Emmanuel Bamgboye, Sophie Banks, Abigail Barber, Thomas Barringer, Daniel Beck, Aishah Begum, Layla Begum, Farhana Begum, Harrison Bellis, Dr Alison Berry, Jim Binks, Jessica Black, Charles Blake, Angelita Bradney, Jessica Braywood, Mark Bridle, Holly Briggs, Ryan Brookes, Fergus Brown, James Brown, Katherine Brown, Jake Browning, Myles Bunclark, Rachel Burns, Nicola Carden, Benjamin Carver, Hannah Chander, Marcus Cheadle, George Child, Dr Gabriel Chow, Romaana Chowdhury, Georgia Christodoulou, Georgia Christopher, Samuel Clark, Paige Clayton, Gemma Cole, Holly Corbett, Michael Cordery, George Cotgrove, James Craig, Charlotte Cranmer, Katie Cremer, Catherine Croome, Luke Danaher, Jack Dandy, William Daughtrey, Emma Davies, Olivia Davis, Lorna Davison, Lezley Dawes, Rachel Dickerson, Cameron Dingle, Kimberley Dunnell, Ellen Eastwood, Jane Eccles, Elizabeth Eyres, Thomas Fagan-Hall, Peter Farmer, Anum Farooq, Jonathan Felfoldi, Dr Clare Fitzsimmons, William Foreman, Ella Forsyth, Amber Foster, Lauren Fox, Wiktoria Frackowiak, Mark Frazer, Ian Garrett, Bethan Gayle, Dzifa Gazo, Alexander Jesse Gilbert, Ben Gilder, William Gould, Tiffany Granville, Anna Gray, Sarah Greengrass, Alexa Gregory, Susan Grove, Anil Gupta, Katie Hancock, Heena Haq, Kim Harmer, Matilda Harper, Thomas Harrison, Joseph Harvey, Shaymaa Hawa, Toby Hawkins, Rebecca Hawkins, Victoria Hayes, Hayley Hedger, Matthew Henley, Selena Hofmann, Aleisha Hogan, Mia Hoggard, Kathryn Holt, Sumayyah Hoque, Ryan Horlock, Afsha Hossain, Leah Houlton, Caiyin Huang-Wright, Victoire Huchet, Robert Hulse, Aqeedah Hussain, Rubiya Hussain, Sian Hyland, Suzanne Ipe, Eshmi Islam, Courtney James, Amaar Jimale, Harry Johnson, Tegfan Jones, Jack Jones, Ryan Kear, Stacey Kelly, Aaron Keys, Ayaz Khan, Frederick Leggate, Charlotte Lucas, Henna Mahmood, Raees Mahmood, Mohammed Tayyub Majeed, Henry Makings, Thomas Mansfield, Hannah Marsden, William Martin, Lucy Mathews, John McAlister, Alfie McDermott, Maire McDevitt, Tejay McGrann, William Merchant, Chidera Metuh, Mohammed Robiul Miah, Benjamin Millard, Justin Minar, Sophie Mitchell, Imogen Mitchell, Thandeka Mkandla, Bradley Moneypenny, Adam Morris, Laura Moss, Keeley Murray, Samuel Murray, Waleed Ahmad Nasir, Awura Ajoa Nuako, Ivy Oldroyd, Bolanle Orimoloye, Bilal Ozturk, Savda Shanavaz Panvala, Libby Jo Parker, Michael Parkinson, Kelly Elizabeth Parra Venegas, Akshay Pathiyath, Jeremy Peirson, Nathaniel Pidcock, Daniel Pope, Danielle Povey, Charlotte Quant, Muhammad Mahfoozur Rahman, Rhys Rawlings, Jefri Reci, Georgina Faith Rees-Evans, Samuel Ridgers, Abigail Riley, Andrew Robinson, Rebecca Roper, Holly Rostill, Andreea Rotaru, Callum Rowe, James Rushton, Chloe Sandford, Robert Sandground, Maisie Savage, William Sawtell, Jaime Scantlebury-Smead, Laura Scrivens-Waghorn, Dr Abdul Khaliq Shah, Neil Shah, Rachael Shepherd, Dr Luke Simmons, Chelsie Sinton, Rebecca Smith, Sarah-Jane Smith, Sophie Smith, Phillip Smith, Natalie Southcott, Tristan Steele-Ewen, Charlotte Stepney, Roger Stone, Erwin Suthagar, Jasmine Swanston, Katie Sweet, Fatima Tabassum, Parker Talbot, Mark Tallintire, Yi Hong Tan, Joseph Taylor, Verity Thomas, Jill Toms, Amelia Turner, Jonah Varney, Anjali Venugopal, Mohammed Maaz Wahid, Megan Walsh, Xincheng (Mark) Wang, Mitchell Ward, Charlotte Webster, Alexander Wedgwood, Elizabeth Wells-Bower, Connor Whelan, Christie Wilce, Naomi Wilson, Emily Woodward, Tegan Worden, Shehla Yakub, Reka Zelena.

Ten members signed the Members' Book and were admitted to the Society.

Professor Jens Marklof (University of Bristol) gave a lecture on Random Lattices.

After tea, LMS Scrutineer Professor Charles Goldie announced the results of the ballot. The following Officers and Members of the Council were elected.

President: Ulrike Tillmann FRS
Vice-Presidents: Catherine Hobbs, Iain Gordon
Treasurer: Simon Salamon
General Secretary: Robb McDonald
Publications Secretary: Niall MacKay
Programme Secretary: Chris Parker
Education Secretary: Kevin Houston
Members-at-Large of Council for two-year terms: Elaine Crooks, Andrew Dancer, Jessica Enright, Frank Neumann, Rachel Newton
Member-at-Large of Council for a one-year term: Brita Nucinkis
Member-at-Large (Women and Diversity): Sara Lombardo

Five Members-at-Large, who were elected for two years in 2020, have a year left to serve: Peter Ashwin, Anne-Christine Davis, Minhyong Kim, Anne Taormina and Amanda Turner.

The following were elected to the Nominating Committee for three-year terms: Nira Chamberlain, Philip K Maini. The continuing members of the Nominating Committee are: Tara Brendle (Chair from 2022), Chris Budd, Beatrice Pelloni, Mary Rees and Gwyneth Stallard. In addition, Council would appoint a representative to the Committee.

Professor Jonathan Keating FRS, handed over the Presidential badge of office to Professor Ulrike Tillmann FRS. The new President thanked members for the honour and privilege of being elected as President and promised to fulfil the Charter, Statutes and By-laws of the Society.

Professor Jonathan Keating FRS, University of Oxford, then gave the Presidential Address 2021 titled, Random Matrices and the Riemann Zeta-Function.

Before closing the meeting, Professor Tillmann thanked the retiring members of Council and the Immediate Past President, Professor Keating FRS.

Professor Tillman also thanked the speaker at the Graduate Student Meeting on Monday 8 November 2021, Professor Nina Snaith (University of Bristol), and congratulated the graduate student speakers.

After the meeting, a reception was held at Goodenough College in the Large Common Room, followed by the Annual Dinner, which was held in the Great Hall at Goodenough College and attended by 92 people. At the start of the Annual Dinner, the Immediate Past President, Professor Jonathan Keating FRS, gave a short speech and proposed a toast to the continued health of the Society and mathematics before the President, Professor Ulrike Tillmann gave a short speech.

Records of Proceedings at LMS meetings
BSHM-IMA-LMS Joint Meeting: Black Heroes of Mathematics; Ordinary Meeting: 5–6 October 2021

This meeting was held virtually on Zoom, as a joint meeting with the British Society for the History of Mathematics (BSHM), the Institute of Mathematics and its Applications (IMA), Isaac Newton Institute (INI), Mathematics Association) and hosted by the International Centre for Mathematical Sciences (ICMS) to celebrate Black Heroes of Mathematics. Over 700 members and visitors were present during all or part of the two-day meeting.

The meeting began at 10.00 am on 5 October with Dr Nira Chamberlain (President, IMA), in the Chair. Dr Chamberlain welcomed guests, thanked the organising parties, and then introduced a recorded welcome message from Angela Tibiri AIMS Ghana.

He then introduced the audience to the subject of the two-day meeting. Most talks given during the two-day meeting had been previously recorded and after each talk, the speaker answered questions, which had been sent in the 'chat' function of Zoom, in real time.

The order of talks for the first day was as follows:
Mathematics is Easy. Teaching it is Hard, Prof Mamokgethi Phakeng (University of Cape Town, South Africa)
Harnessing Artificial Intelligence & Big Data techniques to monitor, manage and forecast an epidemic: the case of COVID-19, Dr Jude Kong (York University, Canada)
Curiously Riding the Waves of Life Towards Success in Mathematical Sciences, Yohance Osborne (University College London, UK)

After a short break, Dr Chamberlain introduced recorded messages from Professor Minhyong Kim (Director, International Centre for Mathematical Sciences) and Professor Sarah Hart (President, British Society for the History of Mathematics) followed by the lecture:

A robust and efficient adaptive multigrid solver for the optimal control of geometric surface evolution laws with applications to cell mitigation, Professor Anotida Madzvamuse (University of Sussex, UK)

Dr Chamberlain then chaired a panel discussion during which the panellists Dr Ejay Nsugbe, Franck Kalala Mutombo, Dr Juliet Ojiako, Takita Bartlett, Dr Jude Kong, Yohance Osborne and Professor Anotida Madzvamuse answered questions from the audience.

The first day of the meeting ended at 3.30 pm and Dr Chamberlain thanked all the speakers and panellists.

The second day of the meeting began at 1.00 pm on 6 October with Dr Nira Chamberlain in the Chair. Most of the talks were pre-recorded, as on the first day, and the speaker answered questions, which had been sent in the 'chat' function of Zoom, in real time. Dr Chamberlain introduced recorded messages from Chris Pritchard (President Mathematics Association) and Professor Jonathan Keating (President, London Mathematical Society). He then introduced the lectures.

The order of talks for the second day was as follows:

Do the Maths Thing, Susan Okereke (Do The Maths Thing, UK)
I Can Be a Hero Too!, Dr Juliet Ojiako (Loughborough University, UK)

After a short break Dr Chamberlain introduced two further lectures:

Heroes of Mathematics: The Power of Narrating their Stories, Maurine Atieno Songa (University of KwaZulu-Natal, South Africa)
Storytelling for Mathematics: Lessons Learned for Teaching, Learning & Research, Professor Erica Walker (Columbia University, USA)

After a break the meeting resumed at 6.00 pm with a message from Isaac Newton Institute Director, Professor Ulrike Tillmann and a panel discussion chaired by Dr Chamberlain, during which the panellists, Dr Mark Richards, Dr Angela Tabiri, Lauryn Mwale, Professor Carla Faria, Professor Clifford V. Johnson and Susan Okereke answered questions from the audience.

The second day of the meeting ended at 7.30 pm and Dr Chamberlain thanked all of the speakers, panellists and organisers.

Jordan Algebras and Symmetric Manifolds

CHO-HO CHU

Jordan algebras correspond to a class of Lie algebras. While the role of Lie algebras in geometry is universally recognised, the same cannot be said about Jordan algebras. We explain in this article the close connection between Jordan algebras and symmetric manifolds.

Jordan and Lie algebras

We are familiar with the role of Lie algebras in geometry, for instance, we know that the smooth vector fields (Figure 1) of a smooth manifold form a Lie algebra. Jordan algebras are close relatives of Lie algebras, but less famous and perhaps even lesser known is their connection to geometry. The revelation below, it is hoped, may help to ameliorate this unfavourable state of affairs and generate wider interest in Jordan algebras.

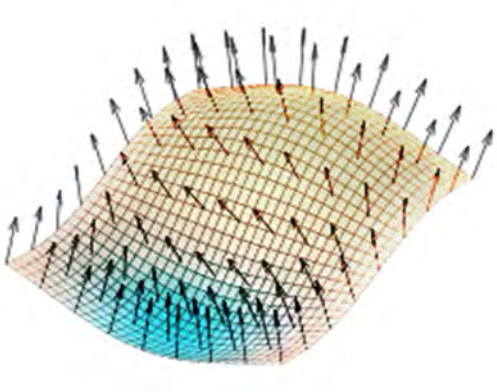


Figure 1. A vector field

Let us begin by introducing Jordan algebras and explain their relationship with Lie algebras. We refer to [7] for a more informative sketch of Jordan algebras. In what follows, all vector spaces are either real or complex. A *Jordan algebra* A is a vector space equipped with a bilinear product

$$(a,b) \in A \times A \mapsto a \circ b \in A$$

which is commutative and satisfies the *Jordan identity*

$$a \circ (b \circ a^2) = (a \circ b) \circ a^2 \qquad (a,b \in A).$$

We do not assume associativity of the product. We call A *unital* if it has an identity.

