LONDON MATHEMATICAL SOCIETY
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NEWSLETTER

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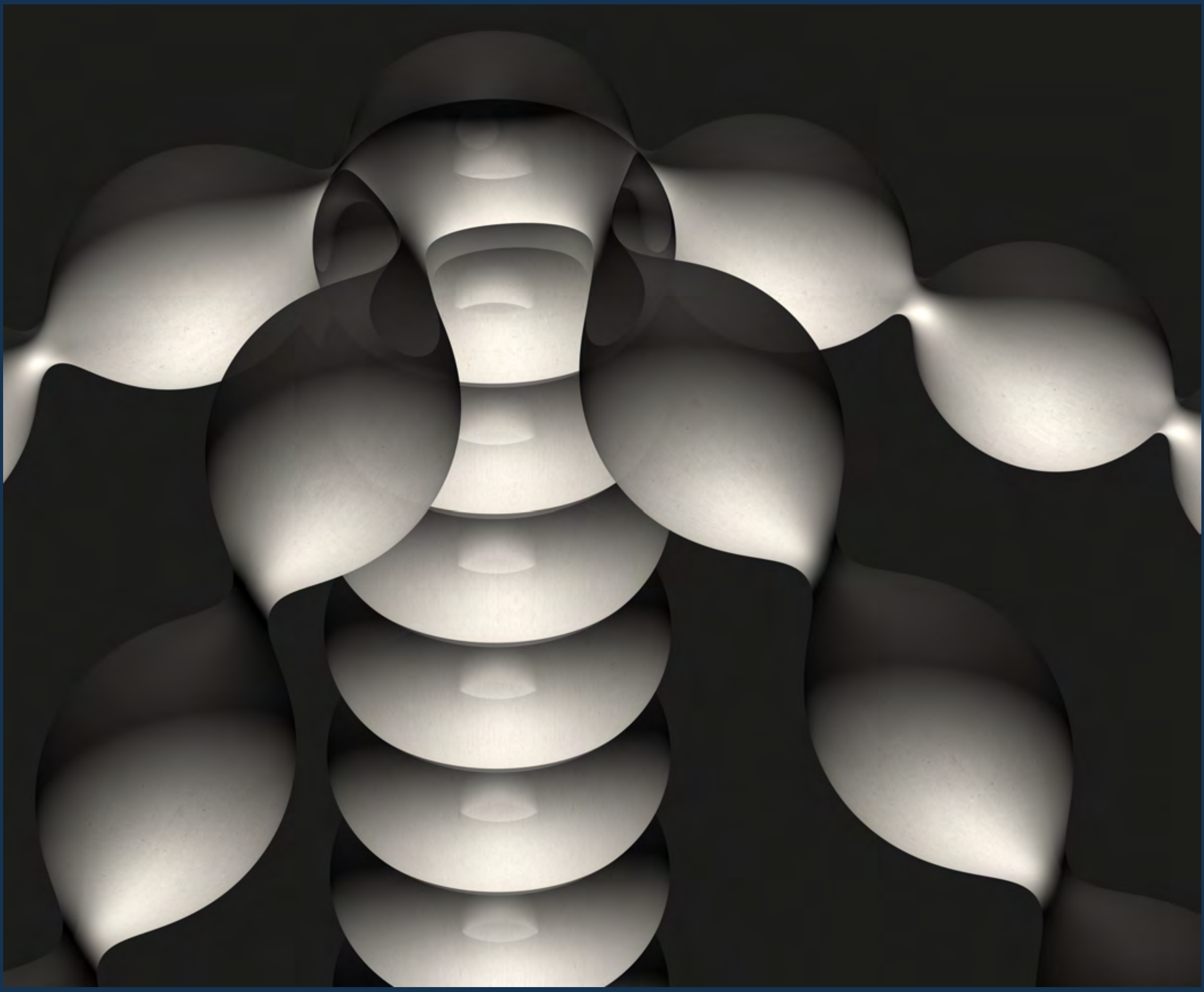

SYMPLECTIC GEOMETRY AND KOOPMAN DYNAMICS

EXTENSIONS OF BERTRANDS POSTULATE

NOTES OF A NUMERICAL ANALYST

EDITORIAL OFFICE

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Do you have an image of mathematical interest that may be included on the front cover of a future issue? Email images@lms.ac.uk for details.

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Feature content should be submitted to the editor-in-chief at newsletter.editor@lms.ac.uk.

News items should be sent to newsletter@lms.ac.uk.

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LMS NEWS

Annual Elections to LMS Council

The LMS Nominating Committee is responsible for proposing slates of candidates for vacancies on Council and vacancies on its own membership. The Nominating Committee welcomes suggestions from the membership.

Anyone who wishes to suggest someone for a position as an Officer of the Society or as a Member-at-Large of Council (now or in the future) is invited to send their suggestions to Professor Tara Brendle, the current Chair of Nominating Committee (nominations@lms.ac.uk). Please provide the name and institution (if applicable) of the suggested nominee, their mathematical specialism(s), and a brief statement to explain what they could bring to Council/Nominating Committee.

It is to the benefit of the Society that Council is balanced and represents the full breadth of the mathematics community; to this end, Nominating Committee aims for a balance in gender, subject area and geographical location in its list of prospective nominees.

Nominations should be received by Friday 21 April in order to be considered by the Nominating Committee.

In addition to the above, members may make direct nominations for election to Council or Nominating Committee. Direct nominations must be sent to the Executive Secretary's office (nominations@lms.ac.uk) before noon on 1 September 2023. For details on making a direct nomination, see lms.ac.uk/about/council/lms-elections.

The slate as proposed by Nominating Committee, together with any direct nominations received up to that time, will be posted on the LMS website in early August.

Retiring Members of Council

Brita Nucinkis
Member-at-Large 2017–2022

Professor Brita Nucinkis stepped down as a Member-at-Large of Council at the Annual General Meeting on 18 November 2022. Brita joined the LMS Council in November 2017, shadowing the then Chair of the Society Lectures and Meetings Committee (SLAM), Professor Iain Stewart. Since 2019, Brita has been (and continues to be) the Chair of SLAM, which is the committee responsible for overseeing our programme of Society Meetings and mathematics lectures, including three regional meetings, the Society Meeting held at the annual BMC, the Society Meeting at the quadrennial ECMs & ICMs, and the General Meetings held in London.

As SLAM Chair, Brita was instrumental in moving Society Meetings online during the Covid-19 lockdown, enabling members of the mathematical community to continue to meet virtually and share ideas during a very difficult time. She continues to work to modernise our programme of events, having already overseen changes to the ways in which we collect demographic data in reports, and on how to improve step-free accessibility at De Morgan House.

Anne-Christine Davis
Member-at-Large 2020–2022

Professor Anne-Christine Davis stepped down as a Member-at-Large of Council at the Annual General Meeting on 18 November 2022, having joined Council in November 2020. Anne has also served as a member of the Committee for Women and Diversity (previously the Women in Mathematics Committee) since 2014, and has been the Chair of the LMS Good Practice Scheme (GPS) Steering Group since 2018. The GPS provides support for departments working towards Athena SWAN Award status via regular workshops on equality, diversity and inclusion (EDI) in academia. In the past, Anne has helped to organise and has chaired workshops on such topics as the Athena SWAN Charter, support for mathematicians during the pandemic, and EDI and the Research Excellence Framework. In addition, Anne has been the LMS representative on the Athena Forum for the past four years.

The LMS is grateful to Brita and Anne for their work on Council and for their ongoing support for the Society over several years.

Katherine Wright
Society Business, Research & Communications Officer

Christopher Zeeman Medal 2020 and 2022: Lectures and Ceremony

The 2020 and 2022 Christopher Zeeman Medals will be awarded to Matt Parker and Simon Singh, respectively, at a ceremony on Wednesday 22 March 2023. The ceremony, which will be held at the Royal Society in London, will be followed by a lecture by each of the medal winners. Details of the lectures will be posted to lms.ac.uk/events/zeemanceremony2023 in due course.

Registration will start at 6pm and the event proper will start at 6:30pm. The ceremony and lectures will be held in the Lecture Theatre and will be followed by a reception with drinks and bar snacks.

If you would like to attend this event, please email Katherine Wright, Society Business, Research & Communications Officer, at the London Mathematical Society (katherine.wright@lms.ac.uk) by 5pm on Wednesday 15 March. Please give a name for each person attending.

New PLMS Managing Editors

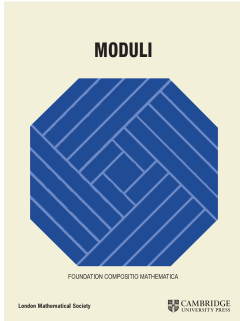
PLMS Managing Editors Jon Brundan (left) and Scott Sheffield

The LMS welcomes Jon Brundan (University of Oregon) and Scott Sheffield (MIT) as the new Managing Editors of the *Proceedings of the London Mathematical Society*.

They take over from Tim Browning and Oscar Randal-Williams, whom the Society thanks for their hard work over the last five years. In particular, thanks are due for their leadership of the relaunch of the *Proceedings* as the Society's flagship title.

For more details on the *Proceedings* see lms.ac.uk/plms.

Launch of *Moduli*



The Society is delighted to announce the launch of *Moduli*, a new open access journal owned by the Foundation Compositio Mathematica and published in collaboration with the LMS and Cambridge University Press.

Now open to submissions, *Moduli* provides a unified forum for significant new results on all aspects of moduli theory or related mathematics, including but not restricted to techniques from: algebraic, differential and arithmetic geometry; combinatorics; dynamical systems; gauge theory; geometric analysis; geometric group theory; pure mathematics inspired by physics; representation theory; and topology.

The journal is led by a team of Managing Editors, supported by an Editorial Board of world-renowned researchers.

The Managing Editors are:

- Jørgen Ellegaard Andersen, University of Southern Denmark, Denmark
- Steve Bradlow, University of Illinois Urbana-Champaign, USA
- Dan Halpern-Leistner, Cornell University, USA
- Vicky Hoskins, Radboud Universiteit Nijmegen, The Netherlands
- Frances Kirwan, University of Oxford, UK
- Margarida Melo, Università di Roma Tre, Italy
- Anna Wienhard, Max Planck Institute for Mathematics in the Sciences, Germany

For information about *Moduli*, including a list of the full Editorial Board, please visit lms.ac.uk/publications/moduli.

To submit your paper visit moduli.nl.

The Foundation Compositio Mathematica also owns two other journals: *Compositio Mathematica* and *Algebraic Geometry*. The former has been published in partnership with the LMS and CUP since 2004.

LMS Hardy Lectureship 2023

The London Mathematical Society is pleased to announce the LMS Hardy Lecturer 2023 is Professor Eva Miranda (UPC and CRM–Barcelona).

The Hardy Lectureship was founded in 1967 in memory of G.H. Hardy in recognition of outstanding contribution to both mathematics and to the Society. The Hardy Lectureship is a lecture tour of the UK by a mathematician with a high reputation in research.

Professor Miranda will undertake three lecture tours of the UK between May and September 2023, which will include the Hardy Lecture at the Society Meeting on Friday 30 June in London.

Further details about the Hardy Lecture Tour 2023 are available on the website at lms.ac.uk/events/lectures/hardy-lectureship.

Forthcoming LMS Events

The following events will take place in the forthcoming months:

LMS CPD Series for Early Career Researchers Session 4: Building your Research Community: 6 March, online (lms.ac.uk/events/CPD-4)

Christopher Zeeman Medal 2020 and 2022: Lectures and Ceremony: 22 March, Royal Society, London (lms.ac.uk/events/zeemanceremony2023)

LMS Midlands Regional Meeting: 27 March, Warwick (bit.ly/3RW0GVj)

LMS CPD Series for Early Career Researchers Session 5: Applying for Jobs in Industry: 30 March, online (lms.ac.uk/events/CPD-5)

LMS Meeting at BMC 2023: 4 April, Bath (tinyurl.com/mr255tpt)

LMS Research School on Adaptive Methods and Model Reduction: 24–28 April, Nottingham (lms.ac.uk/events/RS202301)

A full listing of upcoming LMS events can be found on page 38.

OTHER NEWS

New Years Honours 2023

Daniel James Abramson (Head Teacher, King's College London Mathematics School) was appointed Officer of the Order of the British Empire for services to Education.

Charlotte Victoria Francis (Head of Mathematics, St Catherine's Catholic School, Bexleyheath, Greater London) was appointed Medal of the Order of the British Empire for services to Education.

Paul Glaister (University of Reading) was appointed Commander of the Order of the British Empire for services to Education. Paul is an LMS member.

Kantilal Vardichand Mardia (Leeds University) was appointed Officer of the Order of the British Empire for services to Statistical Science.

Julie Maxton CBE (Executive Director, The Royal Society) was appointed Dame Commander of the British Empire for services to Science and Law.

Academy for the Mathematical Sciences Website

The Academy for the Mathematical Sciences (AcadMathSci) website is now live, with a joint quote from the learned societies on the front page. The website includes an explanation of the Academy's main aims and concerns, and details of the Executive Board for the proto-Academy. See the website at acadmathsci.org.uk. The Academy also has a LinkedIn profile: bit.ly/3RvGNFN.

MATHEMATICS POLICY DIGEST

School Students to Study Mathematics Until Age 18

On 4 January 2023, Prime Minister Rishi Sunak set out his vision for the year ahead. One of his primary policy changes was to introduce compulsory mathematics education up to the age of 18. The Prime Minister argued that many of the UK's children are being "let down" by leaving their education without the numeracy and analytical skills needed for today's workplace.

Read the Protect Pure Maths campaign's response to this announcement at bit.ly/3wlB9qd.