One can turn any associative algebra A into a Jordan algebra by defining a new product

$$a \circ b = \frac{1}{2}(ab + ba) \qquad (a,b \in A)$$

with which A becomes a Jordan algebra, where the product on the right-hand side is the original product of A. Subalgebras of $(A,\circ)$ are called *special* Jordan algebras. For example, the associative algebras $M_n(\mathbb{R})$, $M_n(\mathbb{C})$ and $M_n(\mathbb{H})$ of $n \times n$ matrices over the reals $\mathbb{R}$, complexes $\mathbb{C}$ and quaternions $\mathbb{H}$, respectively, are special Jordan algebras in the product $\circ$. However, the Jordan algebra $(H_3(\mathbb{O}),\circ)$ of 3×3 Hermitian matrices over the Cayley algebra $\mathbb{O}$ is not special.

Although the concept of a Jordan algebra was first introduced by Jordan, von Neumann and Wigner [5], under the name of an *r-number system*, to formulate an algebraic model for quantum mechanics, unexpected connections with Lie algebras and geometry were soon discovered.

How are Jordan algebras related to Lie algebras? A Lie algebra L is also a vector space equipped with an *anti-symmetric* bilinear product, usually denoted by the brackets $[a,b]$, not assumed to be associative, which satisfies the Jacobi identity

$$[[a,b],c] + [[b,c],a] + [[c,a],b] = 0.$$

Comparing definitions, one sees no obvious relationship between the two, e.g. one is commutative but the other anti-commutative.

In fact, it is well-known to algebraists that several exceptional Lie algebras can be constructed from $H_3(\mathbb{O})$. Apart from this and, what is more relevant to our discussion, is the fact that there is a 1-1 correspondence between a class of Lie algebras and Jordan algebras (actually, Jordan triples, which are slightly more general than Jordan algebras).

A *Jordan triple* is a vector space V equipped with a *Jordan triple product*

$$(a,b,c) \in V \times V \times V \mapsto \{a,b,c\} \in V$$

which is linear and symmetric in the outer variables, but conjugate linear in the middle variable, and satisfies the *Jordan triple identity*

$$\{a,b,\{x,y,z\}\} = \{\{a,b,x\},y,z\} - \{x,\{b,a,y\},z\} + \{x,y,\{a,b,z\}\}.$$

Example. A real Jordan algebra $(A,\circ)$ is a Jordan triple in the canonical triple product

$$\{a,b,c\} = (a\circ b)\circ c + a\circ(b\circ c) - b\circ(a\circ c). \qquad (1)$$

If $(A,\circ)$ is a complex Jordan algebra equipped with an involution *, then it is also a Jordan triple in the canonical triple product

$$\{a,b,c\} = (a\circ b^*)\circ c + a\circ(b^*\circ c) - b^*\circ(a\circ c).$$

Jordan algebras are just Jordan triples $(V,\{\cdot,\cdot,\cdot\}_V)$ containing a *unit element* e, the latter means $\{e,a,e\}_V = a$ for all $a \in V$. Indeed, for such V, we can define a product

$$a\circ b := \{a,e,b\}_V \qquad (a,b \in V)$$

with which V becomes a Jordan algebra, and $\{\cdot,\cdot,\cdot\}_V$ is exactly the canonical triple product defined by the Jordan product $\circ$ as in (1).

Example. According to the preceding example, the real Jordan algebra $(M_n(\mathbb{R}),\circ)$ is a Jordan triple in the canonical triple product. The complex Jordan algebra $(M_n(\mathbb{C}),\circ)$ has an involution defined by conjugate transpose : $(a_{ij})^* := (\overline{a}_{ji})$, it is therefore a complex Jordan triple in the canonical triple product, which can be rewritten as

$$\{a,b,c\} = \frac{1}{2}(ab^*c + cb^*a) \quad (a,b,c \in M_n(\mathbb{C})).$$

The subspaces $Sk_n(\mathbb{C})$ and $S_n(\mathbb{C})$ of $M_n(\mathbb{C})$, consisting of skew-symmetric and symmetric matrices respectively, are also complex Jordan triples in the above triple product.

Example. The complex vector space $M_{mn}(\mathbb{C})$ of $m\times n$ complex matrices has no natural Jordan product if $m \neq n$. However, it is a complex Jordan triple in the triple product

$$\{a,b,c\} := \frac{1}{2}(ab^*c + cb^*c) \quad (a,b,c \in M_{mn}(\mathbb{C})).$$

A Lie algebra L is called *3-graded* if there is a 3-grading $L = L_{-1} \oplus L_0 \oplus L_1$, where the summands are subspaces of L satisfying $[L_i,L_j] \subset L_{i+j}$ or $\{0\}$ if $i+j = \pm 2$.

One can find in [2] some references for the following correspondence between Jordan triples and Lie algebras, which can be infinite dimensional.

Theorem. *(Tits–Kantor–Koecher) There is a* 1-1 *correspondence between Jordan triples V and* 3*-graded Lie algebras $L = L_{-1} \oplus L_0 \oplus L_1$ with $L_0 = [L_{-1},L_1]$ and an involution $\theta : L \to L$ satisfying $\theta(L_j) = L_{-j}$.*

In this correspondence, we have $V = L_{\pm 1}$ and

$$\{a,b,c\} = [[a,\theta(b)],c] \qquad (a,b,c \in V)$$

which relates the Jordan triple identity in V and the Jacobi identity in L.

Symmetric manifolds

Our ensuing discussion of Jordan algebras and geometry can be summarised briefly by saying that Jordan algebras appear as tangent spaces of symmetric manifolds (Figure 2).

That Jordan algebras have something to do with symmetry may not be a surprise, given that they correspond to a class of Lie algebras and Lie theory describes fundamental laws of symmetry.

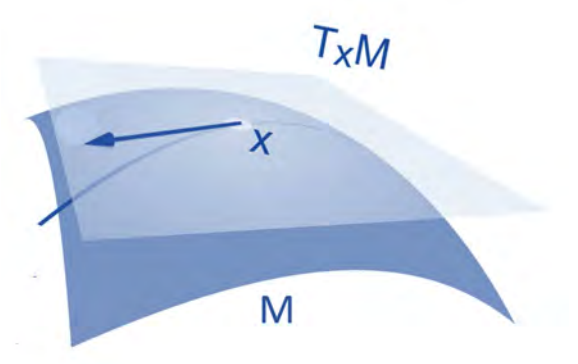


Figure 2. The tangent space T_xM at a point x in a symmetric manifold M carries a Jordan algebraic structure

A tangent space T_xM (Figure 2) of a manifold M at a point $x \in M$ is the vector space of all tangent vectors at x, it is a higher dimensional generalisation of a tangent line to a curve and a tangent plane to a surface. A tangent vector field X on M is a

selection of a tangent vector $X(p) \in T_pM$ at each point $p \in M$.

A connected manifold M with a Riemannian metric g is called *symmetric* if it has a (unique) symmetry s_p at (about) each point $p \in M$, where a *symmetry* is an isometry $s_p : M \to M$ with respect to g satisfying two conditions below:

(i) $s_p \circ s_p$ is the identity map;

(ii) p is the only fixed-point of s_p in a neighbourhood of p.

Such a manifold is called a (Riemannian) *symmetric space*. Symmetric spaces are an important class of Riemannian manifolds and appear in many fields.

Example. The Euclidean space $\mathbb{R}^d$ is a symmetric space. The symmetry s_p about $p \in \mathbb{R}^d$ is given by $s_p(x) = 2p - x$.

A *Hermitian symmetric space* is a Riemannian symmetric space, which carries the structure of a complex manifold such that the given Riemannian metric is Hermitian, and the symmetries are holomorphic isometries. How are they related to Jordan algebras?

Let M be a Hermitian symmetric space. Fix a symmetry $s_p : M \to M$, which is an element in the automorphism group $\mathrm{Aut}\, M$, consisting of biholomorphic isometries of M. Here a *biholomorphism* $f : M \to M$ is a bijective holomorphic map whose inverse f^{-1} is also holomorphic. $\mathrm{Aut}\, M$ is a real Lie group with real Lie algebra $\mathfrak{L}$. Each element in $\mathfrak{L}$ is a (holomorphic) tangent vector field X on M, with $X(p) \in T_pM$. The symmetry s_p induces an involution $\sigma : \mathfrak{L} \to \mathfrak{L}$ for which $\mathfrak{L}$ has an eigenspace decomposition

$$\mathfrak{L} = \mathfrak{k} \oplus \mathfrak{p}$$

where

$$\mathfrak{k} = \{X \in \mathfrak{L} : \sigma(X) = X\},$$
$$\mathfrak{p} = \{X \in \mathfrak{L} : \sigma(X) = -X\}$$

and the map $X \in \mathfrak{p} \mapsto X(p) \in T_pM$ is a real linear isomorphism.

The complexification $\mathfrak{L}_c$ of $\mathfrak{L}$ is a 3-graded Lie algebra

$$\mathfrak{L}_c = \mathfrak{p}_+ \oplus \mathfrak{k}_c \oplus \mathfrak{p}_-$$

with an involution θ satisfying $\theta(\mathfrak{p}_\pm) = \mathfrak{p}_\mp$ and $\theta(\mathfrak{k}_c) = \mathfrak{k}_c$, where $\mathfrak{p}_+$ is complex linear isomorphic to T_pM.

By the Tits-Kantor-Koecher theorem above, $\mathfrak{p}_+$ is a Jordan triple and therefore the tangent space T_pM inherits the Jordan triple structure from $\mathfrak{p}_+$ via the linear isomorphism between them.

Further, T_pM is a so-called *Hermitian* Jordan triple, meaning that $[\mathfrak{p}, \mathfrak{p}] \subset \mathfrak{k}$ in the eigenspace decomposition $\mathfrak{L} = \mathfrak{k} \oplus \mathfrak{p}$.

Conversely, given a complex Jordan triple V, Kaup [6] has shown that one can construct a real Lie algebra L with decomposition $L = k \oplus p$, and if V is Hermitian, meaning $[p, p] \subset k$, then there is a Hermitian symmetric space D such that V identifies in a natural way with a tangent space T_aD of D. We have therefore establish the following.

Theorem. *There is a* 1-1 *correspondence between Hermitian Jordan triples and Hermitian symmetric spaces.*

This theorem offers us an extra tool - to wit, Jordan triples, to study symmetric spaces. Indeed, it can even be extended to infinite dimension, which will be discussed briefly later. Thus we have a unified approach, using Jordan triples, to both finite and infinite dimensional symmetric spaces.

Symmetric spaces have been classified by É. Cartan using Lie theory. Let us consider the example of *nonpositively curved* Hermitian symmetric spaces and offer a Jordan perspective. The Hermitian Jordan triples corresponding to this class can be classified. They are finite direct sums

$$V_1 \oplus \cdots \oplus V_n$$

of six basic types of Jordan triples V_j $(j = 1, \ldots, n)$. Each V_j is one of the following:

$$(1)\ M_{mn}(\mathbb{C}), \quad (2)\ Sk_n(\mathbb{C}), \quad (3)\ S_n(\mathbb{C}),$$
$$(4)\ Spin, \quad (5)\ M_{12}(\mathbb{O}), \quad (6)\ H_3(\mathbb{O})$$

where $H_3(\mathbb{O})$ consists of 3×3 Hermitian matrices over the complex Cayley algebra $\mathbb{O}$ and $M_{12}(\mathbb{O})$ consists of 1×2 matrices over $\mathbb{O}$. Each irreducible nonpositively curved Hermitian symmetric space in É. Cartan's classification list is biholomorphic to the open unit ball of one of these six types of Jordan triples of matrices.

Jordan algebras and symmetric cones

The real Jordan algebras classified in [5] are finite dimensional and assumed to be *formally real*, that is,

they satisfy the condition

$$a_1^2 + \cdots + a_n^2 = 0 \Rightarrow a_1 = \cdots = a_n = 0.$$

We now reveal their relationship with Riemannian symmetric spaces. In short, they are in 1-1 correspondence with a class of cones which are Riemannian symmetric spaces.

Since all finite dimensional (Hausdorff) topological vector spaces are linearly homeomophic to a Euclidean space of the same dimension, there is only one (Hausdorff) topology on a finite dimensional vector space V making addition and scalar multiplication continuous. Therefore, there is no ambiguity to say that a set is *open* in V without referring to this topology.

A nonempty set Ω in a vector space V is called a *cone* if $\Omega + \Omega \subset \Omega$ and $\alpha\Omega \subset \Omega$ for all $\alpha > 0$.

Let V be finite dimensional. An open cone Ω in V is called *proper* if

$$\overline{\Omega} \cap -\overline{\Omega} = \{0\}$$

where $\overline{\Omega}$ is the closure of Ω. The cone $\overline{\Omega}$ induces a partial ordering $\leq$ in V so that $x \leq y \Leftrightarrow y - x \in \overline{\Omega}$.