Joint Open Letter to Birkbeck Vice-Chancellor

The LMS, Royal Statistical Society, Institute of Mathematics and its Applications and Edinburgh Mathematical Society have written an open letter to the vice-chancellor of Birkbeck, University of London expressing serious concern about proposed cuts to the Mathematics and Statistics department.

In the letter to Professor David Latchman, LMS President, Professor Ulrike Tillmann, alongside presidents from the other mathematical societies, point out that the proposed reductions in the teaching capacity will mean the department will become too small to be viable, resulting in a desert of provision for mature students with mathematics.

The letter notes that Birkbeck has a unique and crucial role in the widening participation of higher education. The Birkbeck student body is diverse, and the UK needs to foster an ongoing flow of talented mathematicians from diverse backgrounds to continue to contribute to social and technical advances in the century ahead.

Read the full letter at bit.ly/3jobkIT.

Digest prepared by Katherine Wright
Society Business, Research and Communications Officer

Note: items included in the Mathematics Policy Digest are not necessarily endorsed by the Editorial Board or the LMS.

OPPORTUNITIES

LMS-Bath Symposia 2024: Call for Proposals

A reminder that proposals are invited from the mathematical community to organise the LMS-Bath Mathematical Symposia to be held at the University of Bath in 2024. It is intended that there will be two Symposia and one of these will be funded by the Isaac Newton Institute. Core funding at approximately £40,000 is available to support each Symposium.

For further details about the LMS–Bath Mathematical Symposia, please visit the Society's website: lms.ac.uk/events/mathematical-symposia or the LMS–Bath Symposia's website: bathsymposium.ac.uk. Before submitting, organisers are welcome to discuss informally their ideas with the Chair of the Research Grants Committee, Professor Andrew Dancer (grants@lms.ac.uk).

Prospective organisers should send a formal proposal to the Grants Team (grants@lms.ac.uk) by 13 March 2023. Proposals are approved by the Society's Research Grants Committee after consideration of referees' reports.

Committee for Women and Diversity in Mathematics: Wider Mission

In 2020, the LMS Women in Mathematics (WiM) Committee expanded its remit from a focus on gender inequalities in mathematics to allow it to take a much broader look at other aspects of diversity within the mathematical community. The committee has been renamed **Committee for Women and Diversity in Mathematics (CWDM)** to reflect its wider mission. The Committee has representatives from the Institute of Mathematics and its Application (IMA), the Royal Statistical Society (RSS), the Operational Research Society (OR), the Edinburgh Mathematical Society (EMS) and European Women in Mathematics (EWM).

The change of remit enables the LMS to work more effectively at addressing such issues as the under-representation of Black academics at the professorial level[1]. Indeed, we want to champion cultural and organisational change to ensure that the UK excels in attracting and supporting Black staff and students as well as those from other minority ethnic backgrounds. We want to support the creation of an environment that encourages an open discussion of race-related issues and challenges racism. In addition, we want to ensure that anyone with an interest in mathematics is supported, encouraged and made to feel welcome, so that they are able to participate and thrive in the profession. Underrepresentation inevitably leads to missed opportunities and the loss of mathematical talent. We are committed to celebrating, recognising and supporting a diverse mathematical community of individuals across the spectrum of age, race, gender, gender identity and expression, national or ethnic origin, religion and religious belief, marital status, parental status, body shape or size, sexual orientation, disability, socioeconomic status, employment status and other aspects of diversity. Moreover, such celebration, recognition and support must be authentic and intersectional in approach and outlook.

Previously, the WiM Committee had offered support for running Women in Mathematics Days and Girls in Mathematics Days[2]. Now, in addition to these events, the Committee is able to offer support for events focused on other aspects of Diversity in Mathematics[3] and has become a regular contributor to the annual LGBT+ STEMinar (lgbtstem.wordpress.com, a cross-discipline conference that brings together LGBTQ+ people working in STEM, providing important networking and professional development opportunities, and visibility for this invisible diversity). More recently, the Committee has collaborated with Numberfit (numberfit.com) in a project to amplify the profiles of mathematicians from underrepresented groups. The LMS is among the organisers of the yearly conference on Black Heroes of Mathematics[4], which the Committee promotes and supports. However, more work is needed, particularly to address racial inequality within the mathematics community and systemic barriers to access. Events targeted at raising awareness and celebrating diversity are an important part of this, but they sit within a broader piece of work aimed at creating genuine positive and inclusive change.

There is still a long way to go in transforming the culture and lived reality of the UK mathematics scene for the benefit of all mathematicians, but the broadening of the remit of the former WiM Committee is an important step in that direction. This journey necessitates opening a conversation with you about your lived experiences within the mathematical sciences community. We invite you to raise with us issues you may be experiencing and that you think CWDM should consider. We invite you also to contact us if you would like to organise events dedicated to diversity in mathematics.

If you have any questions or comments about the work of CWDM, please email the Committee Secretary, Katherine Wright: womeninmaths@lms.ac.uk.

Sara Lombardo
Chair, LMS Committee for Women and Diversity in Mathematics

[1]hesa.ac.uk/news/19-01-2021/he-staff-statistical-bulletin-released
[2]Women in Maths Days: lms.ac.uk/women/women-maths-day; Girls in Maths Days: lms.ac.uk/women/girls-maths-day
[3]lms.ac.uk/women/diversity-maths-day
[4]lms.ac.uk/events/black-heroes-mathematics

Maximising your LMS Membership: International Connections with other Mathematical Societies

Discounted membership of the European Mathematical Society (EMS)

Members of the LMS can join the European Mathematical Society at a 50% discount of the full EMS membership fee: currently €25.00 instead of €50.00. Further information about the European Mathematical Society is available at euro-math-soc.eu.

To join the EMS, members can contact the EMS direct and mention their LMS membership or add EMS membership to their LMS membership record when signing in via the LMS website www.lms.ac.uk/user, or complete and return the LMS subscription form. Payment for EMS membership can also be made via the LMS and over 150 LMS members already pay for EMS membership alongside their LMS membership. If you have any queries, please contact membership@lms.ac.uk.

Note: LMS members who are students may be interested in the EMS' free membership for students while LMS members who are aged 60+ may be interested in the EMS' lifetime membership. Further details about EMS membership rates are available at euro-math-soc.eu/individual-members.

Option to pay for membership of European Women in Mathematics (EWM)

Members of the LMS who are also members of European Women in Mathematics have the option to pay for their EWM membership via their LMS Membership account. Over 30 EWM members already pay for their EWM membership alongside their LMS Membership. To do so, you must first be a member of the EWM; further details about EWM membership and how to join are available at tinyurl.com/23ujdzv8. EWM members can then add their EWM membership to their LMS membership account either by logging in via the LMS websiten at lms.ac.uk/user or by completing and returning the LMS subscription form. If you have any queries, please contact membership@lms.ac.uk.

Reciprocal Agreements with 23 International Mathematical Societies

The Society has reciprocal agreements with the following mathematical societies through which LMS members can join those societies at a 50% discount on the full membership fee of each society, if they are not normally resident in the same country as the society. For example, a UK-based Ordinary Member could join the Finnish Mathematical Society at their reciprocal membership rate. For further information about these societies through links to their webpages visit tinyurl.com/yu6px2mj:

Allahabad Mathematical Society, American Mathematical Society, Australian Mathematical Society, Belgian Mathematical Society, Canadian Mathematical Society, Dansk Matematisk Forenig, Deutsche Mathematiker-Vereinigung, Société Mathématique de France, Finnish Mathematical Society, Indian Mathematical Society, Indonesian Mathematical Society, Irish Mathematical Society, Unione Matematica Italiana, Mathematical Society of Japan, KoninklijkWiskundig Genootschap, New Zealand Mathematical Society Nigerian Mathematical Society, Norsk Matematisk Forening, Real Sociedad Matemàtica Española, Singapore Mathematical Society, South East Asian Mathematical Society, Svenska Mathematikersamfundet, Swiss Mathematical Society.

In return, members of the above societies who are not normally resident in the UK can join the LMS as Reciprocity Member and receive a 50% discount on the Ordinary Membership fee only (other subscription rates such as Associate Membership are already discounts on the Ordinary Membership rate and the Society does not offer 'double discounts'). If you have any queries contact membership@lms.ac.uk.

Note: LMS members based in either Northern Ireland or the Republic of Ireland and who are also members of the Irish Mathematical Society can choose whether to be a Reciprocity Member of either the LMS or the Irish Mathematical Society.

Elizabeth Fisher
Membership & Grants Manager

LMS Council Diary — A Personal View

Council met in Goodenough College in a hybrid format, with some members in person and others joining remotely via videoconference, on the morning of Friday 18th November. The meeting began with the President's business. This included a commitment for a further donation for the LMS/INI Solidarity Programme, which supports research mathematicians who have had to leave their county of residence, and the fact that a first-ever parliamentary debate on mathematics, for which a full transcript is available online, had taken place on 15 November. A detailed update on the recent activities of the Protect Pure Maths Campaign was noted by Council later in the meeting.

After agreeing to set up a Council working group to monitor and respond to the proposed Academy of Mathematical Sciences from the perspective of the LMS, Council discussed plans for its upcoming Strategic Retreat, which will take place in Spring 2023.

The Publications Secretary then gave an update on Publications Committee business, including the discussions that had taken place during the Publications Strategic Retreat in Summer 2022. Under financial matters, the Treasurer reported that the Investment Sub-Committee had recommended that the Society should increase the proportion of its investments in responsible multi-asset funds to 50%, which was agreed, together with the aim of increasing this proportion further in the next few years. This fund has stronger responsible investing policy than the standard charity multi-asset fund.

Other business included various committee reports and memberships. The meeting concluded with the President giving thanks on behalf of Council to outgoing Council Members and wishing luck to those members up for election.

Elaine Crooks
Member-at-Large

Growing the LMS Membership

The standing and usefulness of the Society depends on the support of a strong membership. Members provide their expertise and participate in running the Society, which enables its many activities in support of the mathematics community. These include publishing, grant-giving, conferences, public policy, influencing government and mathematics education in schools.

If current members know of friends or colleagues who would like to join the Society, please encourage them to complete the online application form (lms.ac.uk/membership/online-application). Note that as a current member, you can act as the proposer or seconder for your friend or colleague's membership application.

Applications received by 19 April 2023 will be presented at the Society Meeting on 9 June 2023 for election to membership.

Benefits of LMS membership include access to the Verblunsky Members' Room at De Morgan House in London, free online subscriptions to selected journals and complimentary use of the Society's Library at UCL, among other LMS member benefits (lms.ac.uk/membership/member-benefits).

New members need to pay fees only after they have been elected to membership and details of the fees for 2022–23 can be found on the LMS website at lms.ac.uk/membership/membership-categories.

Contact membership@lms.ac.uk for advice on becoming an LMS member.

Longstanding LMS Members

The London Mathematical Society greatly values the contributions made by all its members and would particularly like to acknowledge the members on the following page, who will be celebrating membership milestones of more than 50 years in 2023.

Elizabeth Fisher
Membership & Grants Manager

Over 75 years of membership: Eric L. Huppert.

75 years of membership: Bernard Fishel and Godfrey L. Isaacs.

70 years of membership: John R. Ringrose.

Over 65 years of membership: John C. Amson, John F. Bowers, Ronald Brown, Aldric L. Brown, Daniel E. Cohen, David E. Edmunds, David A. Edwards, Hanafi K. Farahat, Ioan M. James, Michael F. Newman, Roger Penrose, Roy L. Perry, Francis Rayner, Dennis C. Russell and G.E. Wall.

65 years of membership: Bryan Birch, Gearoid De Barra, Donald Keedwell and I.G. Macdonald.

Over 60 years of membership: Benjamin Baumslag, David A. Burgess, John H.E. Cohn, Hallard T. Croft, Roy O. Davies, Ian M.S. Dey, Vlastimil Dlab, James O.C. Ezeilo, Roland F. Hoskins, Glenys Ingram, John F.C. Kingman, Joseph F. Manogue, Alun Morris, Albert A Mullin, John E. Peters, Frank Rhodes, Joseph B. Roberts, John Rutter, Eira J. Scourfield, Dona Strauss, Anthony C. Thompson, Ronald F. Turner-Smith, C. Terence C. Wall, Alan West and Sheila O. Williams.

60 years of membership: J. Clifford Ault, Alan F. Beardon, Thomas S. Blyth, Charles W. Curtis, J. Keith Dugdale, L.C. Eggan, David Epstein, David J.H. Garling, Robin E. Harte, Philip Heywood, J. David Knowles, Frederick C. Piper, John S. Pym, H. Peter Rogosinski, John F. Watters, Alfred Weinmann, David J. White and Joyce E. Whittington.

Over 55 years of membership: John C.R. Batty, M.C. Bramwell, William Brown, Roger M. Bryant, Allan G.R. Calder, Munibur R. Chowdhury, Michael J. Collins, P. Laurie Davies, M.A.H. Dempster, M.M. Dodson, Martin J. Dunwoody, Roger H. Dye, Barry G. Eke, K. David Elworthy, John Erdos, Edward A. Evans, W. Desmond Evans, Roger A. Fenn, James W.M. Ford, Peter Giblin, Keith E. Hirst, Terence H. Jackson, Otto H. Kegel, E. Christopher Lance, David G. Larman, W.B. Raymond Lickorish, Malcolm T. McGregor, Ian M. Michael, Hugh Morton, Oliver Pretzel, George A. Reid, John F. Rennison, Derek J.S. Robinson, James Edward Roseblade, Colin P. Rourke, Keith Rowlands, Stephan M. Rudolfer, Rodney Sharp, Brian F. Steer, Nelson M. Stephens, Bill Stephenson, W. Brian Stewart, Anthony E. Stratton, David Tall, Graham F. Vincent-Smith, Grant Walker, Martin Antony Walker, Bertram Wehrfritz, Christopher M. Williams and Geoffrey V. Wood.

55 years of membership: Irene A. Ault, Anthony D. Barnard, Sheila Carter, Donald J. Collins, Colin R. Fletcher, Charles Goldie, Wilfrid A. Hodges, Graham J.O. Jameson, Michael E. Keating, Thomas J. Laffey, Earl E. Lazerson, Bernard L. Luffman, Bob Margolis, Peter McMullen, William Moran, Kung-Fu Ng, David R. Page, Philip Samuels, David Singerman, Brian Thorpe and Douglas R. Woodall.

Over 50 years of membership: R.B.J.T. Allenby, David H. Armitage, Mark A. Armstrong, Roger C. Baker, Earl R. Berkson, Thomas G.K. Berry, W. Meurig Beynon, Nicholas H. Bingham, Béla Bollobás, David A. Brannan, F. Trevor Brawn, Colin Campbell, Roger J. Cook, Alan Cornish, Richard Delanghe, Peter Dixon, John D. Dixon, Christopher Terence Dodson, John Duncan, Brian Fisher, Leslie R. Fletcher, John B. Fountain, T. Alastair Gillespie, Paul R. Goodey, John A. Haight, F. Mary Hart, Allan Hayes, Anthony J.W. Hilton, Jonathan Hodgson, William Holcombe, Chris Houghton, Tiao-Tiao Hsu, Ignacio Iribarren, John E. Jayne, David L. Johnson, Cherry Kearton, Monsur A. Kenku, Arthur Knoebel, P. Ekkehard Kopp, Charles Leedham-Green, Hendrik W.E. Leemans, T.H. Lenagan, Frederick Long, John H. Loxton, Zafer Mahmud, Vassilis Mavron, Harold V. McIntosh, Paul R. Meyer, Paul Milnes, Peter E. Newstead, Peter J. Nicholls, Adam J. Ostaszewski, Fredos Papangelou, Jon V. Pepper, Roger J. Plymen, Timothy Porter, Hilary A. Priestley, John R. Quine, Andrew Rae, Nigel Ray, Helen D. Robinson, J. Christopher Robson, Harvey Rose, Peter Rowlinson, David L. Salinger, Jean F.H. Schmets, Klaus Schmidt, G. Peter Scott, Dan Segal, John R. Shackell, John Silvester, Allan M. Sinclair, David B. Singmaster, Patrick F. Smith, Linda R. Sons, Gabrielle A. Stoy, R.F. Streater, Elizabeth Taylor, Donald Taylor, R. Kenneth Thomas, R.C. Vaughan FRS, Michael R. Vaughan-Lee, Richard Walton, Jeffrey R.L. Webb, David B. Webber, Dominic J.A. Welsh, Christopher Wensley, Anthony Wickstead, Graham D. Williams, Stephen Wilson, R.M.W. Wood, Christopher F. Woodcock, J.D.M. Wright and Abraham Zaks.

50 years of membership: John Bolton, Douglas S. Bridges, Geoffrey Burton, E. Brian Davies, Richard Haydon, Michiel Hazewinkel, John Hubbuck, Donal Hurley, Aldo J. Lazar, John C. Lennox, John C. McConnell, Thomas McDonough, John R. Merriman, David M. Rodney, Robert Scruton, Jean-Pierre Serre, Fenny Smith, Robert C. Solomon, Carol A. Whitehead, Robin Wilson and Christopher K. Wright.

LEVELLING UP

Dr Helen Cramman (School of Education, Durham University and Head of the Evaluation and Innovation in Science Education (EISE) Research Group) reflects on the first year of the pilot cohort of the Levelling Up Widening Participation Programme.

What have we learned from the pilot cohort of the Levelling Up Widening Participation Programme?

The first cohort of students on the Levelling Up Widening Participation Programme completed their time on the programme in Spring 2022 and the evaluation of their experiences has recently been published. The evaluation found that the programme had successfully accomplished the aims it set out to achieve at the beginning of the programme, and also presented some interesting and important suggestions to aid future implementation.

What is the Levelling Up Programme?

The Levelling Up Pilot Widening Participation Programme is a national academic and pastoral online support programme spanning two academic years, targeted at Year 12 students in England who have an interest in potentially pursuing the study of Chemistry, Maths or Physics at University. There were six spokes in the pilot programme, with three hubs coordinating the overall running of each subject strand. Durham University led the chemistry strand (1 spoke), the LMS led the mathematics strand (2 spokes), and the Institute of Physics led the physics strand (3 spokes). The pilot cohorts started the programme between February 2021 and July 2021 and the last sessions ran between March 2022 and June 2022. In total, 226 students accepted places on the pilot cohort of the programme.

Each programme had a different combination of activities for the students (see table below).

Defining success criteria

Before the Levelling Up Programme started, a detailed theory of change model was created in collaboration with the chemistry, maths and physics hubs. This identified what the programme sought to achieve (what impact did it hope to have), what outcomes it needed to happen in order for those impact aims to be met, what activities would help achieve those outcomes and impact and, importantly, what assumptions were being made in the design of the programme.

Student activities by university

	Chemistry: Durham	Maths: Durham	Maths: Leicester	Physics: Birmingham	Physics: Durham	Physics: Oxford
Subject specific tutorials	Yes	Yes	Yes	Yes	Yes	Yes
Separate mentoring sessions	Yes	No	No	Yes	Yes	Yes
Guest lectures	Yes	Yes	Yes	No	Yes	Yes
Onsite/remote university visits	Yes	Yes	Yes	No	Yes	No
Guaranteed conditional university offer	Yes	Yes	Yes	No	Yes	No
Graphics tablets provided to students	No	Yes	Yes	No	No	No

Using the theory of change model, it was then possible for the evaluation to design a series of measures that would identify whether those impact aims and outcomes had been successfully achieved.

The seven aims for the Levelling Up Pilot Programme were:

(1) Participants aspire to study chemistry, physics, mathematics or a directly related STEM discipline to their programme subject at university.

(2) Participants apply to a high ranked university as listed in the Times Good University Guide.

(3) Participants aspire to study at their Levelling Up host university.

(4) Participants aspire to study at university (in any subject).

(5) Participants consider that the programme has helped them achieve higher grades at A-level in their subject.

(6) (Chemistry and physics): students consider that the programme has helped them achieve higher grades at A-level in maths within their subjects.

(7) Participants received offers to study the courses for which they have applied on their UCAS applications.

The evaluation collected information at multiple timepoints across the full duration of the students' time on the programme. This included application form data, surveys, interviews and focus groups, and observing training sessions for tutors and mentors. Permission from participants was sought for their inclusion in the evaluation and the use of their data.

Were the activities delivered as planned?

The first question to be asked by the evaluation was whether all activities had been delivered as planned. The findings showed that, yes, to some extent, all activities were delivered as planned. There were, however, variations in delivery across spokes as well as in the engagement from participants.

Did the expected outcomes occur and were the impact aims achieved?

The next question was to look at whether the expected outcomes and impact aims had been achieved. The evaluation found evidence that the intended outcomes had taken place, with four out of the six spokes meeting all the impact aims. However, achieving the aim of participants applying to a high ranked university as listed in the Times Good University Guide was slightly weaker for the Maths (Leicester) and Physics (Birmingham) spokes. Maths (Leicester) and Physics (Oxford) did not meet the aim of students applying to their host university; however, this was not of concern to the two spokes as this was not a driver within their university widening participation remits.

Could anything else have impacted on the programme?

The final question to investigate was whether there were any external contextual factors that may have negatively affected the intended outcomes or impact for the programme. None were found from the evidence collected. However, the data did highlight the complexity of differing needs of the students.

What was the final conclusion?

Based on the evidence collected, the final conclusion from the evaluation was that it is reasonable to conclude that the Levelling Up programme contributed to achieving the stated impact aims for the programme.

Five recommendations for future refinement of the programme were made:

(1) Tutoring and mentoring provision should run with small group sessions (max 6 students), with the same students, tutors/mentors in a group week on week.

(2) The opportunity should be provided for students to communicate with each other outside the organised weekly sessions.

(3) Support should be provided to tutors and mentors in tailoring topics and differentiating difficulty of activities for the interests and needs of students within their group.

(4) Graphics tablets should be provided for all subjects to support drawing graphs and writing equations.

(5) Best practice should be considered for most effectively working with students that are not visible on screen (e.g. camera off) or audible (e.g. microphones off).

For more information on the evaluation, contact the lead evaluator, Dr Helen Cramman, at levelling.up@durham.ac.uk. For more information on Levelling Up Maths, contact education@lms.ac.uk.

The full report can be downloaded at tinyurl.com/levellingupreport.

REPORTS OF THE LMS

Report: Integrable and Conformal Field Theory Meeting

This was the 24th edition of a series of short annual meetings on integrable and conformal field theories held in several UK universities according to a rota. The very first such meeting was held in King's College London in 1997. Since then, the Universities of Sheffield, Oxford, York, Durham, Kent, Hertfordshire, Glasgow, Leeds, Cardiff as well as City University, King's College London and the ICMS in Edinburgh have been hosts, often repeatedly. The organising committee has representatives in most of these universities and has regularly enjoyed support from the LMS. This year was no exception, and we are grateful to the LMS for a Scheme 1 conference grant which, alongside support from the Mathematical and Theoretical Group of the Institute of Physics, enabled us to organise a very fruitful meeting.

This year's edition, held on 16–17 December at Durham University, was special as it was the first opportunity we had since the beginning of the pandemic to organise an in-person event, a format the organising committee felt strongly about. However, some last-minute amendments to the schedule were forced upon us by the disruption caused by the railway strikes and this led to some talks being delivered online. Fortunately, the four main speakers were able to attend in person and contributed greatly to the success of the meeting, which once again brought together participants with a broad spectrum of expertise. Some have contributed significantly to the advances in the field of integrability and conformal field theory over many years while others were early career mathematical physicists and PhD students who benefited from the event through contributed talks and the opportunity to network within a thriving community of researchers. The quality of all presentations was exceptional, and it was particularly uplifting to hear the progress made by PhD students Arpit Das (Durham University), Max Downing, Loukas Grimanellis, Nicolo Primi, Julius Julius and Paul Ryan (all King's College London), on techniques that allow to probe integrability in a variety of contexts where field theory is the common language. We enjoyed hearing from probabilist Dr Ellen Powell (Durham University), who works, among other topics, on Schramm–Loewner evolutions and their relation to the Gaussian free field model and Gaussian multiplicative chaos. Her very clear presentation prompted interesting exchanges at the interface of two disciplines and promises to create further opportunities for discussion. Dr Benoît Vicedo (University of York) mastered the art of a white board talk and described how finite-dimensional integrable systems can be obtained either from Gaudin models associated with finite-dimensional Lie algebras or from a certain three-dimensional variant of the so-called BF theory. He also reviewed recent progress towards constructing more general integrable field theories from four-dimensional Chern–Simons theory. Professor Rinat Kedem (University of Illinois) delivered a well-received presentation on the integrability of quantized Q-systems and found opportunities to discuss her work at length with Dr Luca Cassia (Durham University), an early career researcher who spoke about q-Virasoro constraints for refined Chern-Simons and refined ABJ theories. The audience was also treated to a very enthusiastic talk by Professor Benjamin Doyon (King's College London) who used techniques from hydrodynamics to calculate fundamental quantities in conformal field theory, namely the correlators of twist fields. This angle of attack is very promising, especially in theories such as the Sine–Gordon model, where certain results are inaccessible via other known techniques.

The event also attracted four graduate students from Birmingham University who attended in person and several mathematical physicists who attended online.

Anne Taormina
Durham University

Records of Proceedings at LMS Meetings
Annual General Meeting and Society Meeting of the London Mathematical Society, Friday 18 November 2022

The meeting was held as a hybrid meeting in person in the Great Hall at Goodenough College, London and via a live stream over Zoom. Over 80 members and visitors were present for all or part of the meeting, in person and online. The meeting began at 3:00pm, with the President, Professor Ulrike Tillmann FRS, in the Chair.

The Minutes of the General Meeting, which was held on 1 July 2022, were circulated to members 21 days in advance of this meeting. Copies of those Minutes were also available at the meeting. The President asked those members present and online if there were any clarifications or corrections to those minutes. There were none and the Minutes were confirmed.

The Vice-President, Professor Iain Gordon, presented a report on the Society's activities in 2021-2022 and the President invited questions. The Treasurer, Professor Simon Salamon, presented his report, online, on the Society's finances during the 2021-22 financial year and the President invited questions. Copies of the Trustees' Report for 2021-22 were made available on the day and the President invited members to adopt the Trustees' Report for 2021-22 by a show of hands for those in person and via a poll for those joining online. The Trustees' Report for 2021-22 was adopted.

The President proposed Messrs Moore Kingston Smith be re-appointed as auditors for 2022-2023 and invited members to approve the re-appointment by a show of hands for those in person and via a poll for those online. Messrs Moore Kingston Smith were re-appointed as auditors for 2022-2023.

22 people were elected to Ordinary Membership: Dr Roseline Abah, Dr Mark Bell, Mr Max Buttinger, Professor Tom Coates, Dr Michele Coti Zelati, Dr Vicky Crockett, Dr Joao Faria Martins, Dr Daniel Galvis, Dr Lorna Gregory, Dr Deborah Kent, Dr Benjamin Lambert, Mr Matthew Meangru, Dr Shihan Miah, Professor Yuji Nakatsukasa, Dr Ioanna Papatsouma, Mr Cesar Antonio Quezada Puebla, Dr Lukas Schimmer, Dr Bismark Singh, Eur Ing Paul Smith, Dr Uchenna Uka, Dr Shahid Wani, and Mr Yashar Zeynalov.