An open cone Ω in V is called *linearly homogeneous* if for any $a, b \in \Omega$, there is a linear automorphism $\varphi : \Omega \to \Omega$ such that $\varphi(a) = b$. Here, a *linear automorphism* is a (continuous) linear isomorphism $\varphi : V \to V$ such that $\varphi(\Omega) = \Omega$.

If we consider an open cone Ω in V as a smooth manifold, then the tangent space $T_e\Omega$ at each point $e \in \Omega$ can be identified with V.

Theorem. *Let Ω be a proper linearly homogeneous open cone in a finite dimensional vector space V. Then Ω carries the structure of a Riemannian symmetric space whose linear automorphisms are isometries if and only if V admits the structure of a formally real Jordan algebra and $\overline{\Omega} = \{a^2 : a \in V\}$.*

Example. Let $V = \mathbb{R}^3$. The *light cone* (Figure 3)

$$\Omega = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_3 > 0, x_3^2 > x_1^2 + x_2^2\}$$

is a linearly homogeneous proper open cone in $\mathbb{R}^3$ and a Riemannian symmetric space. The corresponding Jordan algebraic structure of $\mathbb{R}^3$ is given by the Jordan product

$$(x_1, x_2, x_3) \circ (y_1, y_2, y_3) = (x_1y_3 + x_3y_1, x_2y_3 + x_3y_2, x_1y_1 + x_2y_2 + x_3y_3).$$

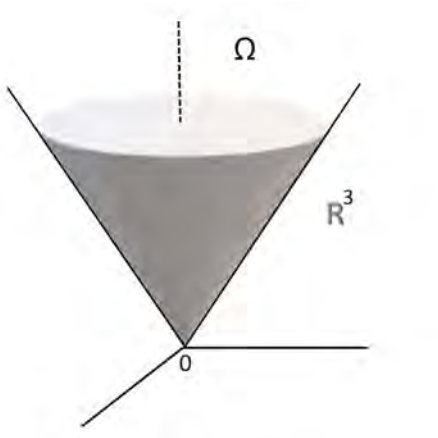


Figure 3. The tangent space R^3 of the symmetric light cone Ω is a formally real Jordan algebra

Infinite dimension

Now a few words about the infinite dimensional case. Essentially, assertions made in the previous discussion, particulary the last two theorems, can be extended to infinite dimension.

First, the toplogical vector spaces we need to consider are the ones equipped with a complete norm, namely, the *Banach spaces*. A norm $\|\cdot\|$ on a vector space V is said to be *complete* if V is a complete metric space in the metric $d(x, y) = \|x - y\|$ defined by the norm.

An infinite dimensional generalisation of a formally real Jordan algebra in finite dimension is the concept of a unital *JB-algebra*, which is a real unital Jordan algebra as well as a Banach space A satisfying

$$\|a^2\| = \|a\|^2, \quad \|a \circ b\| \leq \|a\|\|b\|,$$
$$\|a^2\| \leq \|a^2 + b^2\| \quad (a, b \in A).$$

A finite dimensional formally real Jordan algebra is a unital JB-algebra in the trace norm.

A finite dimensional manifold is modelled locally on $\mathbb{R}^d$ or $\mathbb{C}^d$. Analogously, a connected manifold can be modelled locally on a real or complex Banach space E. Such a manifold is called a real or complex connected *Banach manifold*, respectively. The manifold is called infinite dimensional if E is infinite dimensional.

In contrast to the finite dimensional manifolds, it is not meaningful to define a Riemannian metric on an infinite dimensional Banach manifold. Instead, one can define the notion of a *Finsler metric* on a Banach

manifold, generalising that of a Riemannian metric (cf [1]). With this metric, we can extend the concept of a symmetric manifold to infinite dimension.

A Banach manifold equipped with a Finsler metric ν is called a *Finsler manifold*. A connected Finsler manifold M is called *symmetric* if there is a (unique) symmetry $s_p : M \to M$ at each $p \in M$, which is a ν-isometry satisfying the same conditions for a symmetry given before.

An open cone Ω in a Banach space V is called *normal* if there is a constant $\gamma > 0$ such that $0 \le x \le y \Rightarrow \|x\| \le \gamma\|y\|$. A finite dimensional proper open cone is normal.

An infinite dimensional extension of the last theorem reads as follows (see [4]).

Theorem. *Let Ω be a normal linearly homogeneous open cone in a real Banach space V. Then Ω carries the structure of a Finsler symmetric manifold whose linear automorphisms are isometries if and only if V admits the structure of a unital JB-algebra and $\overline{\Omega} = \{a^2 : a \in V\}$.*

A final remark. Kaup [6] actually proved his aforementioned theorem for Hermitian Jordan triples in infinite dimension (see also [3]).

Theorem. (Kaup) *There is a 1-1 correspondence between complex symmetric Finsler manifolds and Hermitian Jordan triples that are Banach spaces with a continuous Jordan triple product.*

FURTHER READING

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[5] P. Jordan, J. von Neumann and E. Wigner, On an algebraic generalisation of the quantum mechanical formalism, *Ann. of Math.* 36 (1934), 29–64.
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[7] K. McCrimmon, Jordan algebras and their applications, *Bull. Amer. Math. Soc.* 84 (1978) 612-627.

Cho-Ho Chu

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Manin matrices, quadratic algebras, and operads

CHARLES A. S. YOUNG

The notion of a *Manin matrix* emerged from the work of Y. I. Manin, beginning in the 1980s, in the context of noncommutative geometry and quantum groups. In this article we'll start with a gentle introduction to Manin matrices and the idea of coactions, and sketch one of the many ways they crop up in the study of quantum integrable models. We'll then broaden the picture to discuss quadratic algebras and their quadratic data, before moving to our main goal: to describe recent work of Vallette and Manin himself, in which quadratic algebras and the dualities between them serve as a framework for organising various important classes of operads originating in topology.

Manin matrices

Let us begin with the simplest example, which illustrates the main idea. Suppose we start with the usual coordinate functions

$$u, v : \mathbb{C}^2 \to \mathbb{C}$$

on two-dimensional complex space, namely $u : (z,w) \mapsto z$ and $v : (z,w) \mapsto w$. These coordinates u and v are mutually commuting,

$$[u,v] := uv - vu = 0,$$

because the product $uv = vu$ is just the usual pointwise product of functions on $\mathbb{C}^2$, which is uncontroversially commutative. We can think of u and v as the generators of the algebra $\mathbb{C}[u,v]$ of polynomials with complex coefficients. Let us consider a "change of coordinates" of the form

$$\begin{pmatrix} \widetilde{u} \\ \widetilde{v} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}. \tag{1}$$

We may ask: under what conditions do these new "coordinates" $\widetilde{u}$ and $\widetilde{v}$ still commute? That is, what does it take to ensure that

$$[\widetilde{u},\widetilde{v}] = \widetilde{u}\widetilde{v} - \widetilde{v}\widetilde{u} = 0\,?$$

At first sight, this seems a strange question. If the matrix entries a,b,c,d are just complex numbers, then $\widetilde{u}$ and $\widetilde{v}$ are again just functions $\mathbb{C}^2 \to \mathbb{C}$, and certainly still commute. But the idea is that this is, in a sense, overkill. Let us suppose instead that a,b,c,d are elements of some associative algebra $\mathcal{A}$ over $\mathbb{C}$. We assume that elements of $\mathcal{A}$ commute with elements of the polynomial algebra $\mathbb{C}[u,v]$ but *not* necessarily amongst themselves. What is then the very least we can get away with? We see that

$$\begin{aligned}[\widetilde{u},\widetilde{v}] &= [au + bv, cu + dv] \\ &= [a,c]u^2 + ([a,d] + [b,c])\,uv + [b,d]v^2.\end{aligned}$$

Thus, the conditions we need are that

$$[a,c] = 0, \qquad [b,d] = 0 \tag{2a}$$

– i.e. entries of the same column must commute – together with a *cross-commutation relation*,

$$[a,d] = [c,b]. \tag{2b}$$

A matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ obeying these conditions is called a *Manin matrix*.

More generally, an $n \times n$ matrix M with entries in an associative algebra $\mathcal{A}$ is called a *Manin matrix* if its entries obey $[M_{ij}, M_{kl}] = [M_{kj}, M_{il}]$ for all $1 \le i,j,k,l \le n$. The elements $\widetilde{u}_i := \sum_{j=1}^n M_{ij}u_j \in \mathcal{A} \otimes_{\mathbb{C}} \mathbb{C}[u_1,\dots,u_n]$ are mutually commuting, $[\widetilde{u}_i,\widetilde{u}_j] = 0$, for all $1 \le i,j \le n$, if and only if M is a Manin matrix. That is, the map $u_i \mapsto \widetilde{u}_i = \sum_{j=1}^n M_{ij}u_j$ extends to an algebra homomorphism

$$\mathbb{C}[u_1,\dots,u_n] \to \mathcal{A} \otimes_{\mathbb{C}} \mathbb{C}[u_1,\dots,u_n]$$

if and only if M is Manin. If so, we say it defines a *coaction* of the associative algebra $\mathcal{A}$ on the polynomial algebra $\mathbb{C}[u_1,\dots,u_n]$.

The reader will have noticed the asymmetry between rows and columns in our definition of a Manin matrix, which arises because the coaction in (1) was on the left. Actually the conditions (2) do also arise from a right coaction, but only if we instead start with *anticommuting* generators ϕ and ψ, obeying

$$\phi\phi = 0, \qquad \phi\psi + \psi\phi = 0, \qquad \psi\psi = 0,$$

and define $\widetilde{\phi}$ and $\widetilde{\psi}$ by

$$\begin{pmatrix} \widetilde{\phi} & \widetilde{\psi} \end{pmatrix} = \begin{pmatrix} \phi & \psi \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}. \tag{3}$$

One can check that the new generators $\widetilde{\phi}$ and $\widetilde{\psi}$ obey the same relations as ϕ and ψ if and only if $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is Manin.

Column determinants and quantum integrable models

The Manin property turns out to be exactly what is needed to make many familiar results from linear algebra continue to hold, suitably interpreted, for matrices with non-commuting entries [1]. For example, let us define the *column-ordered determinant* of a 2×2 matrix by

$$\operatorname{cdet} : \operatorname{Mat}_{2\times 2}(\mathcal{A}) \to \mathcal{A}; \qquad \operatorname{cdet}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - cb.$$

It is manifestly skew-symmetric under interchange of rows – but it is skew-symmetric under the interchange of columns if and only if $\begin{pmatrix} a & b \\ c & c \end{pmatrix}$ is Manin. This definition, and property, generalizes to $n\times n$ matrices for all positive integers n. Another familiar property is this: suppose $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ is the block form of a Manin matrix of any size; then for any block matrix E of the right shape,

$$\operatorname{cdet}\left(\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} \mathbf{1} & E \\ 0 & \mathbf{1} \end{pmatrix}\right) = \operatorname{cdet}\begin{pmatrix} A & B \\ C & D \end{pmatrix}.$$

(Try the easy case in which A, B, C, D, E are all 1×1. Note that the product of matrices on the left need not in general be Manin.) If A has a right inverse then we get, in particular, that

$$\begin{aligned}\operatorname{cdet}\begin{pmatrix} A & B \\ C & D \end{pmatrix} &= \operatorname{cdet}\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} \mathbf{1} & -A^{-1}B \\ 0 & \mathbf{1} \end{pmatrix} \\ &= \operatorname{cdet}\begin{pmatrix} A & 0 \\ C & D - CA^{-1}B \end{pmatrix} \\ &= \operatorname{cdet} A \operatorname{cdet}(D - CA^{-1}B).\end{aligned}$$

Equally, if D has a right inverse then, using the freedom to permute rows and columns, we get

$$\begin{aligned}\operatorname{cdet}\begin{pmatrix} A & B \\ C & D \end{pmatrix} &= \operatorname{cdet}\begin{pmatrix} D & B \\ C & A \end{pmatrix} \\ &= \operatorname{cdet} D \operatorname{cdet}(A - BD^{-1}C).\end{aligned}$$

Now, as a concrete example of a matrix with the Manin property, consider

$$\begin{pmatrix} z & \frac{\partial}{\partial x} \\ x & \frac{\partial}{\partial z} \end{pmatrix}$$

where z, x are commuting variables, and $\partial_x = \frac{\partial}{\partial x}$ and $\partial_z = \frac{\partial}{\partial z}$ are their derivatives. It is Manin, for indeed $\partial_x x f(x) - x\partial_x f(x) = f(x) + xf'(x) - xf'(x) = f(x)$, i.e. $[\partial_x, x] = 1$, and so

$$[z, \partial_z] - [x, \partial_x] = -1 + 1 = 0.$$

It is a remarkably short jump from this example to recent work on quantum integrable models. To get a flavour of why that is so, note that the following matrix is also Manin, and for much the same reason:

$$T = \begin{pmatrix} z - z_1 & & 0 & \partial_1^1 & \dots & \partial_1^M \\ & \ddots & & \vdots & & \vdots \\ 0 & & z - z_N & \partial_N^1 & \dots & \partial_N^M \\ x_1^1 & \dots & x_N^1 & \partial_z - \lambda_1 & & 0 \\ \vdots & & \vdots & & \ddots & \\ x_1^M & \dots & x_N^M & 0 & & \partial_z - \lambda_M \end{pmatrix}.$$

Here $z_1, \dots, z_N$ and $\lambda_1, \dots, \lambda_M$ can be any complex numbers, and we replaced x and ∂_x by a collection of variables x_j^a and their derivatives $\partial_j^a = \frac{\partial}{\partial x_j^a}$. Using the "$\operatorname{cdet} A \operatorname{cdet}(D - CA^{-1}B)$"-formula above, we see that

$$\operatorname{cdet} T = \left(\prod_{i=1}^{N}(z - z_i)\right) \operatorname{cdet}\left(\partial_z - L(z)\right)$$

for some matrix $L(z)$ whose entries are meromorphic functions of z with poles at the points z_i. This $L(z)$ is in fact the *Lax matrix* for a certain *quantum Gaudin model* associated to the Lie algebra $\mathfrak{gl}_M$. Of course, evaluating the column determinant this way was a choice: we could equally have used the "$\operatorname{cdet} D \operatorname{cdet}(A - BD^{-1}C)$" formula above, or just evaluated it directly from the definition. By comparing the resulting three expressions – roughly speaking – one obtains the $(\mathfrak{gl}_M, \mathfrak{gl}_N)$-duality for quantum Gaudin models, due to Mukhin, Tarasov and Varchenko.

That duality can be seen as generalization of the classical Capelli identities, which emerge as the special case in which one sets all $z_1, \dots, z_N$ and $\lambda_1, \dots, \lambda_M$ to zero.[1]

Other important applications of Manin matrices to quantum integrable models have been given by Chervov, Falqui, Molev, Nazarov, Ragoucy, Rubtsov and many other authors. Notably, the Manin property is key to the explicit formula due to Talalaev for the generators of the space of singular vectors of the vacuum Verma module at critical level over the affine Lie algebra $\widehat{\mathfrak{gl}}_M$. For references, and a discussion of applications of Manin matrices and their generalizations in quantum integrable systems, see the nice recent paper [3] by Silantyev.

In the rest of this article, however, we'll pivot away from quantum integrable systems, and sketch some recent progress closer to the original line of thinking in which Manin matrices first arose.

Quadratic Algebras and Koszul duality

The notion of Manin matrix above is the tip of an interesting iceberg, because one can replace the commutative algebra $\mathbb{C}[u, v]$ of polynomial functions with any of the rich class of *quadratic algebras*: algebras defined by quadratic relations, which include for example q-commutativity ($uv = qvu$).

Formally, an associative algebra $\mathbb{Q}$ is a *quadratic algebra* if it is of the form

$$\mathbb{Q} = T(V)/\langle R \rangle\,,$$

where $T(V) = \bigoplus_{n=0}^{\infty} V^{\otimes n}$ is the free algebra on V and $\langle R \rangle$ is the two-sided ideal generated by a subspace $R \subset V \otimes V$. We call (V, R) the *quadratic data* defining the quadratic algebra $\mathbb{Q}$.

For the polynomial algebra $\mathbb{C}[u, v]$, the quadratic data are $V = \mathbb{C}u \oplus \mathbb{C}v$ and $R = \mathbb{C}(u \otimes v - v \otimes u)$. The Grassmann algebra above generated by ϕ and ψ is also quadratic: its quadratic data consist of the space $\mathbb{C}\phi \oplus \mathbb{C}\psi$ and the quadratic relations given by $\mathbb{C}(\phi \otimes \phi) \oplus \mathbb{C}(\psi \otimes \phi + \phi \otimes \psi) \oplus \mathbb{C}(\psi \otimes \psi)$.

Given any quadratic algebra $\mathbb{Q} = T(V)/\langle R \rangle$ generated by a finite-dimensional vector space V, one can define the *Koszul dual* quadratic algebra,

$$\mathbb{Q}^! = T(V^*)/\left\langle R^\perp \right\rangle.$$

For example, the polynomial algebra and Grassmann algebra are each other's Koszul duals, as one can see by identifying $\phi = u^*$ and $\psi = v^*$ with the dual basis to the basis u, v of V.

Actually, there is one other example of a quadratic algebra implicit in what we've already done: let $\mathcal{U}$ be the associative algebra generated by a, b, c, d and subject *only* to the relations (2), i.e.

$$[a, c] = [b, d] = 0, \qquad [a, d] - [c, b] = 0.$$

Since these relations are quadratic, $\mathcal{U}$ is itself a quadratic algebra. It is sometimes called a *(right) quantum algebra*. Observe that giving a homomorphism $\mathcal{U} \to \mathcal{A}$ to an associative algebra $\mathcal{A}$ amounts precisely to picking a Manin matrix with entries in $\mathcal{A}$. (To state the universal property of $\mathcal{U}$ formally is a cute application of standard machinery from category theory; see e.g. [3].)

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[1] Indeed, it is a fun exercise (which the reader may like to try) to evaluate $\operatorname{cdet} T$ three ways in this special case. One finds that the *Capelli invariants*

$$\Gamma_r = (-1)^r \sum_{\substack{A \subset \{1,\dots,M\} \\ |A|=r}} \sum_{\substack{I \subset \{1,\dots,N\} \\ |I|=r}} \det X_{AI} \det D_{AI},$$

where X_{AI} and D_{AI} are minors of the matrices

$$X = \begin{pmatrix} x_1^1 & \dots & x_N^1 \\ \vdots & & \vdots \\ x_1^M & \dots & x_N^M \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} \partial_1^1 & \dots & \partial_N^1 \\ \vdots & & \vdots \\ \partial_1^M & \dots & \partial_N^M \end{pmatrix},$$

can be expressed in terms of either of the following sets of generators

$$E_{ij} := \sum_{a=1}^{M} x_i^a \partial_j^a \quad \text{and} \quad E^{ab} := \sum_{i=1}^{N} x_i^a \partial_i^b.$$

These generate the natural actions of $U(\mathfrak{gl}_N)$ and $U(\mathfrak{gl}_M)$ respectively on the (Weyl) algebra generated by the x_i^a and ∂_i^a, and Howe duality is the statement that they are one-another's commutants.

Black and white products

Suppose $\mathcal{Q} = T(V)/\langle R\rangle$ and $\mathcal{P} = T(W)/\langle S\rangle$ are two finitely-generated quadratic algebras. It is natural to ask what quadratic algebras are generated by the tensor product $V \otimes W$. The question is what quadratic relations to impose, and Manin observed that there are *two* natural choices. Intuitively speaking, we can either impose the relations R and S separately, or only together in parallel. More precisely, define

$$\mathcal{Q} \circ \mathcal{P} = T(V \otimes W) \Big/ \left\langle \sigma_{23}(R \otimes W^{\otimes 2} + V^{\otimes 2} \otimes S)\right\rangle$$
$$\mathcal{Q} \bullet \mathcal{P} = T(V \otimes W) \Big/ \left\langle \sigma_{23}(R \otimes S)\right\rangle .$$

(Here σ_{23} is the isomorphism $V \otimes V \otimes W \otimes W \xrightarrow{\sim} V \otimes W \otimes V \otimes W$.)

As the prototypical example of the black product in action, consider $\mathbb{C}[u,v] \bullet \mathbb{C}[u,v]^!$. It is generated by $u\phi, v\phi, u\psi, v\psi$. The relations include

$$(u\phi)(v\phi) - (v\phi)(u\phi) = (uv - vu)\phi\phi = 0$$

but only – to stress the point – because *both* $uv - vu$ *and* $\phi\phi$ are zero. In fact, if we set $a = u\phi$, $b = u\psi$, $c = v\phi$, and $d = v\psi$, then – lo and behold – we find that the defining relations of $\mathbb{C}[u,v] \bullet \mathbb{C}[u,v]^!$ are precisely the Manin relations (2)! That is, there is an isomorphism of quadratic algebras

$$\mathcal{U} \cong \mathbb{C}[u,v] \bullet \mathbb{C}[u,v]^!.$$

In this way, the black product can be used to re-write $\mathcal{U}$ in terms of the simpler quadratic algebra $\mathbb{C}[u,v]$ and its Koszul dual. Manin showed that various well-known quantum groups arise this way, i.e. are of the form

$$\mathcal{Q} \bullet \mathcal{Q}^!$$

for some quadratic algebra $\mathcal{Q}$. (In particular, $\mathcal{Q} \bullet \mathcal{Q}^!$ gets a Hopf algebra structure.) Moreover, Manin showed that there is an adjunction in the category $\mathbf{QA}$ of finitely-generated quadratic algebras,

$$\mathrm{Hom}_{\mathbf{QA}}(\mathcal{Q} \bullet \mathcal{P}^!, \mathcal{R}) \cong \mathrm{Hom}_{\mathbf{QA}}(\mathcal{Q}, \mathcal{P} \circ \mathcal{R}).$$

One also has $(\mathcal{Q} \circ \mathcal{P}^!)^! = \mathcal{Q}^! \bullet \mathcal{P}$.

Both $\bullet$ and $\circ$ give well-defined symmetric monoidal structures on the category $\mathbf{QA}$ of finitely-generated quadratic algebras. One should think of $\mathcal{R}^! \circ \mathcal{P}$ as an internal Hom object, $\underline{hom}(\mathcal{R}, \mathcal{P})$, in the symmetric monoidal category $(\mathbf{QA}, \bullet)$; dually, $\mathcal{P} \bullet \mathcal{R}^!$ is an internal coHom object, $\underline{cohom}(\mathcal{P}, \mathcal{R})$ in $(\mathbf{QA}, \circ)$.

Operads

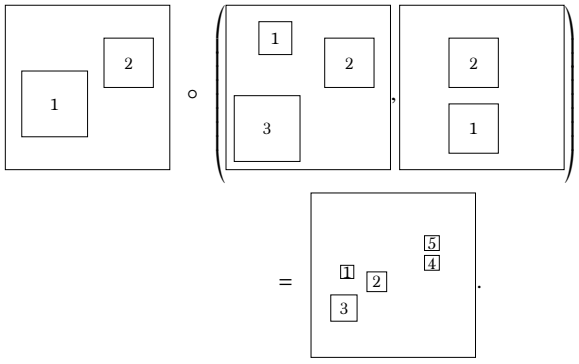
Since Manin first introduced them in the context of associative algebras, these ideas have been extended to the broader context of *operads* by authors including Ginzburg, Kapranov, Loday, Vallette, and Manin himself. We'll give a flavour of this progress, drawn from the recent paper [2], which the reader can consult for references.

Operads model algebraic structures with various kinds of associativity. More precisely, an *operad* in the category $\mathbf{Set}$ consists of, for each non-negative integer n, a set $\mathcal{P}(n)$ of *n-ary operations*, together with rules for composing these operations, which amount to functions

$$\circ : \mathcal{P}(n) \times (\mathcal{P}(k_1) \times \cdots \times \mathcal{P}(k_n) \to \mathcal{P}(k_1 + \cdots + k_n).$$

Compositions of operations are required to satisfy some natural associativity laws. The operad is *symmetric* if, for every n, the set $\mathcal{P}(n)$ of n-ary operations admits a right action of the symmetric group S_n, subject again to various natural relations.

A prototypical example is the *little-squares* operad $\mathbf{Squares}$, which is best described using pictures. Here, for instance, is the composition in $\mathbf{Squares}$ of a 3-ary operation and a 2-ary operation, using another 2-ary operation, to form a 5-ary operation:



The definition of operad generalizes from $(\mathbf{Set}, \times)$ to any symmetric monoidal category. For example $\mathbf{Squares}$ is an operad in sets, but also in topological spaces.

Now, how does the world of operads connect to the notion of quadratic data we discussed above? In (at least) two interesting ways:

(1) quadratic operads, and

(2) operads in quadratic data.

For both of these, we want to consider operads in the category $(\mathbf{Vect}, \otimes)$ of complex vector spaces with tensor product. Such an operad consists of, for each non-negative integer n, a vector space $\mathcal{P}(n)$ of n-ary operations, together with linear maps describing composition of operations,

$$\mathcal{P}(n) \otimes (\mathcal{P}(k_1) \otimes \cdots \otimes \mathcal{P}(k_n)) \to \mathcal{P}(k_1 + \cdots + k_n),$$

obeying certain associativity axioms. It is symmetric if each $\mathcal{P}(n)$ is a right module over $\mathbb{C}[S_n]$, subject again to various natural relations.