36 people were elected to Associate Membership: Mr Ibai Aedo, Mrs Mnerh Alqahtani, Mr Giancarlo Antonino Antonucci, Mr Samuel Charles, Mr Luke Collins, Mr George Cooper, Mr Nicholas Daultry Ball, Mr Torin Fastnedge, Mr Leo Gitin, Ms Gonzalo Gonzalez de Diego, Mr Shuchen Guo, Mr Jordan Haden, Miss Nikita Handel, Miss Gemma Hood, Mr John Hughes, Mr Akif Ince, Dr Nikoleta Kalaydzhieva, Mr Zain Kapadia, Mr Adam Klukowski, Mr Min Chul Lee, Mr Samuel Lloyd-Lindholm, Miss Kaitlyn Louth, Ms Mariem Magdy Ali Mohamed, Mr Adrià Marin Salvador, Mx Sofia Marlasca Aparicio, Mr Anthony Miller, Ms Thiziri Nait Saada, Mr Guy Parker, Mr Ioan Laurentiu Ploscaru, Mr Prama Putra, Mr Thomas Read, Mr John Stokes-Waters, Mr Daniel Turaev, Mr Harkaran Uppal, Mr Shizhe Xu and Mr Hao Yang.

11 people were elected to Associate (undergraduate) Membership: Mr Zayd Aboulazhar, Mr Mohammed Alhashem, Mr Ioannis Charalampous, Mr Keith Nicholas Gironella, Dr Andreas Katsaros, Ms Sara Luder, Mr Urfan Nawaz, Mr Pier Giorgio Peroni, Mr Adam James Sheppard, Mr James Tomas and Mr Aziz ul Islam. 5 people was elected to Reciprocity Membership: Mr V Madhukar Chowdary, Mr Indunil Hewage, Dr Muhammad Kamran, Professor Yohanna Tella and Dr Edwin Valeroso.

160 people were elected to Associate Membership for Teacher Training Scholars: Mr Harry Acott, Miss Chlöe Allason, Miss Kaia Allen, Dr Mascia Amici, Miss Thea Andrews, Mr Taimoor Answer, Mr Ethan Axon, Mrs Mridula Babu, Miss Hannah Bagley, Mr Calum Baker, Miss Fatima Bax, Mr Marc Baylis, Mr Thomas Bell, Miss Sian Bennett, Mr Paul Best, Miss Eleanor Bines, Mr Andrew Brassington, Mrs Lejla Brook, Miss Samantha Brown, Miss Lauren Burd, Mr Nathan Burke, Mr Peter Callow, Mr Fidel Angel Casado Castillo, Mr Christian Clarke, Miss Victoria Clarke, Miss Emily Clough, Mr Philip Cohen, Mr Daniel Cole, Mr Christopher Collins, Miss Ava Cookson, Miss Brooke Cope, Mrs Laura Corbett, Ms Maeve Corcoran, Miss Rebecca Coward, Mr Joseph Coyle, Mr Alexander Crane, Mr Taran Davies, Mr Jacob Dobson, Mr Jack Drewitt, Miss Natoyia Dunstan, Ms Theone Ellis, Miss Imogen Elnaugh, Miss Caitlin Evans-McCallion,

Mr Eaad Fara, Mr Yousof Fazelpoor, Miss Megan Field, Mrs Karen Foxwell-Moss, Mr Stanislaw Franc, Mr James Franklin, Mr Isaac Gibbons, Mr Juskarun Gill, Mr Varun Gill, Miss Laura Gilyead, Miss Sian Goodwin, Mr Peter Graham, Miss Lauren Griffiths, Mr Thomas Gyllenship, Miss Jessica Hale, Miss Bethany Hamilton, Mrs Delia Hardyman, Dr Joseph Harris, Dr Nicola Hawkins, Mr Lewis Hilton, Miss Cassandra Hotchkies, Miss Shengwen Huang, Mrs Aleen Hussain, Mr Callum Ilkiw, Miss Charlotte Isaacs, Miss Saoirse Jackman, Mr Adam Jasko, Mr Shay Jordan, Ms Isabel Joyce, Miss Lucy Judd, Miss Emily Keating, Miss Catherine Keeble, Mr Johar Khan, Mr Zane Khan, Miss Sarah Kingsley, Miss Elise Lansdowne, Mr Nathan Laurie, Miss Katherine Lay, Mr Junjie Li, Mr Joseph Livesey, Mr Robert Lockett, Ms Lizzie Lomax, Mr Marcos Lopez Lema, Mr James Loveday, Mr Aaron Macintyre, Mr John Mann, Mr Freddie Mansfield, Mr Leigh Martin, Mr Ben McAuley, Miss Tiffany Meeley, Miss Lauren Miles, Miss Natalia Nowicka, Mr Connor O'Hare, Mr Joseph Ostler, Mr Wasim Oulabi, Miss Caitlin Park, Miss Amy Partridge, Mr Lloyd Pennington, Mr Finn Peronius, Miss Stefania Petrescu, Miss Emily Pomering, Miss Alicia Port, Dr Jasmine Pratt, Mr Jack Preece, Mr Said Raii, Miss Maisie Ramsey, Mr Ben Rawcliffe, Miss Kate Rigby, Miss Sara Rigby, Miss Melissa Rix, Miss Jessica Roberts, Miss Sally Roberts, Miss Sarah Rogers, Mr James Rolls, Mrs Sharmin Rumi, Miss Thadshagini Sagthivel, Mr Jake Sandy, Mr William Scopsi Goodey, Mr Lee Searle, Mr Ben Shailes, Miss Susannah Shanly, Miss Alice Simmons, Miss Natalie Slatford, Miss Victoria Smart, Miss Chloe Smith, Miss Elizabeth Smith, Mr Jack Spencer, Mr Peter Stead, Mr Nicholas Stewart, Miss Holly Sturton, Miss Emily Sugden, Miss Aishah Tabor, Miss Francesca Taylor, Mr Jordan Taylor, Mr Matt Teale, Mr Thabit Mohamed Thabit, Mr Edward Thomas, Mr Liam Tillin, Ms Yana Todorova, Mr Matthew Tomlinson, Dr Hossein Towsyfyan, Mr Antonio Tuccinardi, Miss Jessica Tuffs, Miss Lauren Ubertowski, Mr Luke von Leuchtenberg, Mr Samuel Wainwright, Mr Benjamin Walter, Mr Joram Wamagui, Miss Victoria Watts, Mr Jamie Westwood, Ms Yattichia White, Dr Rachael Windsor, Ms Charlotte Withyman, Mr Jonathan Wong, Miss Katy May Woodwards Bloomer, Mrs Damla Yagmur Khazma and Mr Najib Zafar.

Professor Josef Málek (Charles University, Prague) gave a lecture on *Beyond the incompressible Navier-Stokes equations: mathematical foundations of models of non-Newtonian fluids*.

After tea, LMS Scrutineer, Professor Chris Lance announced the results of the ballot. The following Officers and Members of the Council were elected.

President: Ulrike Tillman FRS
Vice-Presidents: Catherine Hobbs, Iain Gordon
Treasurer: Simon Salamon
General Secretary: Robb McDonald
Publications Secretary: Niall MacKay
Programme Secretary: Chris Parker
Education Secretary: Kevin Houston
Members-at-Large of Council for two-year terms: Minhyong Kim, Jason Lotay, Anne Taormina, Amanda Turner and Sarah Whitehouse.
Member-at-Large of Council for a one-year term: Peter Ashwin

Five Members-at-Large, who were elected for two years in 2021, have a year left to serve: Elaine Crooks, Andrew Dancer, Jessica Enright, Sara Lombardo (Member-at-Large Women and Diversity), Frank Neumann and Rachel Newton.

In addition, it was announced that Professor Jens Marklof FRS had been voted President-Elect, taking office as President in November 2023.

The following were elected to the Nominating Committee for three-year terms: Laura Ciobanu and Helen Wilson. The continuing members of the Nominating Committee are: Tara Brendle (Chair), Chris Budd, Nira Chamberlain, Philip K Maini and Gwyneth Stallard. In addition, Council would appoint a representative to the Committee.

Professor Endre Süli (University of Oxford) gave the Naylor Lecture 2022 titled *Hilbert's 19th Problem and Discrete De Giorgi–Nash–Moser Theory: Analysis and Applications*.

The President, on Council's behalf, presented certificates to the 2022 Society Prize-winners:

De Morgan Medal: Professor Sir John Ball FRS (University of Oxford)
Senior Berwick Prize: Jointly awarded to Professor John Greenlees (University of Warwick) and Professor Brooke Shipley (University of Illinois Chicago). Unfortunately, Professor Shipley was unable to collect her certificates and so her certificate has been sent to her.
Fröhlich Prize: Professor Richard Thomas, (Imperial College, London)
Whitehead Prize: Professor Euan Spence (University of Bath)

The **Anne Bennett Prize** was awarded to Dr Asma Hassannezhad (University of Bristol), and **Whitehead Prizes** were awarded to Professor Jessica Fintzen (University of Cambridge, Duke University and Universität Bonn), Professor Ian Griffiths Farrell (University of Oxford), Professor Dawid Kielak (University of Oxford), Professor Chunyi Li (University of Warwick) and Professor Tadahiro Oh (University of Edinburgh). However, they were unable to collect their certificates and so their certificates have been sent to them.

Before closing the meeting, Professor Tillmann thanked the retiring members of Council (Anne Christine Davis and Brita Nucinkis), the retiring Librarian (Mark McCartney), the previous Executive Secretary, Caroline Wallace, and the retiring Scrutineer (Chris Lance).

After the meeting, the new Executive Secretary, Simon Edwards, announced that the reception would be held at Goodenough College in the Vestibule outside the Great Hall, followed by the Annual Dinner, which was held in the Large Common Room at Goodenough College and attended by 63 people. When announcing the dinner, the Executive Secretary thanked the LMS staff. At the start of the Annual Dinner, the President, Professor Ulrike Tillmann FRS, gave a short speech and proposed a toast to the continued health of the Society.

Records of Proceedings at LMS Meetings
Ordinary Meeting: 17 January 2023

This meeting was held on 17 January 2023 at the University of Southampton and via Zoom, as part of the South West & South Wales Regional Meeting & Workshop on *Geometric Group Theory*. Over 50 members and guests were present for all or part of the meeting.

The meeting began at 13:15 with The President, Professor Ulrike Tillmann FRS in the Chair.

There were no members elected to Membership at this Society Meeting.

Six members signed the Members' Book and were admitted to the Society.

Dr Ashot Minasyan, University of Southampton, introduced the first lecture given by Professor Karen Vogtmann (University of Warwick) on *Fixed Points in the Outer Space of a RAAG*.

After tea, Dr Minasyan introduced the second lecture by Professor Ian Leary (University of Southampton) on *Mod-p Analogues of* L^2*-Betti Numbers*.

Dr Minasyan introduced the third lecture by Professor Peter Kropholler (University of Southampton) on *Three Open Problems about Groups, Group Rings and Group Actions*.

Afterwards, a wine reception was held at the Student Centre on campus at the University of Southampton.

Symplectic Geometry and Koopman Dynamics at the Quantum–Classical Interface

CESARE TRONCI

Going back to the early days in the history of quantum mechanics, the interaction of quantum and classical systems stands among the most intriguing open questions in science and makes its appearance in several fields, from physics to chemistry. Recently, a new perspective on this problem was unfolded by an unprecedented combination of symplectic geometry and Koopman's formulation of classical mechanics.

How does a classical system interact with a quantum system? This simple question is among the most problematic in science and has been puzzling the community ever since the early discussions among the founding fathers of quantum theory. Nevertheless, attempts to construct dynamical models of mixed quantum-classical (QC) systems are still ubiquitous in several fields, from theoretical chemistry to solid state physics. Indeed, the complexity of fully quantum many-body simulations stimulates the search for approximate models in which part of a quantum system is treated classically while the remainder remains quantum. The multiscale nature of this problem requires powerful mathematical structures beyond established methods in semiclassical analysis.

Despite several efforts, current hybrid QC models suffer from various consistency issues and no general consensus has been reached. A well-known hybrid QC model is given by the system

$$\dot{q} = \partial_p \langle \widehat{H} \rangle, \qquad \dot{p} = -\partial_q \langle \widehat{H} \rangle, \qquad i\hbar \partial_t \psi = \widehat{H}\psi\,, \quad (1)$$

where $\langle \hat{A} \rangle := \langle \psi | \hat{A} \psi \rangle$ denotes the expectation value, $\langle \psi_1 | \psi_2 \rangle$ is the inner product, and $\widehat{H} = H(q, p, \hat{\mathsf{x}}, \hat{\mathsf{p}})$ is a quantum Hamiltonian operator depending on the classical phase-space coordinates (q, p). Notice that $(\hat{\mathsf{x}}, \hat{\mathsf{p}})$ denote the quantum position and momentum operators such that $[\hat{\mathsf{x}}, \hat{\mathsf{p}}] = i\hbar$. Despite their mathematical appeal, equations (1) often fail to produce realistic results.

Over the years, it has been recognized that QC coupling requires a probabilistic description in both quantum and classical sectors in such a way that statistical correlation effects are retained in the treatment. This means that, no matter the underlying construction, classical dynamics must ultimately be given in terms of a probability distribution $\rho_c(q, p)$ and quantum evolution in terms of a von-Neumann density operator $\hat{\rho}$ so that, for example, $\langle \hat{A} \rangle = \mathrm{Tr}(\hat{\rho}\hat{A})$. However, blending these essentially different descriptions in such a way to retain statistical QC correlations is far from easy. A new strategy that is recently making its way in QC coupling comes from results in pure mathematics, upon blending van Hove's prequantum geometry with Koopman's unitary flows on phase-space.