Just as a quadratic algebra was the quotient of a free algebra by quadratic relations, one has the notion of a *(binary) quadratic operad* in $(\mathbf{Vect}, \otimes)$: start with the *free operad* $\mathcal{F}(V)$ generated by a vector space V of binary operations (one should picture binary rooted trees with copies of V at their branch-points) and then quotient by the ideal generated by a subspace R of the space $\mathcal{F}(V)(3)$ of all cubic operations. The definitions of the Koszul dual, the black and white products, and the adjunction relating them, all go over to this setting.

Here, though, we want to concentrate on the second possibility above:

Operads in quadratic data

The starting observation is that many of the most important operads in the literature originate from topological operads but are themselves algebraic in character. Not only that, these algebras are often quadratic.

One doesn't need to look beyond **Squares** for an example. The arity n component of **Squares** is homotopy equivalent to the configuration space of n points in the complex plane,

$$\mathbf{Squares}(n) \sim \mathbf{Conf}_n(\mathbb{C}),$$

as one sees from the pictures above. By a result due to Arnold, its cohomology $H^\bullet(\mathbf{Conf}_n(\mathbb{C}))$ is isomorphic to the algebra of wedge products of the closed holomorphic differential forms

$$\omega_{ij} = \mathrm{d}\log(z_i - z_j) = \frac{\mathrm{d}z_i - \mathrm{d}z_j}{z_i - z_j}$$

on $\mathbb{C}^n \setminus \{\text{diagonals}\}$. That algebra is called the Orlik-Solomon algebra, $OS(n)$, and it turns out to be quadratic: the only relations obeyed by these ω_{ij}, besides usual the graded-commutativity of the wedge product, are

$$\omega_{ij} \wedge \omega_{jk} + \omega_{jk} \wedge \omega_{ki} + \omega_{ki} \wedge \omega_{ij} = 0. \qquad (4)$$

So we get a quadratic algebra – specifically, a graded-commutative quadratic algebra – but why do we get any operadic structure? Well, taking cohomologies is a contravariant functor $(\mathbf{Top}, \times) \to (\mathbf{ComAlg}, \otimes)$ to the category of graded-commutative associative algebras with tensor product. So the topological operad **Squares** gives rise to a *co*operad – i.e. the dual of an operad – in $(\mathbf{ComAlg}, \otimes)$. The Orlik-Solomon algebras $OS(n)$ are the spaces of (co)operations of this cooperad. Since they are quadratic algebras, we arrive at a cooperad OS in the category $(\mathbf{QA}, \otimes)$ of quadratic algebras with tensor product.

(We should stress that the monoidal product on **QA** is now the usual tensor product $\otimes$ rather than either of the products $\bullet$, $\circ$ from before. One can ask in what sense these various monoidal structures are compatible, and that leads to Vallette's notion of *lax 2-monoidal categories*.)

The Koszul dual of the Orlik-Solomon algebra $OS(n)$ is the *Kohno-Drinfeld* algebra $KD(n)$:

$$OS(n)^! = KD(n).$$

$KD(n)$ has generators $t_{ij} = t_{ji}$, with $1 \le i < j \le n$, and the quadratic relations given by the span of

$$\begin{aligned} [t_{ij}, t_{kl}] &= 0 \quad \text{for distinct } i,j,k,l, \\ [t_{ij}, t_{ik} + t_{jk}] &= 0 \quad \text{for distinct } i,j,k \end{aligned} \qquad (5)$$

where $[a,b] = a \otimes b - b \otimes a$. Because these relations involve only skew products, the associative algebra $KD(n)$ is actually the envelope of a *Lie* algebra, the quotient of the free Lie algebra in the t_{ij} by the relations above. That Lie algebra is called the Kohno-Drinfeld Lie algebra, or the Lie algebra of infinitesimal braids, and it is the *holonomy Lie algebra* of $\mathbf{Conf}_n(\mathbb{C}) \sim \mathbf{Squares}(n)$. Functoriality once more ensures that the quadratic algebras $KD(n)$ combine to form an operad in $(\mathbf{QA}, \otimes)$.

Now, since we actually got (co)operads in specifically *quadratic* algebras, one can and really should "lift" them to a simpler category of quadratic *data* defining those algebras. Namely, one should work in the symmetric monoidal category $(\mathbf{QD}, \oplus)$ whose objects are quadratic data (V,R) as defined above, and where we now take the monoidal product $\oplus$ to be the notion of direct *sum* given by

$$(V,R) \oplus (W,S) := (V \oplus W, R \oplus S \oplus (\mathrm{id} - \sigma_{12})(V \otimes W)).$$

Here $(\mathrm{id} - \sigma_{12})(V \otimes W) \subset V \otimes W \oplus W \otimes V$ embeds in $(V \oplus W)^{\otimes 2}$, as do $R \subset V \otimes V$ and $S \subset W \otimes W$. This monoidal product is cooked up to ensure that the passage $(\mathbf{QD}, \oplus) \to (\mathbf{QA}, \otimes)$ is functorial.

Moreover, we saw that the algebra $OS(n)$ was graded-commutative, while $KD(n) = OS(n)^!$ was the envelope of a Lie algebra. One can bake that into the constructions by introducing monoidal subcategories of suitably *symmetric* and *skew-symmetric* quadratic data, with Koszul duality exchanging the two, as is done in [2].

The point of all this, finally, is that our starting topological operad **Squares** was just one example.[2] The pattern of Koszul dualities between (co)operads in (skew) symmetric quadratic data provides a powerful organising principle which holds in many other important cases – and might well be suggestive of new ones.

Examples given by Manin and Vallette include the topological operad made up of the complex loci $\overline{\mathcal{M}}_{0,n}(\mathbb{C})$ of the Deligne-Mumford compactifications of the moduli spaces of stable curves of genus zero with marked points (this is the setting for quantum cohomology); non-commutative versions thereof; graph operads; and hypergraph operads.

Let us end with an example of the last of these, chosen because from the perspective of quadratic data it appears, very intriguingly, to be the "next" case after **Squares**. There is a topological operad whose space of n-ary operations is the *real* locus $\overline{\mathcal{M}}_{0,n+1}(\mathbb{R})$ of the Deligne-Mumford compactification of the moduli space of stable curves of genus zero with $n+1$ marked points. Again we can take cohomologies to obtain a cooperad in graded-commutative quadratic algebras and – it turns out, due to a deep result of Etingof, Henriques, Kamnitzer and Rains – that $H^\bullet(\overline{\mathcal{M}}_{0,n+1}(\mathbb{R}))$ is isomorphic to an algebra $EHKR(n)$ on certain generators ω_{ijk}, subject to relations

$$\omega_{ijk}\wedge\omega_{klm}+\omega_{jkl}\wedge\omega_{lmi}+\omega_{klm}\wedge\omega_{mij}+\omega_{lmi}\wedge\omega_{ijk}+\omega_{mij}\wedge\omega_{jkl}=0$$

– compare (4)! The Koszul dual, $EHKR(n)^!$, is the quadratic algebra on generators t_{ijk} subject to

$$[t_{ijk},t_{lmn}]=0 \quad \text{for distinct } i,j,k,l,m,n,$$
$$[t_{ijk},t_{lmi}+t_{lmj}+t_{lmk}]=0 \quad \text{for distinct } i,j,k,l,m$$

– compare (5)! – and it is again the envelope of the holonomy Lie algebra, now of $\overline{\mathcal{M}}_{0,n+1}(\mathbb{R})$.

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[1] A. Chervov, G. Falqui, and V. Rubtsov. "Algebraic properties of –Manin matrices. I". In: *Adv. in Appl. Math.* 43.3 (2009), pp. 239–315. doi: 10.1016/j.aam.2009.02.003 (cit. on p. 2).

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[2] Albeit a very important one: The space $\mathrm{Conf}_n(\mathbb{C})$ can also be viewed as the complement $\mathbb{C}^n \setminus \{\text{diagonals}\}$ of a *discriminantal arrangement* of hyperplanes in $\mathbb{C}^n$. Both the Orlik-Solomon algebra of forms, and realisations of the Kohno-Drinfeld Lie algebra, are used in the construction of solutions to the Knizhnik-Zamolodchikov equations on this complement, in terms of multidimensional hypergeometric integrals. Those solutions play the key role in the explicit geometric proof due to Varchenko of the celebrated Kohno-Drinfeld theorem which describes the monodromy of Knizhnik-Zamolodchikov equations in terms of quantum groups.

Notes of a Numerical Analyst

Hermite polynomial surprises

NICK TREFETHEN FRS

Among the oldest tools in the box are Hermite polynomials, which are used for working with functions of a real variable that decay as $|x| \to \infty$. Hermite polynomial expansions and numerical methods are derived from conditions of optimality. Yet they are far — very far! — from optimal.

We can illustrate the issue by looking at a problem of quadrature. Suppose a function f is given and we want to calculate the integral

$$I = \int_{-\infty}^{\infty} f(x) e^{-x^2} dx. \qquad (1)$$

A quadrature formula is an approximation

$$I_n = \sum_{k=1}^{n} w_k f(x_k) \qquad (2)$$

for some nodes $x_1, \ldots, x_n$ and weights $w_1, \ldots, w_n$. Suppose we ask, what $\{x_k\}$ and $\{w_k\}$ are optimal in the sense that (2) gives exactly the correct answer, $I = I_n$, whenever f is a polynomial of the highest possible degree? There is a unique such choice, and it is called *Gauss–Hermite quadrature (GH)*, integrating (1) exactly whenever f is a polynomial of degree $\le 2n-1$.

For example, for $f(x) = e^x$, the integral is $I = e^{1/4}\pi^{1/2} \approx 2.275875794469$. With just $n = 9$, GH gives the approximation $I_n \approx 2.275875794454$, accurate to better than 10^{-10}.

But the story changes for a more complicated function like $f(x) = \cos(x^3)$. To get $|I - I_n| < 10^{-10}$ with GH now, we need $n \ge 606$. Yet this integral is not really as hard as that, for although $\cos(x^3)$ wiggles a lot, the factor $\exp(-x^2)$ damps it down. In fact, if we chop the interval to $[-5,5]$ and apply ordinary Gauss(–Legendre) quadrature, $n \ge 89$ is enough to give ten digits.

Figure 1 shows this effect for varying n. The nodes of GH span a range of order $\exp(Cn^{1/2})$. This is so wide that if f is a bounded analytic function on the real line, the outer samples contribute negligibly to (2), and the accuracy is only $O(\exp(-Cn^{1/2}))$. (With $n = 606$, 476 of the weights are below the standard machine precision of $\approx 10^{-16}$!) By contrast if we apply Gauss–Legendre quadrature on a narrower interval of size $\exp(Cn^{1/3})$, the accuracy improves to $O(\exp(-Cn^{2/3}))$.

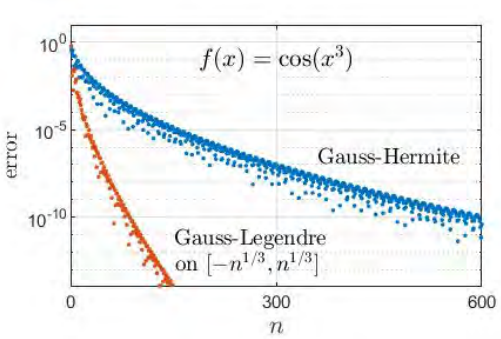

Figure 1. Gauss–Hermite quadrature, notwithstanding its optimality, converges far more slowly than chopping the real axis to a finite interval and applying a simpler formula.

How can an optimal formula be so far from optimal? The explanation is that polynomial exactness implies very little about accuracy. Polynomials must grow as $|x| \to \infty$, and loosely speaking, a formula that treats them exactly wastes most of its effort managing that growth.

We mathematicians have a way of proving theorems that are literally true, yet miss the point — I call them "inverse Yogiisms". GH is 140 years old, but although many theorems have been published, its optimality has rarely been questioned.

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[2] L. N. Trefethen, Exactness of quadrature formulas, *SIAM Review*, to appear.

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Mathematics News Flash

Jonathan Fraser reports on some recent breakthroughs in mathematics.

Rigidity of Riemannian embeddings of discrete metric spaces

AUTHORS: Matan Eilat and Bo'az Klartag
ACCESS: https://arxiv.org/abs/2004.08621

This paper, published in *Inventiones Mathematicae* in 2021, considers the following intriguing question: Can one identify a 2-dimensional manifold simply by studying the distances determined by a discrete subset? Of course, the answer is no in general. However, the main result states that if the manifold is a complete connected Riemannian surface and the discrete subset gives rise to distances corresponding to a 2-dimensional lattice, then the manifold must be isomorphic to the Euclidean plane. As such, 'flatness' is observed by a discrete object! A corollary, which at first sight may seem surprising, is that a subset of the 3-dimensional integer lattice $\mathbb{Z}^3$ which *strictly* contains $\mathbb{Z}^2 \times \{0\}$ cannot embed isometrically in a complete connected Riemannian surface.