In 1931 Koopman made the remark that classical mechanics can be formulated as a unitary flow on the Hilbert space of square-integrable complex functions (i.e. wavefunctions, WFs) on phase-space [5]. This intuitive result follows by observing that, if $\{\ ,\ \}$ denotes the standard Poisson bracket, one has

$$i\hbar \partial_t \chi = i\hbar \{H, \chi\} \quad \Longrightarrow \quad \partial_t |\chi|^2 = \{H, |\chi|^2\}. \quad (2)$$

Then, since the Liouvillian operator $\widehat{L}_H := i\hbar\{H,\ \}$ is Hermitian, the classical Liouville equation for $\rho_c = |\chi|^2$ can be realized in terms of a unitary evolution on the Hilbert space $L^2(T^*Q)$. As customary, here we identify the phase-space with the cotangent bundle T^*Q of the classical configuration manifold Q.

The above remark has made many appearances in the literature, with several eminent scholars apparently being unaware of the early works by Koopman and von Neumann. These days, the *Koopman-von Neumann equation* (KvN) (2) for the WF $\chi \in L^2(T^*Q)$ is attracting increasing attention. The idea of using KvN to formulate hybrid QC models goes back to George Sudarshan's work from 1976: if classical mechanics has a Hilbert-space formulation, then one can also take the tensor product of classical and quantum Hilbert spaces, and construct the dynamics of such hybrid QC WFs. However, so far this direction has failed to produce consistent models. Indeed, Koopman's approach involves several challenges that can hardly be tackled without resorting to powerful mathematics.

Within an international collaboration [1], we recently showed how unprecedented insights into hybrid QC dynamics may be uncovered by exploiting the symplectic geometry of Koopman WFs [3]. After restoring the information on classical phases, a first Koopman-based model was obtained via a partial quantization process upon starting with two fully classical systems. Then, the quantum and classical densities are recovered as momentum maps associated to specific unitary representations corresponding to the classical and quantum motions.

Koopman-van Hove prequantum geometry

As the standard KvN construction failed to produce consistent QC models, we proposed an alternative approach to Koopman WFs based on *prequantum theory*. First formulated in van Hove's thesis and further developed by Kostant and Souriau, prequantization is a niche branch of symplectic geometry, whose potential in other fields has not been previously realized. Here, we will simply point out that the prequantum evolution differs from the first (KvN) equation in (2) by the insertion of a phase term retaining the information on the Lagrangian function $L = p \cdot \partial_p H - H$. Then, upon restricting to a one-dimensional configuration space for convenience, the KvN equation becomes

$$i\hbar\partial_t\chi = \widehat{\mathscr{L}}_H\chi\,, \quad \text{with} \quad \widehat{\mathscr{L}}_H := i\hbar\{H,\,\} - (p\partial_p H - H)\,. \tag{3}$$

The *prequantum operator* $\widehat{\mathscr{L}}_H$ has been introduced for later purpose. At this point, the polar form $\chi = \sqrt{D}e^{iS/\hbar}$ returns $\mathrm{d}D/\mathrm{d}t = 0$ and $\mathrm{d}S/\mathrm{d}t = L$ along $(\dot{q},\dot{p}) = (\partial_p H, -\partial_q H)$. The second equation is a phase-space version of the general Hamilton-Jacobi equation for Hamilton's principal function. Indeed, unlike the original KvN construction, prequantum theory carries the information on the classical phase.

Equation (3) was dubbed *Koopman-van Hove* (KvH) equation in [1]. Much insight on (3) is obtained by looking at its canonical Hamiltonian structure, which appears from the variational principle

$$\delta\int_{t_1}^{t_2}\mathrm{Re}\int\bar{\chi}\Big(i\hbar\partial_t\chi - i\hbar\{H,\chi\} + \chi L\Big)\mathrm{d}q\,\mathrm{d}p\,\mathrm{d}t = 0\,. \tag{4}$$

Here, the Hamiltonian functional of the system is identified with the last two integral terms. If we insist that this Hamiltonian must coincide with the physical energy $\int \rho_c H\,\mathrm{d}q\mathrm{d}p$, then an integration by parts yields the following expression of the Liouville density in terms of the KvH WF:

$$\rho_c = |\chi|^2 + \partial_p(p|\chi|^2) + \hbar\,\mathrm{Im}\{\bar{\chi},\chi\}\,. \tag{5}$$

While the first term coincides with the original KvN prescription, the remaining two terms appear mysterious. The major breakthrough in [1, 3] was to recognize that the expression above identifies a *momentum map* structure, thereby ensuring that (5) indeed satisfies the classical Liouville equation. We will now give a quick review of this result.

What is a momentum map?

Momentum maps are a crucial ingredient in symplectic geometry. Specifically, the canonical (left) action of a Lie group G with Lie algebra $\mathfrak{g}$ on a symplectic manifold $\mathcal{S}$ induces a momentum map $J : \mathcal{S} \to \mathfrak{g}^*$ generalizing Noether's conserved quantity. Indeed, the latter occurs in the particular case of a symmetry group. When $\mathcal{S}$ is a linear symplectic space (V,Ω), one has

$$\Omega(\xi_V(x),x) = 2\langle J(x),\xi\rangle\,, \quad \forall\xi\in\mathfrak{g}\,,\ \forall x\in V, \tag{6}$$

where ξ_V denotes the infinitesimal $\mathfrak{g}$–action on V and $\langle\,,\rangle$ denotes the duality pairing. Any complex Hilbert space with inner product $\langle\,|\,\rangle$ has the canonical symplectic form $\Omega(\chi_1,\chi_2) = 2\hbar\,\mathrm{Im}\langle\chi_1|\chi_2\rangle$. When a Hamiltonian can be entirely written in terms of a momentum map, this Hamiltonian is called 'collective' and the dynamics comprises a Lie-Poisson system [4, 6]. Then, the KvH Hamiltonian functional is collective for the momentum map given by (5).

A geometric picture of the KvH equation is achieved by observing that the Hermitian operator $\widehat{\mathscr{L}}_H$ in (3) identifies the infinitesimal generator of the *van Hove representation* on Koopman wavefunctions. This is a more technical aspect that is reviewed in the box below.

Hybrid QC dynamics I: quantization

But how do we use KvH for devising a mixed quantum-classical model? An important hint is given by the fact that, at least for Hamiltonians of the type $H = T + V$, the standard Schrödinger equation can

The van Hove representation

Consider the group $G = \{(\eta, e^{i\varphi}) \in \mathrm{Diff}(T^*Q) \circledS \mathcal{F}(T^*Q, S^1) \mid \eta^*\theta + \mathrm{d}\varphi = \theta\}$, where $\mathrm{Diff}(T^*Q)$ is the group of diffeomorphisms of T^*Q and $\mathcal{F}(T^*Q, S^1)$ denotes the space of S^1–valued functions on T^*Q. Also, $\circledS$ denotes a semidirect product and θ is the symplectic potential $\theta = p\mathrm{d}q$. The Lie algebra of G is identified with the space of scalar functions, endowed with the canonical Poisson bracket. At this point, one shows that the operator $-i\hbar^{-1}\widehat{\mathscr{L}}_H$ generates the unitary *van Hove representation* of G on $L^2(T^*Q)$, that is $\chi_0(z_0) \mapsto \chi_0(z_0)e^{-i\varphi(z_0)/\hbar}|_{z_0=\eta^{-1}(z)}$. Then, the definition (6) returns the momentum map (5), whose Lie-Poisson dynamics gives the classical Liouville equation.

be obtained from (3) by simply applying canonical quantization, i.e. by enforcing $\partial_p \chi = 0$ and replacing $(q, p) \to (\hat{x}, -i\hbar\partial_x)$. Then, in [1], a first QC model was obtained by simply starting with a two-particle KvH equation and then quantizing one of them. As a result, the hybrid QC wavefunction $\Upsilon(q, p, x)$ obeys the quantum-classical wave equation (QCWE)

$$i\hbar\partial_t\Upsilon = \{i\hbar\widehat{H}, \Upsilon\} - \left(p\partial_p\widehat{H} - \widehat{H}\right)\Upsilon\,, \qquad (7)$$

where the Hamiltonian $\widehat{H}$ is the same as in (1). Once again, the RHS identifies a Hermitian operator on the hybrid Hilbert space $\mathscr{H} = L^2(T^*Q \times M)$, where M is the manifold comprising the quantum coordinates x. Thus, the QCWE has again a canonical Hamiltonian structure that is provided by a variational principle analogous to (4). The quantum density matrix $\hat{\rho}$ and classical Liouville density ρ_c are found as momentum maps corresponding to the actions of quantum unitary operators and van Hove transformations on $\mathscr{H}$, respectively. More explicitly, we have

$$\hat{\rho}(x, x') = \int \Upsilon(q, p, x)\bar{\Upsilon}(q, p, x')\, \mathrm{d}q\mathrm{d}p \qquad (8)$$

$$\rho_c = \int \left(|\Upsilon|^2 + \partial_p(p|\Upsilon|^2) + \hbar\,\mathrm{Im}\{\bar{\Upsilon}, \Upsilon\}\right)\mathrm{d}x \qquad (9)$$

Notice that the quantum density matrix is positive semidefinite. This is the first success of this approach: as several previous models were unable to retain this property, the current approach represents a substantial step forward.

As an example, we consider a quantum subsystem given by a $1/2$–spin. In standard Pauli matrix notation, we have $\hat{\rho} = (\mathbf{1} + \mathbf{n} \cdot \widehat{\boldsymbol{\sigma}})/2$ with $n^2 \leq 1$. Then, Hamiltonians of the type $\widehat{H} = H_0(q, p)\mathbf{1} + H_I(q, p)\widehat{\sigma}_z$ lead to an exactly solvable QCWE (7) for $\Upsilon \in L^2(\mathbb{R}^2) \otimes \mathbb{C}^2$, when both H_0 and H_I are quadratic. In this case, the classical density (9) coincides at all times with the results obtained from the fully quantum theory, while a good agreement is also observed for the rotational motion of the *Bloch vector* $\mathbf{n} = \langle\widehat{\boldsymbol{\sigma}}\rangle$. In turn, the QCWE predicts slightly lower levels of *quantum purity* $\|\hat{\rho}\|^2 = (n^2 + 1)/2$.

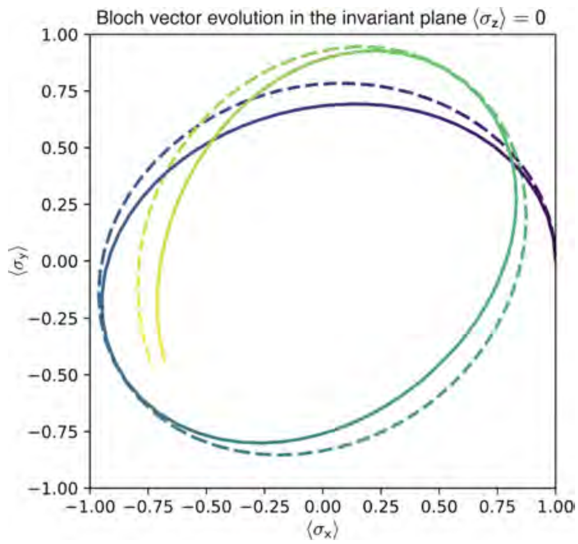


Figure 1. Bloch vector evolution for $H_0 = (p^2 + q^2)/2$ and $H_I = (q^2 - p^2)/4 + 1/2$. The dashed line corresponds to the fully quantum dynamics, while the thick line identifies the QCWE results. The initial time is coloured in purple and the final state (t=10) appears in yellow. The factorized initial condition $\Upsilon_0(q, p) = \chi(q, p)(1, 1)/\sqrt{2}$ corresponds to $\mathbf{n}_0 = (1, 0, 0)$ and $\rho_{c0} = e^{-2H_0}/\pi$. *Courtesy of Giovanni Manfredi and Icare Morrot-Woisard (IPCMS, Strasbourg).*

Despite these encouraging results, we observe that in the case of QC evolution the expression (9) of the classical distribution is generally sign-indefinite. Is the sign of ρ_c preserved by its time evolution? While this is always the case for classical KvH dynamics, a positive answer in the QC context has been found only for an certain infinite family of Hamiltonians $\widehat{H}$ including those discussed above [3]. However, no statement of general validity is currently available. The next section addresses this point by presenting an upgrade model of the QCWE.

Hybrid QC dynamics II: S^1–symmetry

So far, things still look rather simple. But how can we restore the positivity of the classical density? A hint is made available by the observation that expressing the KvH equation (3) by using the polar form $\chi = \sqrt{D}e^{iS/\hbar}$ leads to the relation $(\partial_t + \pounds_{X_H})(\mathrm{d}S - \theta) = 0$. Here, $X_H = (\partial_p H, -\partial_q H)$ is the Hamiltonian vector field and $\pounds$ denotes the Lie derivative. Thus, if we could set $\mathrm{d}S = \theta$, the KvH momentum map (5) would reduce to the KvN prescription $\rho_c = |\chi|^2$. Unfortunately, things are not that simple because this would introduce challenging topological singularities. In turn, upon using the polar form $\chi = \sqrt{D}e^{iS/\hbar}$, we might want to replace $\mathrm{d}S = \theta$ in the variational principle (4). This step has a two-fold effect: a) the Lagrangian becomes trivially S^1–invariant, and b) the variational principle (4) returns an alternative formulation of the KvN equation, thereby retaining a positive density $\rho_c = |\chi|^2$. Thus, KvN arises from KvH upon enforcing an S^1–symmetry [2].