Patterns in thick compact sets

AUTHORS: Alexia Yavicoli
ACCESS: https://arxiv.org/abs/1910.10057

Given a compact subset of the line and a finite configuration of points in the line, is it possible to find a scaled and translated copy of the finite set inside the compact set? The Lebesgue density theorem guarantees that this is always possible provided the compact set has positive Lebesgue measure. For sets with zero Lebesgue measure, one might hope that having large enough Hausdorff dimension might be sufficient to guarantee the existence of finite configurations, but this is not true in general. (Keleti constructed a compact subset of the line with Hausdorff dimension 1 which fails to contain any 3-term arithmetic progressions.)

In this paper, which is to appear in *Israel Journal of Mathematics*, conditions guaranteeing the existence of finite configurations are given in terms of *Newhouse thickness*. More precisely, a compact subset of the line will contain scaled and translated copies of arbitrary configurations of n points, provided the Newhouse thickness is large enough in terms of n.

Parabolic orbits in celestial mechanics: a functional-analytic approach

AUTHORS: Alberto Boscaggin, Walter Dambrosio, Guglielmo Feltrin and Susanna Terracini
ACCESS: https://arxiv.org/abs/1903.07849

In celestial mechanics, the N-body problem is to determine the individual motions of N celestial bodies interacting with each other via gravitational attraction. This is a notorious problem which remains open for $N \geq 3$. This paper, published in *Proceedings of the London Mathematical Society* in 2021, studies the equation

$$\ddot{x} = \nabla U(x) + \nabla W(t,x)$$

for $x \in \mathbb{R}^d$, $d \geq 2$, where U is a positive potential subject to certain constraints, and W is a lower order term for $|x| \to \infty$. The existence of half-entire parabolic solutions, asymptotic to a prescribed central configuration is established. The proof relies on formulating the problem in a suitable functional space.

Many well-known problems in celestial mechanics, including the N-body problem, may be expressed in this way. For example, the results of the paper apply to the 3-body problem with a collinear configuration consisting of two unit masses and one suitably bounded mass.

Jonathan Fraser is a Professor at the University of St Andrews and an Editor of this Newsletter. He is pictured here with Dylan, Rayna and Reuben.

Microtheses and Nanotheses provide space in the Newsletter for current and recent research students to communicate their research findings with the community. We welcome submissions for this section from current and recent research students. See newsletter.lms.ac.uk for preparation and submission guidance.

Microthesis: Combining data sources to develop a fertility projection model

JOANNE ELLISON

Fertility projections are a key determinant of population projections, which are widely used by government policymakers and planners. They are also vital to anticipate demand for maternity services and school places. My thesis presents a fertility projection model that combines individual-level and population-level data sources to exploit their opposing strengths.

Individual-level fertility data

Individual-level fertility data is often extracted from surveys collecting retrospective fertility histories and additional information from women in the population. The date of birth tends to be recorded imprecisely to the nearest month or year. A key feature is the presence of the five clocks, namely age, period, cohort, time since last birth (TSLB) and parity. Parity is the number of children previously borne, and we take the reproductive age range to be 15-44.

We illustrate three of the clocks in Table 1 (rows 2-4) for a woman aged 41 who had single births at ages 25 and 27, and twins at age 31. As the woman is currently 41, this year of age is not fully observed, and so she contributes 26 sets of observations (or person-year records) out of 30, from ages 15-40. Each person-year record has a corresponding binary response variable B equal to 1 if the woman had a birth at that age and 0 otherwise (see row 1 of Table 1). The person-year record then also contains the values of any extra variables included in the model.

B	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
Age	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
Parity	0	0	0	0	0	0	0	0	0	0	0	1	1	2	2
TSLB	-	-	-	-	-	-	-	-	-	-	-	1	2	1	2
B	0	1	0	0	0	0	0	0	0	0	0				
Age	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
Parity	2	2	4	4	4	4	4	4	4	4	4				
TSLB	3	4	1	2	3	4	5	6	7	8	9				

Table 1. Binary response variable B and three of the five clocks for a hypothetical woman aged 41.

Population-level fertility data

Whereas individual-level data only covers a sample of the population, population-level data includes everyone. The Office for National Statistics (ONS) publishes fertility rates by parity for England and Wales (tinyurl.com/rpz2f44x). This dataset uses mid-year population estimates and birth registration data. The rates are indexed by age, period, cohort and parity only, unlike the individual-level data where we have time since last birth and a range of survey variables. In this way the two levels of data complement each other, with a shortcoming of one (e.g. sample size, detail) being a strength of the other.

Modelling individual-level data

Individual-level data informs the base of our model, namely responses from Wave 1 (2009-11) of a survey called the UK Household Longitudinal Study or UKHLS (tinyurl.com/3kf2n6t3). Our sample of 18,218 women (357,287 person-years) were born in 1945-1992 and resided in England or Wales when interviewed. For parities 0, 1, 2 and 3+, we learn about the smooth dependence of B on age, cohort and time since last birth, as well as the effect of highest educational qualification. We do this by fitting logistic generalized additive models (GAMs) (see "GAMs", and [1] for an excellent introduction).

We henceforth focus on parity 0 and model the probability of a first birth. Our chosen model includes smooth effects of age, cohort, and their interaction; it also includes the categorical highest qualification variable and its smooth interaction with age.

GAMs

Just as a generalized linear model (GLM) generalizes linear regression to allow the response variable distribution to be non-normal, a GAM generalizes a GLM to allow the response to depend on smooth functions of covariates in some way.

Let Y_i be the ith response variable, following an exponential family distribution and with $\mu_i \equiv E(Y_i)$. A GAM has the following form

$$g(\mu_i) = \mathbf{X}_i\boldsymbol{\beta} + f_1(x_{1i}) + f_2(x_{2i}) + f_3(x_{3i}) + f_4(x_{1i}, x_{3i}) + \dots$$

where g is a monotonic link function, $\mathbf{X}_i$ is the ith row of the fixed effects model matrix, $\boldsymbol{\beta}$ are the corresponding parameters, and the f_j are smooth functions of covariates x_k. These can be one-dimensional curves or higher-dimensional surfaces.

Integrating population-level data

To incorporate the ONS data into our individual-level model, we marginalize over the single additional covariate, i.e. qualification (Q). For each age and cohort, this amounts to taking a weighted average of the Q-specific probabilities, where the weights are the probabilities of a woman belonging to each Q category given her age and cohort. We model these probabilities using multinomial logistic regression. Lastly, we weight the contributions of the two data sources according to our prior beliefs about their relative importance. Terming these 'integrated models', we fit several of them, varying the weights.

Results

In Figure 1 we plot the mean probabilities of a first birth by age and cohort for all women and two of the qualification categories (GCSE, Degree). The model in the first row only uses UKHLS data (100% UKHLS, 0% ONS), while the second balances the information in the data sources roughly equally (50/50).

Incorporating the ONS data closely aligns the "All" probabilities to the observed ONS rates. In particular, we see that the quickly increasing trends forecast by the 100/0 model completely reverse to slow declines.

The Q-specific forecasts show us how changes are shared across population subgroups. The recent teenage fertility declines are strongly felt by the GCSE category, whose bimodal curves change to unimodal for the youngest cohorts, approaching those of the Degree category. Such additional insights demonstrate the value and richness of integrating different levels of data for demographic forecasting. For further details on this work, see [2].

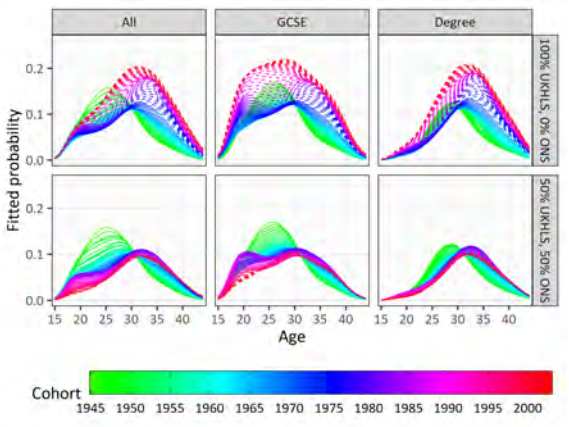


Figure 1. Mean conditional probabilities of a first birth for various integrated models; dashed lines indicate forecasts.

Acknowledgements

This work was funded by EPSRC (award 1801045), and partly supported by the ESRC FertilityTrends project (grant ES/S009477/1) and the ESRC Centre for Population Change - phase II (grant ES/K007394/1).

FURTHER READING

[1] S. N. Wood. *Generalized Additive Models: An Introduction with R, Second Edition*. Chapman & Hall/CRC Press, 2017.

[2] J. Ellison. Stochastic modelling and projection of age-specific fertility rates. PhD Thesis, *University of Southampton*, 2021. eprints.soton.ac.uk/450468/

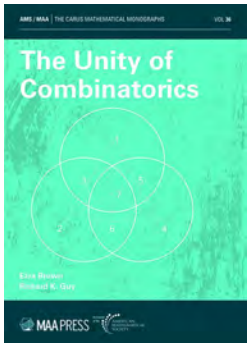
Joanne Ellison

Joanne is a Research Fellow at the University of Southampton, and is currently working on the ESRC-funded FertilityTrends project. Her main research interests are in developing statistical models for forecasting demographic processes. Despite being a big tennis fan, she is yet to pick up a racket herself.

The Unity of Combinatorics

by Ezra Brown and Richard K. Guy, Carus Mathematical Monographs, Vol. 36, MAA Press, 2020, US$ 65, ISBN 978-1-4704-5279-7

Review by Robin Wilson



"One reason why combinatorics has been slow to become accepted as part of mainstream mathematics is the common belief that it consists of a bag of tricks in many areas ... with little or no connection between them." So begins the Introduction to this book, where the gap in this opening sentence lists fifteen different subject areas that range from packing and covering and projective geometry to lattices and error-correcting codes. The paragraph triumphantly concludes: "We shall see that they have numerous threads weaving them together into a beautifully patterned tapestry."

Combinatorialists have often made bold claims for their subject, ever since Leibniz, on the cover of his youthful *Dissertation on the Combinatorial Art* on permutations and combinations, claimed to demonstrate the existence of God with complete mathematical certainty. The authors of this book do not make this particular promise, but are more successful in justifying those that they do. To achieve this, they make use of copious examples, the "life-blood of combinatorics", often developing their narrative from a recreational puzzle to topics of a more technical nature.

The idea for this unusual book originated in a lecture given by Richard Guy in 1994 and written up as a 30-page article shortly after. As a first example of the threadweaving mentioned above, the opening chapter begins with Langford sequences, which consist of two copies of the numbers 1 to n arranged so that the two occurrences of each number k appear k spaces apart (such as 312132 or 41312432). After proving that these sequences are possible only when $n = 0$ or $3 \pmod 4$, the authors consider related sequences and prove Beatty's theorem, that if α and β are irrational numbers, greater than 1, with $1/\alpha + 1/\beta = 1$, then the disjoint sequences $\{\lfloor n\alpha \rfloor\}$ and $\{\lfloor n\beta \rfloor\}$ together include every positive integer just once. Choosing $\alpha = \phi$ and $\beta = \phi^2$, where ϕ is the golden ratio $(1+\sqrt{5})/2$, then links up with geometric patterns ('Conway worms') appearing in Penrose aperiodic tilings of the plane. These sequences $\{\lfloor n\alpha \rfloor\}$ also feature in Chapter 2 in connection with combinatorial games, such as Nim, and the Sprague–Grundy theorem on possible positions in finite two-person games.

The next three chapters also range widely, linking Fibonacci numbers, Pascal's triangle, and Catalan numbers with walks on an infinite square lattice and other regular arrangements of points, while Langford sequences reappear in Chapter 6 in connection with triples of integers (x, y, z) with $x + y = z$. These then lead in two directions: first, to the Ringel–Youngs theorem on the number of colours required for colouring maps on orientable surfaces, and in turn to the problem of splitting a square with integer-length sides into smaller squares all of different sizes, and to packings and coverings of the complete graph K_n with triangles.

Consideration of these triangles leads in Chapter 7 to Steiner triple systems such as the system with triples $\{124, 235, 346, 450, 561, 602, 013\}$, and the next two chapters then connect these triple systems with difference sets and with symmetric block designs and finite projective planes, such as the Fano plane with seven points $\{0, 1, 2, 3, 4, 5, 6\}$ and seven lines given by the triples above. Two later chapters then present further connections with a wide range of other topics, such as Latin squares and tournaments, normed algebras, sums of squares, quaternions and octonions. The Fano plane also reappears as an example of a matroid, a simultaneous generalisation of 'independence' as it arises in both graph theory and linear algebra.