At this point, one may apply the same argument to mixed QC dynamics and enforce the S^1–symmetry on the QCWE variational principle. Upon denoting $\widehat{\mathcal{P}}(q,p,\mathrm{x},\mathrm{x}') = \Upsilon(q,p,\mathrm{x})\bar{\Upsilon}(q,p,\mathrm{x}')$, one obtains

$$i\hbar\partial_t\widehat{\mathcal{P}} + i\hbar\,\mathrm{div}\big(\widehat{\mathcal{P}}\langle X_{\widehat{\mathscr{H}}}\rangle\big) = \big[\widehat{\mathscr{H}},\widehat{\mathcal{P}}\big]\,, \qquad (10)$$

where $X_{\widehat{A}} = (\partial_p\widehat{A}, -\partial_q\widehat{A})$ and $\langle\widehat{A}\rangle = \mathrm{Tr}(\widehat{A}\widehat{\mathcal{P}})/\rho_c$. Also, $\widehat{\mathscr{H}} = \widehat{H} + \hbar\widehat{F}$, where $\widehat{F} = \widehat{F}(\widehat{\mathcal{P}}, \{\widehat{\mathcal{P}},\widehat{H}\})$ is a prescribed function [2]. Despite its formidable appearance, the nonlinear equation (10) appears the first to ensure several consistency properties beyond positivity of the quantum density (8) and the classical density $\rho_c = \int|\Upsilon|^2\mathrm{dx}$. For example, (10) leads to a QC Poincaré invariant extending the classical quantity $\oint_{c(t)} p\,\mathrm{d}q$. Hence, one obtains the typical ingredients of symplectic geometry including a symplectic form, a Liouville volume, and Casimir functions.

Starting with the prequantum geometry of Koopman WFs in QC coupling, we have unfolded the emergence of a very rich mathematical structure combining celebrated concepts in the symplectic geometry of quantum and classical systems. In turn, these geometric structures are capable of accommodating stringent consistency requirements such as quantum and classical positivity, and the reduction to uncoupled quantum and classical dynamics in the absence of a coupling potential.

Computational efforts are underway, along with the development of fluid closure models. In addition, the present models can be adapted to couple quantum and classical spin systems. An entire new direction is unfolding in front of us and right now we are only scratching the surface of a new ground at the boundary between mathematics, physics, and chemistry.

Acknowledgements

These results were obtained within an international and interdisciplinary collaboration involving Denys Bondar (Tulane), François Gay-Balmaz (Paris), Ilon Joseph (Livermore), and Giovanni Manfredi (Strasbourg). The author thanks Paul Bergold and Tom Bridges for their valuable feedback. This work was made possible through the support of Grant 62210 from the John Templeton Foundation as well as the Royal Society Grant IES\R3\203005.

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Cesare Tronci

Cesare is an Associate Professor in mathematics at the University of Surrey with a strong international profile. His research interests are in applications of symplectic geometry, variational fluid models, and multiscale dynamics. Most recently, Cesare became interested in problems of mathematical chemistry.

Extensions of Bertrand's Postulate

ETHAN S. LEE

In general, extensions of Bertrand's postulate seek values $a < b$ such that there is at least one prime in the interval $(ax, bx]$ for all sufficiently large x. This note describes several results which provide the shortest intervals $(ax, bx]$ containing at least one prime when x is small, medium-sized, or large. We also prove some conjectures that Zucker proposed in an earlier feature article.

Bertrand's postulate from 1845, which was proved five years later by Chebyshev, states that there is always a prime in the interval $(n, 2n-2)$ for every integer $n > 3$. As a standalone result, this isn't too impressive, but it has kick-started an area of research on primes in intervals that is still active today.

For every $x > 0$, we know that there exist values a and b such that $a < b$ and there always contains a prime in the interval $(ax, bx]$, because there are infinitely many primes. Moreover, Oliveira e Silva *et al.* have already computed all the gaps between primes up to $4 \cdot 10^{18}$ in [6], so we can safely restrict our attention to $x \geq 4 \cdot 10^{18}$. It follows that finding the shortest interval $(ax, bx]$ that always contains a prime for every $x \geq x_0 \geq 4 \cdot 10^{18}$ is an interesting problem; this is an extension of Bertrand's postulate.

Results on the existence of primes in intervals are important, because they are useful for problems in additive number theory. For example, results of this type were used to verify the ternary Goldbach conjecture, which states that every odd integer $n > 5$ can be expressed as the sum of three primes, up to certain heights.

One extension of Bertrand's postulate is to find integers $k > 1$ such that there is always a prime in the interval $(kn, (k+1)n)$ for integers $n > 1$. To this end, Shevelev *et al.* [10] have showed that the only integers $k \leq 10^8$ such that this holds are $k \in \{1, 2, 3, 5, 9, 14\}$. This form will not give the shortest interval estimates, but are interesting results none-the-less, because they hold for $n > 1$.

Another extension is to find the *smallest* h such that there is always a prime in the interval $(x, x+h]$ for all sufficiently large x; Baker, Harman, and Pintz proved $h = x^{0.525}$ is admissible in [1]. This is the best unconditional result for large x, although finding an *explicit* range for x such that their h is admissible is a *difficult* open problem, so we don't know how large x needs to be. If one assumes the Riemann hypothesis is true, then Cramér showed that $h = c\sqrt{x}\log x$ is admissible for some constant $c > 0$ and Carneiro *et al.* [2] most recently showed that $c = 22/25$ is admissible in Cramér's result for $x \geq 4$.

The preceding form of result yields the shortest intervals (in a numerical sense) when x is *large*. Here, large could mean $x \geq \exp(100\,000)$, so it is important to consider more interval results which will be shorter when x is not large. For example, in the following theorem, we prove (and in some cases refine) certain conjectures that Zucker proposed in an earlier feature article.

Theorem 1. *There is a prime in the interval $(x, \ell x)$ for all $x \geq x_0$, where*

$$(\ell, x_0) \in \{(\tfrac{23}{19}, 24), (\tfrac{17}{11}, 7.2), (\tfrac{11}{7}, 3.2), (\tfrac{5}{3}, 2)\} := \mathscr{L}.$$

Proof. For each pair $(\ell, x_0) \in \mathscr{L}$, we used Python to verify the result for $x_0 \leq x \leq 100\,000$, so it is sufficient to prove that there is a prime in $(x, 1.09x]$ for all $x \geq 100\,000$. To this end, we introduce the Chebyshev θ-function

$$\theta(x) = \sum_{p \leq x} \log p;$$

here p are prime numbers. It is an equivalent problem to show $\theta(1.09x) - \theta(x) > 0$ and we know from [8, Thm. 4] that $|\theta(x) - x| \leq x/(2\log x)$ for $x \geq 563$, so $x \geq 100\,000$ and $\ell \geq 1.09$ imply

$$\frac{\theta(\ell x) - \theta(x)}{x} > \ell - 1.08686 > 0. \qquad \square$$

The results in the preceding theorem are interesting because they hold for $x \geq x_0$ where x_0 is small. However, we already saw that it is safe to consider $x_0 \geq 4 \cdot 10^{18}$. In this case, we expect the corresponding ℓ to be *much* closer to one than $23/19$. Therefore, we consider one more extension: what is the *largest* Δ such that there is a prime in the interval $(x(1 - \Delta^{-1}), x]$ for all $x \geq x_0 \geq 4 \cdot 10^{18}$?

The latest admissible values for Δ at several choices of x_0 have been established by Cully-Hugill and the author in [3], which implements some corrections to their published paper [4]; this work refines previous works from [5], [7], and [9]. The table below compares values for Δ at several choices of x_0.

$\log x_0$	Δ		
	[7, Thm. 2]	[5, Thm. 1.1]	[3, Thm. 1]
46	$8.13 \cdot 10^7$	$1.48 \cdot 10^8$	$1.63 \cdot 10^8$
50	$1.90 \cdot 10^8$	$7.53 \cdot 10^8$	$1.06 \cdot 10^9$
55	$2.07 \cdot 10^8$	$1.77 \cdot 10^9$	$1.02 \cdot 10^{10}$
60	$2.09 \cdot 10^8$	$1.96 \cdot 10^9$	$7.69 \cdot 10^{10}$
150	$2.12 \cdot 10^8$	$2.44 \cdot 10^9$	$2.07 \cdot 10^{11}$

Similar to before, this problem is equivalent to finding Δ such that $\theta(x) - \theta(x(1 - \Delta^{-1}))$ is positive for $x \geq x_0$. Now, we know that studying $\theta(x)$ is explicitly connected to studying the non-trivial zeros of the Riemann zeta-function $\zeta(s)$. Moreover, we can use the smoothing argument introduced in [7] to obtain stronger lower bounds. Therefore, we can determine there is at least one prime in $(x(1 - \Delta^{-1}), x]$ for $x \geq x_0$ using results on the zeros of $\zeta(s)$ and a smoothing argument. It follows that one can improve the values of Δ in the table by implementing better explicit knowledge about the zeros of $\zeta(s)$.

In summary, the extensions of Bertrand's postulate we have seen provide the shortest intervals that contain a prime for every $x > 0$. First, we know every gap between the primes $p \leq 4 \cdot 10^{18}$ exactly. Next, we split the range $x \geq 4 \cdot 10^{18}$ into two ranges: the medium values of x are $4 \cdot 10^{18} < x < x_S$ and the large values of x satisfy $x \geq x_S$, where x_S is sufficiently large; note that x_S could be significantly larger than $4 \cdot 10^{18}$. If $x \geq x_S$, then the shortest interval containing a prime is $[x, x + x^{0.525}]$. If x is medium-valued, then the shortest intervals have the form $(x(1 - \Delta^{-1}), x]$, where admissible Δ are known.

Before concluding, I would like to note that this is still a rich area of research, with plenty of open problems and opportunities to refine existing results, beyond those that have already been mentioned in this note. For example, one could consider the last extension of Bertrand's postulate in reverse. That is, for fixed $\Delta > 1$, what is the *smallest* x_0 such that there is at least one prime in the interval $(x(1 - \Delta^{-1}), x]$ for all $x \geq x_0$? To tackle this problem, it should be possible to adapt the methods used in [3], [5], and [7].

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Ethan is a research fellow at the University of Bristol. His research interests include the distribution of primes and special values of L-functions. He was recently awarded his PhD by the University of New South Wales in Australia, where he also conquered his fear of spiders.

Notes of a Numerical Analyst

Randomness and Certainty

NICK TREFETHEN FRS

Here is a curious symmetry. To achieve randomness, our standard strategy is exponentials. You can toss a coin, but the outcome isn't so random because it is only algebraically sensitive to the initial conditions. For truer randomness you need a chaotic system with exponential sensitivities, like the logistic equation or the Lorenz equations or a pinball machine (Figure 1). Run such a system for a moment and your randomness might be 99%. If that's not enough, run it a little longer to get 99.99%. With each step, your knowledge of the state of the system shrinks by a multiplicative factor, soon reaching zero for practical purposes.

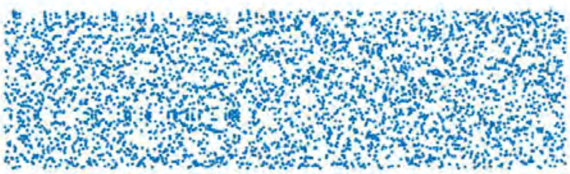

Figure 1. 4000 numbers $y_k \in [0,1]$. The first 2000 are the values $\pi^{-1}\cos^{-1}(y_k)$ for 2000 samples of the logistic equation $y_{n+1} = 1 - 2y_n^2$. The next 2000 are uniformly distributed random numbers in $[0,1]$. One can hardly tell the difference.

And to achieve certainty, our standard strategy is exponentials again! At the level of physics, almost anything can happen because of quantum tunnelling. But some things "never" happen in practice, such as the radioactive decay of an iron-56 atom or an error in a integrated circuit gate. Why? Because the frequency of quantum events shrinks exponentially with the width of a potential barrier. With each slight thickening of a circuit element, your uncertainty about the state of the system shrinks by a multiplicative factor, soon reaching zero for practical purposes.

Figure 2 illustrates the approach to certainty with a solution to the bistable ODE

$$y' = y - y^3 + Cf(t) \qquad (1)$$

driven by f, a smooth random function which approximates standard white noise. Transition from one metastable state to the other depends, loosely speaking, on a succession of random impulses happening to align. As C is reduced, the odds of sufficient alignment shrink exponentially and the half-lives increase to ∞.

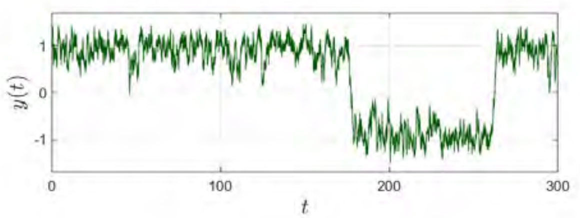

Figure 2. Solution to the random ODE (1) with $C = 0.4$. By reducing C, one lengthens the metastable periods exponentially.

These principles have roots long before chaos theory or quantum mechanics. Think of a combination lock with wheels from 0 to 9. Three wheels gives some security, but four or five are better. The challenge of risk assessment ranges from simple stories like this to large deviations, anomalous diffusion, fat tails and black swans.

Figure 1 generates randomness from certainty, and Figure 2 generates certainty from randomness—or would, with a smaller value of C. The fact that one happens to be set in discrete time and the other in continuous time is not important.

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Mathematics News Flash

Jonathan Fraser reports on some recent breakthroughs in mathematics.

A constant lower bound for the union-closed sets conjecture

AUTHORS: Justin Gilmer
ACCESS: https://arxiv.org/abs/2211.09055

A collection of subsets of $\{1, ..., n\}$ is called *union-closed* if the union of any two sets from the collection yields another set from the collection. For example, the power set is union closed. The union-closed sets conjecture asserts that, for a collection of subsets to be union-closed, there must be an element which belongs to at least half of the sets in the collection. Note that the power set is a motivating and (conjecturally) sharp example since every element is in precisely half of the sets in the power set. If you have digested the somewhat amusing concept of considering 'collections of subsets' (or, sets whose members are themselves sets), then this conjecture sounds fairly harmless. However, it turns out to be very difficult and remains open to this day.