As would be expected from two authors who are so well known for their expository writing, all these

topics are lucidly described and extensively illustrated with examples and with many helpful diagrams. One could easily go on to list connections with other topics, but it should already be clear that this is a remarkable book in both its scope and its clarity and attractiveness of presentation. As the authors note, parts of the book can be read in bed or while commuting or travelling, while other sections may require a fair amount of work at a board or a table, and they encourage their readers when pursuing any topic to "Go For It."

If I may be permitted to conclude on a personal note, I first met Richard Guy over fifty years ago during a month's visit to Hungary, and my memories of him are of pads of paper covered with numbers as he worked on problems from number theory or combinatorics. The last time we met was at a Mathematical Association of America meeting at which he presented an impressive extension of the nine-point circle of geometry in the month before his 100th birthday, and he continued his daily visits to the mathematics department of the University of Calgary until shortly before his death last March at the age of 103. This attractive and highly recommended book, which sadly he did not live long enough to see, is a fitting memorial to his memory.

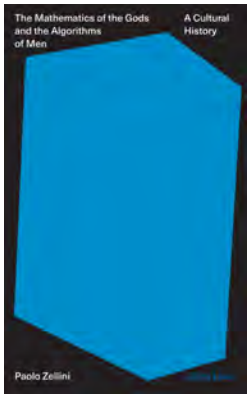
Robin Wilson

Robin Wilson is Emeritus Professor of Pure Mathematics at the Open University and Emeritus Professor of Geometry at Gresham College, London. A former President of the British Society for the History of Mathematics, he has written and edited over 40 books on graph theory, combinatorics, and the history of mathematics.

The Mathematics of the Gods and the Algorithms of Men: A Cultural History

by Paolo Zellini, Allen Lane, 2020, £18.99, ISBN: 978-0241312179

Review by Snezana Lawrence



This pleasingly narrated and structured book was a surprise to me in many ways. Firstly, although I expected a historical work, this can safely be described as a book in the philosophy of mathematics, drawing a thread between the intuitionism, logicism, and nominalism in tracing the development of mathematical sciences. Secondly, with an extended introduction, the author explains how, in order to see how mathematics deals with reality and our understanding of mathematics, we need to go back to antiquity. The surprise here is that the book, from antiquity, then manages to cover more or less all periods of mathematical history up to the present day. Finally, this read was a surprise to me as I expected the book to be more about the links between mathematics and theology, and instead found that it concentrates on our understanding of mathematical thinking. Of course, in this instance, it touches upon the religious, but rather in passing. I will add that this should have been obvious to me from the title: after all, the author clearly suggests that the algorithms of men in dealing with mathematics are his real interest. So, for the hopeful that the book is a study of links between mathematics and theology I would have to say: "do not be disappointed, for you will find what you seek, albeit in different and unexpected ways". And for the reader who dreads links between mathematics and theology I too can safely recommend this enjoyable study in the cultural history of mathematics and philosophy.

The book opens up with a question "about which reality does mathematics speak to us?" and answers are given through the many examples of how that reality is conjured through mathematical works of many cultures and peoples throughout the centuries. It is a small volume of 256 pages, and within it there are nineteen chapters as well as an introduction; notes and index are also present.

The place where Gods make an appearance is the second chapter, 'Mathematics of the Gods'. Here the narrative is focused on how mathematical algorithms feature in the various world religions. In this chapter, one can notice some echos of his previous work, *A Brief History of Infinity*, which treated the development of the concept of infinity that included philosophy, psychology, as well as spirituality. In the current work though, Zellini stays more clear of the spiritual and theological and instead stays focused on the algorithmic nature of theological texts or concepts. But here also lies an important recounting (to coin a phrase) of how and where there lies the root of the connections between mathematics and algorithms to which the book is dedicated: the process of enumeration. To make things 'real' has always been somehow to render them actual, to name them. Zellini gives examples of enumerations, censuses, catalogues from Homer to the Old Testament and other sacred texts of the world religions in this chapter.

The development and use of algorithms to solve the types of problems that mathematics gives us or that is used to solve the problems in turn from other areas of life is looked at from many perspectives. To give you a flavour of the style, I begin with the beginning:

> So where to start? One way is to go look at how the question of the reality of numbers was posed in ancient Greece, at least implicitly, in attempts to understand what relations mathematical entities have with the infinite, with the non-being of the ápeiron , or infinite. Infinity was absence (stéresis), pure potentiality — and everything, to exist and to endure, had to oppose itself

> against the negativity of the limitless. This was, in Greek mathematics, the task of the lógos, of proportion, in which the precursors of modern numbers were found. The phenomenon of relation, and what derived from it, was an entity close to the gods. (pp. 17–18)

I deliberately chose this passage as it also shows the nature of the narrative as it is developed by Zellini. If I could describe it in a literary sense, it would be that it is akin to the 'stream of consciousness' type of writing that is sometimes found in great novels (for example, Philip Roth's *American Pastoral* is one such), but in our case the central character is the mathematical algorithm which we look at from all the different angles the author is able to show us. The author's knowledge is, without a doubt, wide and deep: the book contains many references from the very many examples scattered throughout the history and geographical areas of mathematical development. In this sense, this book is neither the faint-hearted nor the novice in the history or philosophy of mathematics.

The nineteen chapters, excluding the Introduction, cover a wide range of topics such as 'Abstraction, Existence and Reality' (Chapter 1), 'The Growth of Numbers' (Chapter 5) or the next chapter (16), 'The Growth of Matrices'. In between, the number of examples, from how mathematical problems have been seen in different cultures and periods, and the algorithms of solutions relating to these, is immense. This is quite a feat as the book itself, as I have already mentioned, is not very long; and when one bears in mind that the period covers Babylonian mathematics to present-day mathematical manifestations, it may give you an idea how dense the text is in terms of the sheer amount of information and references.

Sometimes, the wide-sweeping conclusions based on the author's enviable knowledge are not necessarily given through a type of description, explanation, or exploration that would make these statements, or rather the route by which the author came to the conclusions, meaningful to those perhaps less well versed in either the history or the philosophy of mathematics. Although praise is due to the author for being able to seemingly, without effort, draw such threads and weave patterns of thought that span such vast mathematical landscapes, and produce a story so beautifully told, I wondered whether the non-historians would be able to put together cognizant pictures of the intended meanings. With this also comes the issue of covering the sources that are not always given. But, if one approaches this work as a follower of an internal dialogue, and in that case the reader also has the necessary knowledge and reflective ability to respond to this in their own way, then this is, without a doubt, an incredibly enjoyable read.

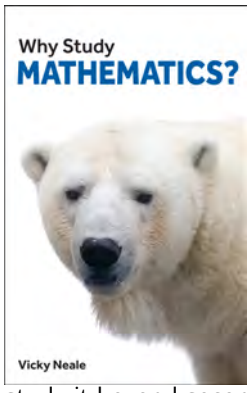
Snezana Lawrence

Snezana Lawrence is a Senior Lecturer in the Department of Mathematics at the University of Middlesex, London, England. She is the Chair of the History and Pedagogy of Mathematics ICMI Affiliated Study Group (2020-2024) and is the IMA Diversity Champion. Snezana has published on the history of mathematics and its relationship to education, and her main focus area within this field is the history of geometry, applied mathematics, and the perception of mathematics and mathematicians in mathematics education and popular culture.

Why Study Mathematics?

by Vicky Neale, London Publishing Partnership, 2020, £12.99 (print), £8.99 (ebook), ISBN: 978-1913019112

Review by Finley Ilett



Why Study Mathematics? is a fantastic guide to this most wonderful of subjects, giving readers the ultimate guided tour of maths, with many aspects to the book that make it ideal for any A-Level student who wants to know more about the paths available for those who wish to study it beyond secondary school.

Right from the opening, there is a clear focus on maths being a 'versatile subject' — already a firm response to the question posed by the title. What's more, the whole book contains an excellent level of detail relating to all aspects of maths-based higher education, as well as a sample of the many career opportunities there are. The result is a book built around clarity, and one that assumes little prior knowledge — this particularly comes into play in the chapter that explains how degrees (and the many titles, qualification names and accreditations) work, because most students are probably not very familiar with much of the terminology (in spite of it being rather important when making decisions about university, as Neale wisely points out). All of this collectively means that this book is perfect for learning what maths as a subject has to offer, and is definitely worth reading even for those students who are only vaguely interested in post-secondary mathematical study.

One part of the book that is much more relevant than many people may think is the often-overlooked content of maths degrees themselves. Some may assume that all degree courses are too similar for this to be of any importance, but that viewpoint is firmly opposed here, ensuring that readers are encouraged to actually look more closely at exactly what they are applying for. While the reality of this may be surprising for students who are used to the GCSE and A-Level system, with relatively little variation between a few exam boards (especially in maths and sciences), the way course content at university works is clearly explained — and a useful point of reference for students is the in-depth section on the four general areas of mathematics that are almost guaranteed to turn up in a degree (and the rough breakdown of each is the perfect way to make readers aware of the range of topics they are likely to encounter). This is important when many students spend too great a proportion of time just looking at post-university career prospects, and then risk regretting their choices at an earlier stage (perhaps by not having a clue what to do when given a choice between modules while at university, for example!).

Something that Neale never forgets is the importance of putting the maths that readers may study at university into a real-world context. There is nowhere near enough focus on this for secondary school maths, so it is a great asset to the book that this information is included throughout — it gives the subject relevance, in the eyes of the reader, thus providing a level of inspiration to those who had perhaps not considered the usefulness of the maths they study as deeply as they probably ought to have done — the section on applied maths within degrees being especially useful in this regard. However, this book is not solely to do with the study of the subject — the second part, aptly titled 'Maths In Action' looks instead at the uses of maths. These include some demonstrations and proofs that serve to remind readers of some of the beautiful qualities of the subject, along with some intriguing applications that few readers will have considered before. This offers a refreshing change from the exam-focused, theory-based type of maths that students are more likely to have already encountered. Leading on from this, emphasis is placed on the way that although some students know what they want to do, other may not — this is especially important for the many readers tired of all the university and

careers advice excessively targeted at people who have already chosen their path, when they, being as-yet-undecided, cannot benefit as much from this advice themselves. It means that they can stop worrying about this, with Neale reminding the reader that in fact, many maths students end up going into a field they didn't even know existed before studying it at university. This might well also be something of an eye-opener to those who thought that they had already encountered all the mathematical fields there are to study.

One other element of this book in particular that must be recognised is the chapter on 'Further Applications Of Mathematics'. Ultimately, some will see this as the most inspiring part of the book, because this is where the importance of maths in our everyday lives is showcased, and the subtext of this chapter is the most valuable message the book has to offer — that of the sheer diversity of applications of maths as a general, all-encompassing whole. The examples of topics — ranging from medicine, computer science and cyber-security, to retail and climate science (as well as the all-important field within any form of maths of studying and reducing errors!) — really show the true scope of opportunity for someone who chooses to go on to study maths, or one of its related sub-groups, at university or beyond.

So overall, this book will encourage students to look further into the many possibilities maths makes available to those who choose to study it, and it can therefore be thoroughly recommended to anyone who is thinking of studying maths after their A-Levels — informative, engaging and inspiring, it serves to help readers make their post-secondary decisions from a much better vantage point — and one which they might not otherwise have imagined existed at all!

Finley Ilett

Finley Ilett is a student at Queen Elizabeth Grammar School in Penrith, and is doing A-levels in Maths and Further Maths, as well as Physics and Chemistry.

Obituaries of Members

Uwe G. Grimm: 1963–2021

Professor Uwe G. Grimm, who was elected a member of the London Mathematical Society on 9 February 2007, suddenly and unexpectedly died on 28 October 2021, aged 58.

Michael Baake and John Hunton write: Uwe Grimm was born and educated in Gütersloh, Germany. He studied physics and mathematics at the University of Bonn, where he completed his PhD in 1991 on quantum spin chains, quantum groups and minimal models, under the supervision of Vladimir Rittenberg. He spent eight years as a postdoctoral researcher in Melbourne, Amsterdam and Chemnitz, and joined the mathematics department of the Open University in late 2000. He became professor of mathematics in 2009.

Uwe's research began with spin systems and solvable models and gradually changed to the theory of aperiodic order around 1990, which was initiated by the discovery of quasicrystals and the important role that symmetries of all kinds played in this new field. His habilitation on the physics of aperiodic order was completed in 2000 in Chemnitz. From then on, he was particularly interested in the fascinating properties of aperiodic tilings, which gave him the opportunity to work at the interface of mathematics and physics, and at that of geometry and dynamics.

He published close to 100 papers in refereed journal and made about 50 further contributions, on problems from mathematics, physics and even genetics. He co-authored the now widely used monograph *Aperiodic Order* (jointly with his friend and colleague Michael Baake from Bielefeld University). It contains a preface by Roger Penrose whose famous tiling still plays an important role in the field today.

At the Open University, Uwe was a valued member of the department, where he recently developed a strong 'aperiodic order' research group, supported by several EPSRC grants. He served the department as head of school and helped the university at large to shape its profile as pro-dean, always with modesty, respect and a good sense of humour — hallmarks of his character that will be remembered clearly by all who interacted with Uwe in any of his various roles.