This paper, posted on the arXiv in November 2022, made a major breakthrough towards this conjecture. It established, for the first time, that there is an integer $M > 0$ such that there must be an element which belongs to at least $1/M$ of the sets in the collection. The important thing here is that M does not depend on n. (In fact the paper proves that $M = 100$ works.) An interesting part of the story is that the author no-longer works (formally) in mathematics. He completed his PhD in 2015 and now works on machine learning with Google. His proof uses ideas from information theory.

Making cobordisms symplectic

AUTHORS: Yakov Eliashberg, Emmy Murphy
ACCESS: https://arxiv.org/abs/1504.06312

Given compact manifolds M_1 and M_2, a *cobordism* between M_1 and M_2 is a compact manifold with boundary given by the disjoint union of M_1 and M_2. A *contact manifold* is a manifold equipped with additional geometric structure and one can consider cobordisms between such manifolds which also respect the additional structure, such as *symplectic cobordisms* or *Liouville cobordisms*.

This substantial paper, published in *Journal of the American Mathematical Society* in 2022, provides new existence results for Liouville cobordisms between certain contact manifolds.

Bertrand's Postulate for Carmichael Numbers

AUTHORS: Daniel Larsen
ACCESS: https://arxiv.org/abs/2111.06963

Bertrand's Postulate is a beautiful result in number theory which states that for all integers $N \geq 2$ there must exist a prime between N and $2N$. This can be interpreted as a quantitative upgrade to the fact that there are infinitely many primes. A *Carmichael number* is a composite number n such that b^n is congruent to b modulo n for all integers b. The smallest Carmichael number is 561 and Carmichael numbers are in some sense the closest a composite number can be to being prime. Alford-Granville-Pomerance proved that there are infinitely many Carmichael numbers in a celebrated article from 1994.

This remarkable paper, published in *International Mathematics Research Notices* in 2022, proves the analogue of Bertrand's postulate for Carmichael numbers: for all sufficiently large integers N there must exist a Carmichael number between N and $2N$ (see 'Extensions of Bertrand's Postulate' on page 22 in this issue). In fact it proves a rather stronger result. The proof uses recent work of Maynard on prime clusters.

Finally, I am very sorry to depress our readers, but Larsen was only 17 years old when he completed the paper. He has since commenced his undergraduate degree in mathematics at MIT. Watch this space!

Jonathan Fraser is a Professor at the University of St Andrews and an Editor of this Newsletter. He is pictured here with Reuben and Dylan during a practical exploration of balancing equations.

Microtheses and Nanotheses provide space in the Newsletter for current and recent research students to communicate their research findings with the community. We welcome submissions for this section from current and recent research students. See newsletter.lms.ac.uk for preparation and submission guidance.

Microthesis: Using functional analysis to design the materials of the future

BRYN DAVIES

Metamaterials are transforming how we interact with and manipulate waves in our environment. However, the complexity of these microstructured media poses ever-increasing challenges for modellers, leading to spiralling costs on expensive simulations. Fortunately, mathematical analysts are here to help, by revealing new asymptotic approaches for characterising the crucial properties of these futuristic materials.

High-contrast metamaterials

Metamaterials are artificial materials that have complex, repeating microstructures which yield properties and device functionalities not available using conventional materials. For example, they can display exotic behaviours corresponding to effectively negative refractive indices or negative effective masses and have been used to design cloaking devices and perfect lenses. The name 'metamaterial' signifies these 'unnatural' properties which are primarily determined by the configuration of the repeating microstructures or 'metaatoms'.

In my thesis, I developed techniques to study high-contrast metamaterials consisting of a collection of material inclusions whose material parameters differ greatly from the background medium. Similar to the resonance of air bubbles in water, known as Minnaert resonance, these metaatoms experience resonance in response to wavelengths much greater than their size and are ideal for accurate analysis using asymptotic methods.

Operator formulation

Consider a metamaterial composed of N metaatoms, occupying $D_1, \dots, D_N \subset \mathbb{R}^3$. We wish to characterise the resonant modes of the metamaterial, which we define as non-zero solutions that exist when there are no sources of energy. If $D = \cup D_i$, then we study solutions to the Helmholtz equation $(\Delta + \omega^2)u = 0$ on $\mathbb{R}^3 \setminus \partial D$. This is obtained by assuming solutions are time harmonic and substituting $e^{-\mathrm{i}\omega t}u(x)$ into the standard wave equation. These solutions should satisfy a radiation condition[1] as $|x| \to \infty$ as well as continuity conditions on the boundary ∂D:

$$u|_+ = u|_- \quad \text{and} \quad \delta \frac{\partial u}{\partial \nu}\Big|_+ = \frac{\partial u}{\partial \nu}\Big|_-. \tag{1}$$

Here, ν is the unit normal vector and the subscript $+$ and $-$ denote limits from outside and inside ∂D.

Boundary integral operators can be used to represent solutions for generally shaped D. In particular,

$$u(x) = \begin{cases} \mathcal{S}_D^\omega[\phi](x) & x \in \mathbb{R}^3 \setminus \overline{D} \\ \mathcal{S}_D^\omega[\psi](x) & x \in D \end{cases} \tag{2}$$

for some density functions $\phi, \psi \in L^2(\partial D)$, where $\mathcal{S}_D^\omega$ is the single layer potential, given by

$$\mathcal{S}_D^\omega[\phi](x) = \int_{\partial D} -\frac{\exp(\mathrm{i}\omega|x-y|)}{4\pi|x-y|}\phi(y)\,\mathrm{d}\sigma(y). \tag{3}$$

This is a Green's function approach: the integrand in (3) is the Helmholtz fundamental solution. A solution of the form (2) necessarily satisfies all the desired properties other than the continuity conditions (1), which reduce to a system of integral equations

$$\mathcal{A}(\omega,\delta)\begin{pmatrix}\phi \\ \psi\end{pmatrix} = \begin{pmatrix}0 \\ 0\end{pmatrix} \tag{4}$$

where, for any $(\omega,\delta) \in \mathbb{C}^2$, $\mathcal{A}(\omega,\delta)$ is a bounded linear operator from $L^2(\partial D) \times L^2(\partial D)$ to $H^1(\partial D) \times L^2(\partial D)$. Thus, we have reframed the Helmholtz resonance problem as a problem to find ω and δ such that the operator $\mathcal{A}(\omega,\delta)$ has a non-trivial kernel.

[1] The radiation condition is the Sommerfeld radiation condition which guarantees energy can radiate outwards to infinity but not inwards from it. It also gives the uniqueness of solutions.

Asymptotic characterisation

The highly contrasting nature of the metamaterial is captured by the parameter δ in (1). In the case of an acoustic metamaterial, δ is the ratio of the material densities inside and outside the metaatoms. We assume that δ is small and consider the asymptotic limit $\delta \to 0$. To study subwavelength resonance (where the metaatoms are much smaller than the wavelength) we consider modes for which ω is a continuous function of δ and $\omega(\delta) \to 0$ as $\delta \to 0$.

Under this asymptotic formulation, we can show that a system of N metaatoms has N subwavelength resonant frequencies which are given by

$$\omega_n = \sqrt{\delta \lambda_n} + O(\delta) \quad \text{as } \delta \to 0 \tag{5}$$

where λ_n are the eigenvalues of the generalized capacitance matrix $C \in \mathbb{R}^{N \times N}$, defined as

$$C_{ij} = -\frac{1}{|D_i|} \int_{\partial D_i} (\mathcal{S}_D^0)^{-1} [\chi_{\partial D_j}] \, \mathrm{d}\sigma. \tag{6}$$

Here, $\chi_{\partial D_j}$ is the indicator function of ∂D_j, $|D_i|$ is the volume of D_i and we have used the fact that $\mathcal{S}_D^0 : L^2(\partial D) \to H^1(\partial D)$ is invertible. We can similarly find an asymptotic expansion for the eigenmode u in terms of the associated eigenvector of C.

Generalized capacitance matrices

Capacitance coefficients were introduced by Maxwell to model the relationship between the distributions of potential and charge in a system of conductors. The role of generalized capacitance coefficients here is similar, with the $1/|D_i|$ factor accounting for differently sized metaatoms. Additional factors can be used to describe differing material parameters in each metaatom.

Applications to metamaterial design

This asymptotic method captures metamaterials with general configurations. For example, a graded metamaterial, as depicted in Figure 1, can be shown to separate frequencies. Further, since the asymptotic formula (5) is very fast to compute, the design of the graded metamaterial can be optimised. For example, it can be tuned to replicate the frequency separation of the cochlea, giving a biomimetic frequency sensor.

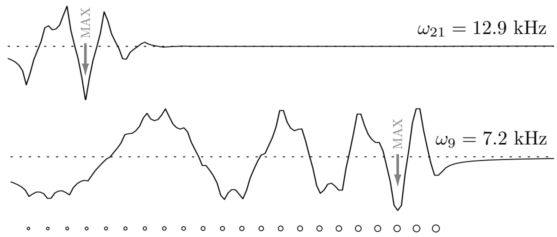

Figure 1. A cochlea-inspired graded metamaterial separates frequencies at different spatial positions

Another important example is waveguides which localise waves at subwavelength scales and, based on notions of 'topological protection', enjoy enhanced robustness to imperfections. Our asymptotic results give concise formulas for the topological indices used in this theory. An example of such a waveguide is shown in Figure 2, along with a localised mode.

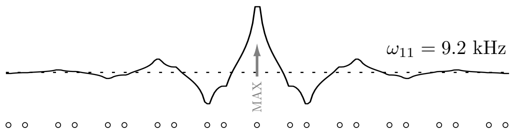

Figure 2. A waveguide with a strongly localised mode; topological protection makes this mode robust to errors

This versatile asymptotic theory gives an approach for studying high-contrast metamaterials very efficiently. Given the broad class of geometries that can be described by these operators, it provides a powerful framework for optimising the design of these futuristic materials.

FURTHER READING

[1] B. Davies. Mathematical analysis of subwavelength metamaterials. PhD Thesis, ETH Zurich, research-collection.ethz.ch.

Bryn Davies

Bryn is a Research Associate at Imperial College London where he works on project BOHEME, which studies bio-inspired hierarchical metamaterials. He has recently completed his PhD under the supervision of Habib Ammari at ETH Zurich. Away from maths, he spends time racing bikes, building rockets and riding the emotional rollercoaster of Welsh rugby fandom.

ADVERTISE IN THE LMS NEWSLETTER

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Information on advertising rates, formats and deadlines are at: lms.ac.uk/publications/advertise-in-the-lms-newsletter.

An example in this issue can be found on page 37.

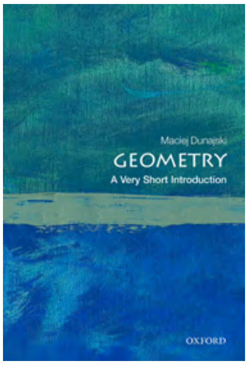
To advertise contact Susan Oakes (susan.oakes@lms.ac.uk).

Geometry: A Very Short Introduction

by Maciej Dunajski, Oxford University Press, 2022, £8.99

ISBN: 978-0199683680

Review by Stephen Huggett



It would probably be a mistake to argue that mathematicians do their best work while in prison, not least because some university administrations might not be able to resist the temptation to lock up their mathematics departments. But it is true that some beautiful mathematics has been done behind bars. The wonderful idea of the cohomology of sheaves came to Jean Leray while he was a prisoner of war in Austria from 1940 to 1945. Jean-Victor Poncelet made his lovely discoveries of the projective properties of conics while a prisoner of war in Russia from March 1813 to June 1814. (There are other examples, not all called Jean.)

Not surprisingly, *Geometry: A Very Short Introduction* does not even touch on the cohomology of sheaves, but it does include a discussion of Poncelet's amazing Porism. More on Poncelet (and porisms) later.

Maciej Dunajski starts with the most familiar part of his subject, Euclidean geometry, but soon leaves the beaten track. For example, in the section on tilings of the plane we find a brief discussion of Penrose's aperiodic tilings. Given a patch P (of diameter d, say) in one such tiling, there are infinitely many ways of extending P to an aperiodic tiling T_1 of the entire plane. But they are all locally indistinguishable, in the following sense. Given an arbitrary point p in any other such tiling T_2, a copy of P can be found in T_2 within $2d$ of p. Actually, this 'local isomorphism theorem' does not appear in the book, but this is not intended as a criticism: of course the author had to make many hard decisions about what to include, and I like his choices.

The Euclidean chapter ends with some intriguing 'hard problems with easy statements', starting with Sylvester's problem on configurations of points and lines. This is: given any configuration of n points in the plane which do not all lie on one line, must there always be a straight line which contains only two of the points? The book includes a lovely elementary proof.

Another such problem concerns the chromatic number of the plane: what is the minimum number of colours needed to paint a plane so that no two points of the same colour are a unit distance apart? This is especially juicy because it is open: all we know is that the answer must be 5, 6, or 7. Introducing novices to open problems is an excellent thing to do, and this book does it well.

Spherical geometry is the first topic in the next chapter, on non-Euclidean geometry. There is a beautifully clear proof, using lunes, of the formula for the area of a spherical triangle. Then we find ourselves in the fascinating world of hyperbolic geometry, crawling about in the Beltrami–Poincaré disc learning how to define distances, and how to tile the disc with regular n-gons.

For me, the highlight of the chapter on curved spaces is the wonderfully clear non-technical discussion of the Gauss–Bonnet Theorem relating the integral of the Gaussian curvature to the genus of a surface. But the chapter also has a great section on manifolds and Riemannian geometry, discussing geodesics and curvature.

Projective geometry is particularly exquisite, and this chapter does it justice. It gives the classical theorems of Desargues and Pascal, followed by Poncelet's wonderful Theorem. Take two conics A and B. (It is easiest to visualise if they are both ellipses, with B inside A.) Choose a point P_1 on A, and choose one of the two tangents to B through P_1. This tangent meets A again at P_2. Take the other tangent to B through P_2. It meets A again at P_3. This procedure either continues indefinitely, or it closes: $P_n = P_1$ for some $n \geq 3$. Both cases occur, and Poncelet's Theorem says that if the procedure does close then you can slide P_1 around A and it carries the whole n-gon with it.