During the pandemic, Uwe started a regular series of informal online meetings, bringing together the UK community in aperiodic order and connecting it with other groups in Europe and the world. He was particularly keen on supporting the wider group of early career researchers in the field. In general, he considered communication very important and had many ongoing co-operations with people all over the globe. He co-organised, jointly with his friend and colleague Ronan McGrath, the *Aperiodic 2009 Conference* in Liverpool, and was a visiting professor at the University of Tasmania in Hobart, Australia, for many years.

Outreach and public engagement activities were a passion for Uwe. The field of aperiodic order gave him a fine, visual angle which he was able to use for this, doing so on many occasions. In conjunction with various collaborators each time, he was chosen twice to exhibit at the prestigious Royal Society Summer Science exhibitions, first in 2004 on the hidden beauty of quasicrystals, and then again in 2010 on the geometry and mathematics behind aperiodic order, the latter exhibit also being showcased by the EPSRC around the same time as part of its display of contemporary research put on for government and policy makers.

Uwe was a true gentleman who could calmly smile at life's irritations. He will be greatly missed at both a personal and intellectual level by all who worked with him. He is dearly missed by his wife Kathrin and his two sons, Jasper and Moritz.

Helge Tverberg 1935–2020

Professor Helge Tverberg, who was elected a member of the London Mathematical Society on 17 October 1975, died on 28 December 2020, aged 85.

Gunnar Fløystad writes: Helge Tverberg was born in Bergen. The University of Bergen was founded in 1946 and Tverberg entered there as a student in 1954. He was the first to obtain a Masters degree in pure mathematics, in 1958, at a time when the department had only two positions in pure mathematics. He quickly became the third to take up a position. In 1970 he was appointed full professor. He was a member of

the LMS since 1977 and of the Norwegian Academy of Science and Letters since 1988.

Tverberg was always a friendly and considerate colleague. He had an encompassing memory for people, stories, and mathematical results. During lunch and social occasions we greatly appreciated his anecdotes, speeches, jokes, and his original observations and views. He was easy to ask and had a wealth of mathematical thoughts and problems, always eager to share. He was broadly oriented in mathematics. As a young man he made a resolution to read all the books in the then department library. Before the internet, and also after, he was therefore a colleague of whom it was natural to inquire about both specific and general problems and directions.

His scientific work was in combinatorics and convex geometry. His foremost result is the celebrated Tverberg's theorem, which he discovered in 1964. Radon's theorem from 1921 says that $d+2$ points in the Euclidean space Rd may be partitioned into two sets such that their convex hulls intersect. (Check $d=2$.) In 1959 Birch had discovered a version in the plane R^2 where you partition $3q-2$ points into q sets. It was natural to seek a simultaneous generalization of Birch and Radon, partitioning point sets in R^d into point sets with intersecting convex hulls. Birch had a conjecture but had given up proving this. Tverberg was visiting colleagues in Manchester, and had devoted much thought to this. "I recall that the weather was bitterly cold in Manchester. I awoke very early one morning shivering, as the electric heater in the hotel room had gone off, and I did not have an extra shilling to feed the meter. So, instead of falling back to sleep, I reviewed the problem once more, and then the solution dawned on me!"

Tverberg's theorem is considered today one of the fundamental results in convex geometry. Ten years ago a large international conference in discrete mathematics was organized in Bergen. The main speaker was Günther Ziegler, president of the German Mathematical Society, giving a large survey talk on Tverberg's theorem and the research in its wake. Tverberg also liked to fundamentally understand and simplify established results. Let us mention a result on decompositions of graphs by Ron Graham and Henry O. Pollak. Tverberg's new proof was exceedingly short, with the argument mainly using linear algebra. Twenty years ago his proof was included in *Proofs from THE BOOK*, the title referring to ErdHos' idea that our Lord had a grand book with the most simple and natural arguments for theorems.

To give another example, Jordan's curve theorem or "the principle behind the prisons" as Tverberg would say, is a theorem with a prehistory of proofs not holding up. Tverberg found a highly original and simple proof for the result.

Besides mathematics he had a continued interest in life in his home town Bergen. He was an eager reader of the local newspaper Bergens Tidende, where he also for some time had a weekly column with mathematical puzzles and problems. With his wife he shared an interest for antiques, and the large second hand markets organized at schools in Bergen each year were likely places to meet him outside of work. Another long-time interest was jazz music where he had a large collection.

After retiring in 2005 he still regularly came to work at his emeritus office. Thus he was active at our department for 60 years. The last few years he could not come and many of us missed him then. His passing marked the end of an era at our department.

Biographical Memoirs and LMS Obituaries

The obituary of Alan Baker (1939–2018) has been published in the *Bulletin of the London Mathematical Society*. All obituaries (both recent and historical) published in the *Bulletin* are free to read and can be accessed at https://tinyurl.com/32mctbk4.

Geometry Workshop in Honour of 75th Birthday of Fedor Bogomolov

Location: University College London
Date: 14–16 January 2022
Website: tinyurl.com/sumeavju

This workshop is in the honour of 75th birthday of Fedor Bogomolov, who has contributed significantly to the study of complex geometry, algebraic geometry and questions of rationality. The workshop features famous mathematicians and leading researchers in the field of geometry. The meeting is supported by an LMS Conference grant.

Discrete Mathematics Conference

Location: University of Essex, Colchester
Date: 4–5 April 2022
Website: tinyurl.com/yusvfefy

This third IMA conference on discrete mathematics will consider a range of aspects of discrete mathematics, both pure and applied. Discrete mathematics is a branch of the mathematical sciences which poses a wide range of challenging research problems in its own right and gives rise to important applications in other fields. It is open to researchers working with mathematical structures and abstract constructs, and to those involved in the theory and practice of discrete mathematics.

BAMC 2022

Location: Loughborough University & online
Date: 11–13 April 2022
Website: bamc2022.lboro.ac.uk

The British Applied Mathematics Colloquium (BAMC) is the UK's largest applied mathematics conference. BAMC 2022 will be hosted at Loughborough University and held in hybrid format with events predominately in person, but enabling online participation. Deadline for abstract submission of contributed talks and in mini-symposia is 17 January. The meeting is supported by the LMS Conference grant, IMA, QJMAM Fund and several other sponsors. For full details, registration and abstract submission visit the conference website.

Functor Categories for Groups Meeting on Beauville Groups

Location: University of Lincoln
Date: 28 January 2022
Website: lancaster.ac.uk/maths/fcg

This meeting will introduce the connection between Beauville groups and Riemann surfaces, and then focus on recent developments involving Beauville groups, such as new links to groups acting on rooted trees. The meeting will take place in hybrid format. To register email anitha.thillaisundaram@math.lu.se.

Probability, Analysis and Dynamics Conference

Location: University of Bristol
Date: 6-8 April 2022
Website: bris.ac.uk/pad21

Leading mathematicians will speak at this in-person conference about recent developments in probability, analysis and dynamics. The talks will be accessible to researchers from all three fields, including graduate students who can apply to have their costs covered due to an LMS Scheme 1 grant. Visit the website to register and for the list of speakers.

UK Association for Computational Mechanics (UKACM)

Location: University of Nottingham
Date: 20–22 April 2022
Website: tinyurl.com/2dcf9swd

This annual conference provides a forum to present recent advances in computational mechanics (solid and structural mechanics, fluid mechanics, computational fluid dynamics, gas, bioengineering, geomechanics, electromagnetics, multi-physics, mesh generation, etc) with a particular emphasis on interdisciplinary aspects. Plenary lectures at the conference will be delivered by the following distinguished researchers: Professor René de Borst (University of Sheffield), Professor Stefanie Elgeti (Vienna University of Technology), Professor Dominik Schillinger (Leibniz Universität Hannover), Dr Emilio Martinez-Paneda (Imperial College London).

Mathematics of Finance and Climate Risk Conference

Location: Holiday Inn, Liverpool
Date: 8–10 June 2022
Website: tinyurl.com/yczj34f3

This IMA conference on financial mathematics will be focusing on pressing challenges in finance and insurance produced by climate change, demographic developments, and the ever-increasing dominance of data and information. The aim is to promote interdisciplinary cooperation bridging mathematics, statistics and computer science with finance, climate science, insurance and economics.

Point Configurations: Deformations and Rigidity Graduate Research School

Location: University College London
Date: 27 June – 1 July 2022
Website: tinyurl.com/2p8a58cf

The main courses will be given by Gero Friesecke, Douglas Hardin and Edward Saff and Danylo Radchenko. The courses will be supplemented by tutorial sessions. Plenary talks given by Keith Ball, Henry Cohn and Sylvia Serfaty. This will be a hybrid in-person/online event; online registration is required and applications for fully-funded in-person attendance will be considered.

Maths in Music Conference

Location: Royal College of Music, London
Date: 13–15 July 2022
Website: https://tinyurl.com/347mjh8b

This IMA conference will focus on the exploration of the connections between mathematics and music, in particular current developments of music theory, music performance, music perception and music technology based on or inspired by mathematical applications and concepts, including (but not limited to) category theory, group theory, topology, differential geometry, combinatorics, analysis, acoustic theory, as well as artificial intelligence, deep learning, language processing and coding.

New Challenges in Operator Semigroups

Location: St John's College, Oxford
Date: 18–22 July 2022
Website: tinyurl.com/ky958fnc

This conference will celebrate Charles Batty's ongoing outstanding contributions to the theory of operator semigroups. The meeting will focus on the mathematical theory of operator semigroups and their applications to linear evolution equations and connected fields. Information on how to register is available on the conference webpage. A limited amount of funding is available to support UK-based PhD students; contact David Seifert (david.seifert@ncl.ac.uk). Partially supported by an LMS Conference Grant and St John's College, Oxford.

Numerical Linear Algebra and Optimization

Location: University of Birmingham
Date: 29 June – 1 July 2022
Website: https://tinyurl.com/mujj3rjh

This is the 7th IMA conference on Numerical Linear Algebra and Optimization. The success of modern codes for large-scale optimization is heavily dependent on the use of effective tools of numerical linear algebra. On the other hand, many problems in numerical linear algebra lead to linear, nonlinear or semidefinite optimization problems. The purpose of the conference is to bring together researchers from both communities and to find and communicate points and topics of common interest. This conference has been organised in co-operation with SIAM.

Microlocal Analysis and PDEs

Location: University College London
Date: 20–22 July 2022
Website: tinyurl.com/m3pjw5er

This workshop is devoted to recent trends and developments in microlocal analysis and partial differential equations. It aims to bring together different communities working in microlocal analysis, to share ideas and establish new collaborations. Deadline for submission of contributed talks: 31 May 2022. Contact Matteo Capoferri (CapoferriM@cardiff.ac.uk) for the limited funds available to support the participation of junior researchers.

Society Meetings and Events

January 2022

4-6 South Wales and South West Regional Meeting and Workshop, Braces in Bracelet Bay, Swansea (online).

February 2022

28-4 Mar LMS Invited Lecture Series 2022: The Mathematics of Deep Learning, Centre for Mathematical Sciences, University of Cambridge

March 2022

4 Diverse Perspectives on Alan Turing and Society Meeting, De Morgan House, London

April 2022

4-7 Midlands Regional Meeting and Workshop, Mirrors, Moduli and M-theory in the Midlands, Birmingham

Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website (www.lms.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

January 2022

4-6 South West & South Wales LMS Regional Meeting, Swansea (496)
14-16 Geometry Workshop in Honour of 75th Birthday of Fedor Bogomolov, University College London (498)
28 Functor Categories for Groups Meeting on Beauville Groups, University of Lincoln and online (498)

April 2022

4-5 Discrete Mathematics Conference, University of Essex (498)
6-8 Probability, Analysis and Dynamics Conference, University of Bristol (498)
11-13 BAMC, Loughborough University
20-22 UK Association for Computational Mechanics, University of Nottingham (498)
25-29 Rational Points on Higher-Dimensional Varieties, ICMS, Edinburgh (495)

May 2022

2-6 Adaptive Methods and Model Reduction for PDEs Research School, Nottingham (496)
18-20 Mathematics in Signal Processing, Aston, Birmingham (495)

June 2022

8-10 Mathematics of Finance and Climate Risk Conference, Holiday Inn, Liverpool (498)
27-1 Jul Point Configurations: Deformations and Rigidity Graduate Research School, University College London (498)
29-1 Jul 7th IMA Conference on Numerical Linear Algebra and Optimization (498)

July 2022

13-15 Maths in Music Conference, Royal College of Music, London (498)
18-22 Rigidity, Flexibility and Applications LMS Research School, Lancaster (498)
20-22 Microlocal Analysis and PDEs, University College London (498)
24-26 7th IMA Conference on Numerical Linear Algebra and Optimization, Birmingham (487)