The theorem has modern consequences: I have been lucky enough to attend a superb lecture given by the author on its use in a construction in twistor theory [2].

Sometimes, it is referred to as Poncelet's Porism. 'Porism' is a lovely word, if somewhat mysterious. Guicciardini [1] suggests that Newton 'speculated on the possibility that by porisms the ancients might have meant theorems related to the projective properties of conic sections'. For John Playfair a porism is a problem which under certain conditions is impossible while under certain other conditions it is indeterminate or capable of an infinite number of solutions.

This projective chapter ends with duality, and Brianchon's Theorem, an excellent coda.

In the penultimate chapter, on other geometries, we find a discussion of Klein's Erlangen programme, followed by an introduction to affine geometry, including Ceva's elegant Theorem. After revisiting projective geometry from this point of view, the chapter touches on topology, even giving a brief description of Perelman's proof of the Poincaré conjecture, and then ends with a section on the geometry of Fermat's Last Theorem.

Finally, there is a lovely chapter on the geometry of the physical world. It starts with a lucid description of special relativity, including the beautiful fact that a Lorentz transformation is a Möbius transformation of the celestial sphere. Appropriately for a book on geometry, it ends where the geometry of the physical world ends: in a singularity. We get there via another luminous description, this time of general relativity, black holes, and the singularity theorems.

The book is not flawless: there are some errors, and the prose is sometimes awkward. Also, I for one would have welcomed more purely geometric arguments. But it is a lovely little book, to be recommended for sixth-formers or first year undergraduates: it will open their eyes to the amazing beauty and power of this ancient and modern subject.

FURTHER READING

[1] Guicciardini, Niccolò, *Isaac Newton on Mathematical Certainty and Method*, Cambridge Massachusetts: MIT Press, 2009.
[2] Dunajski, Maciej, and Tod, Paul, *Conics, twistors, and anti-self-dual tri-Kähler metrics*, Asian Journal of Mathematics **24**, 621–652, 2020.

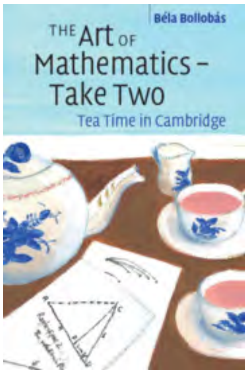
Stephen Huggett

Stephen Huggett is a mathematician living in Somerset, quite near the vertex of degree three formed by its borders with Devon and Dorset. His main research interests are in twistor theory and graph theory, but he is also fascinated by the history of geometry. So far, he has avoided being locked up.

The Art of Mathematics — Take Two: Tea Time in Cambridge

by Béla Bollobás, Cambridge University Press, 2022, £19.99, ISBN: 978-1108978262

Review by Dominic Thorrington



Béla Bollobás, a fellow of Trinity College, Cambridge, has collected 128 mathematical problems for a very engaging book that will give the reader plenty of fun moments playing with some recreational mathematics. This book is a sequel to the author's previous book *The Art of Mathematics: Coffee Time in Memphis*, which was a collection of mathematical problems that the author and his friend would pose to each other over coffee. The new book follows a similar format, this time taking those problems posed between colleagues over tea.

Other mathematicians like Ian Stewart and even organisations such as GCHQ have produced similar problem books for the general audience to dip into every now and again to rescue them from an unproductive afternoon with work or studies, and this book is best enjoyed in the same manner. If you find that you have some free time, perhaps a few more minutes to kill on your commute, keep your phone in your pocket and try to tackle one of these problems instead. Indeed, during the process of reviewing the book I wrestled with a few problems whilst sitting in a parked car waiting for my daughter's swimming class to end.

The book is divided into three sections: the first lists the 128 problems succinctly and clearly for the reader, followed by a chapter with hints, finishing with a chapter with full solutions. That first chapter with all of the problems has been structured to avoid a gradient of difficulty as the reader progresses through the book. Instead they will weave a path through some problems that might take a few minutes followed by ones that will require an awful lot more pondering. The author was also keen to avoid grouping problems on the same topics, ensuring that the reader bounces effortlessly from one branch of mathematics to another. The result is that if the reader wants to systematically work through all of the problems from beginning to end then they are guaranteed variety in both mathematical content and complexity.

Not all problems have hints provided in the second chapter, though probably not because the problem was thought to be too easy because none of them are. For those problems with hints, some of the advice is very succinct (e.g., 'Use induction') whilst other hints go into much more detail to get the reader started on the right path to solving the problem.

The chapter of solutions is very thorough indeed, and the reader will appreciate the scrupulous arguments provided for each solution, as well as the occasional reference for further reading provided by the author.

Bollobás' book is not aimed at a general audience, more so a specialist audience. The author writes that he had three main target audiences in mind: students who enjoy mathematics, professional mathematicians, and anyone who loved mathematics in their younger days. I sit somewhere within the latter two groups and I had to dig out their old notes from university to be able to tackle some of the problems in the book. A pre-university reader might not be able to get through many of the problems provided by the author, and indeed a lot of the material will only be familiar to someone who's done at least a few terms of a mathematics degree. The author has also been careful to avoid any suggestion that this book is a reference book for certain topics — the reader is invited to follow some references in the chapter of solutions, indeed, but this book should not and does not serve as a study guide.

What sort of problems has the author provided for the reader? To my delight, the Monty Hall Problem appears here as the 59th problem, and to add to that delight Bollobás has provided a lengthy discussion about the problem that includes the rather shameful saga surrounding the first publication of the problem. There's also the occasional problem from the Mathematical Tripos at Cambridge. Other less well-known problems cover topics such as Vieta jumping (I found it difficult to put that one down), geometry, number theory, and a lovely little problem where the reader is invited to provide three proofs relating to properties of the reciprocals of the primes.

This is a fun book to have. It's pitched at a higher level of difficulty than other mathematical problem books such as the aforementioned publications by Ian Stewart, and requiring a deeper knowledge of mathematics than the problem books on broader topics made by GCHQ. It deserves a place on your shelf to rescue you from the times you find yourself stuck with the mathematics you're dealing with at work or during your studies. For the reader who has long since finished formal mathematical studies but enjoys jumping back into recreational mathematics, this book will provide a great excuse to go back over old topics, without being a reference book.

Dominic Thorrington

Dominic is a mathematical modeller and health economist at the Haute Autorité de Santé in Saint-Denis, France, where he evaluates the impact of vaccination programmes using disease transmission models. He keeps the cogs in his head turning thanks to a few problem books both on his desk at work and on his shelves at home, though he hasn't managed to convince work that these books should be put on expenses.

Obituaries of Members

Garth Dales: 1944–2022

Professor H. Garth Dales, who was elected a member of the London Mathematical Society on 20 November 1969, died on 8 October 2022, aged 78.

Joel Feinstein writes: Garth Dales was a tremendously productive mathematician throughout the fifty plus years of his career. He is particularly well known for disproving (under the assumption of the continuum hypothesis) Kaplansky's conjecture concerning the automatic continuity of algebra homomorphisms from C(X). In 1980 the London Mathematical Society awarded him a Whitehead Prize for this work.

MathSciNet currently lists 94 of Garth's mathematical publications, including joint papers with 55 different co-authors. The publications include several books, as well as many long papers and mathematical memoirs. Another book (joint with Ali Ülger) is currently in press. It is impossible to do justice to his prolific output in a short space, but this may give some idea of the scale of his energy and ingenuity.

Garth was born in Cleethorpes, Lincolnshire in 1944. His mother was a primary school teacher and his father was a clerk on the railway. He lived in Lincolnshire with his parents until he went to Queens' College, Cambridge in 1963 with a scholarship to read mathematics. He gained a first in the 3-year Mathematical Tripos, and then stayed on in Cambridge for another year to do Part III of the Tripos. He then studied for a PhD with Graham Allan, first in Newcastle, and then in Cambridge when Graham Allan moved back there in 1969.

After gaining his PhD, Garth was appointed Lecturer in Mathematics at the University of Glasgow (1970-73). In 1974 he moved to a lectureship at the University of Leeds, which he took up after returning from a visiting professorship at UCLA. At Leeds he was promoted to Reader in Analysis in 1980, and to Professor of Mathematics in 1986. He served as Head of the Department of Pure Mathematics for two separate three-year terms. He also had several visiting positions in California (UCLA, Berkeley and the California Institute of Technology), where he developed fruitful research collaborations with Philip Curtis, Jr., William C. (Bill) Bade, and Hugh Woodin. When Garth retired from Leeds in 2011, he took up a position as part-time professor at the University of Lancaster, where he worked until his second retirement in early 2021.

In 1974 Garth helped found what became a series of (biennial) international conferences on Banach algebras. He attended every one of these meetings until illness prevented his attending the 25th conference (Granada, Spain, 2022). He served as chairman of the associated International Steering Committee on Banach Algebras from 1988-2008.

Garth had close connections with the London Mathematical Society for many years. He served as editorial adviser (1980-89), member of the Publications Committee (1990-98), executive editor of LMS Monographs (1988-2000), Council member (2005-11), and as obituaries editor (2020-22).

Garth supervised many research students, first at Leeds and then at Lancaster. To my knowledge, at least fourteen of his research students have obtained PhDs. In particular, Garth was my own PhD supervisor in Leeds (1986-89). He was an invaluable source of advice, both mathematical and practical. He had a vast knowledge of the mathematical literature on Banach algebras, and even if he did not know the answer to a question himself, he would know who the best person to ask would be. I always enjoyed working with him on problems of mutual interest, though it was not until 2008 that we published our first joint paper.

Garth enjoyed hiking and climbing (he climbed all of the Scottish Munros). He was also very knowledgeable on a wide range of topics outside mathematics. I particularly enjoyed his recounting historical events from around the world that I often knew little about.

Garth is survived by his wife Joanna, his two sons John and Andrew, and four grandchildren Yve, Kiran, Nayan and Arjun. I am very grateful to Garth's family, and especially to Joanna, for providing me with much of the biographical material here and with permission to use the photograph above.

Death Notice

We regret to announce the following death:

- Yuri Manin, of the Max Planck Institute for Mathematics and Northwestern University, who was elected an LMS Honorary Member on 1 July 2011 and was the LMS Hardy Lecturer in 2006, died on 7 January 2023 aged 85.

LMS Meeting

Midlands Regional Meeting 2023

27 March, University of Warwick

Website: tinyurl.com/keyyh3f9

This meeting forms part of the Midlands Regional Workshop on Ergodic Theory of Group Extensions (28–30 March).

The meeting will open with Society Business, during which LMS members will have the opportunity to sign the Members' Book, which dates from 1865 and includes the signatures of Augustus De Morgan, Henri Poincaré, Mary Cartwright and others. The following speakers will then give talks:

- Viveka Erlandsson (Bristol University), *Counting Geodesics on Surfaces*
- Michael Magee (Durham University), *Spectral Properties of Hyperbolic Surfaces*
- Mark Pollicott (Warwick University), *Counting Geodesic Arcs on Surfaces with no Conjugate Points*

A drinks reception will be held at the Warwick Mathematics Institute, followed by a Society Dinner at the Scarman Conference Centre, costing £45 (including drinks). To reserve a place at the dinner, email mrc@maths.warwick.ac.uk.

Funds are available for partial support to attend the meeting and workshop. Requests for support with an estimate of expenses should be addressed to the organiser, Richard Sharp: MRC@warwick.ac.uk.

These lectures are aimed at a general mathematical audience. All interested, whether LMS members or not, are welcome to attend.

LMS Professional Development Series for Early Career Researchers

Session 5: Applying for Jobs in Industry

30 March, 12–1pm; Zoom, hosted by the LMS

Website: lms.ac.uk/events/CPD-5

LMS Early Career Researcher (ECR) Professional Development sessions are free online panel discussions aimed at ECRs and run by the LMS ECR Committee. Each session focuses on a particular element of career development for ECRs.

In Session 5, the panel will discuss how to find a first job in industry (broadly defined), and offer the opportunity to ask more detailed questions. The panellists are to be announced. To attend the Continuing Professional Development sessions in 2022/23, please register once at bit.ly/3Y0p0c5 to receive Zoom links to all sessions. The Zoom link will be circulated the day before each session so if this is your first time registering, please register by 3pm on Wednesday 29 March.

Geometry and Analysis meet at Brunel

Location: Brunel University, London
Date: 15 March
Website: tinyurl.com/GeometryAnalysisBrunel

This meeting is focused on themes surrounding global analysis. It aims to bring together harmonic analysts, geometers and mathematical physicists to explore mutual intersections between these fields. The event will feature invited talks by Martin Dindoš (Edinburgh), Kasia Rejzner (York) and Zoe Wyatt (KCL). In addition there will be time for interaction and discussion, particularly at the social event that will be held after the talks. This meeting is partly funded using an LMS 'Celebrating New Appointments' grant.

UK Easter Probability Meeting

Location: University of Manchester
Date: 27-31 March
Website: tinyurl.com/4by2cdp5

This meeting features three minicourses given by Nathanaël Berestycki, Nathan Ross and Amandine Veber, as well as 12 invited talks from the probability community. Register at the URL above by 15 March for general registration.

At the Interface of Asymptotics, Conformal Methods and Analysis in General Relativity

Location: Royal Society, London and online
Date: 9-10 May
Website: tinyurl.com/m4zy5bec

This Royal Society meeting aims to bring together researchers working on asymptotic analysis in general relativity and other areas that build on Penrose's insight that conformal geometry could be used to study the global structure of fields and spacetimes. Both in-person and online attendance is available. There is no registration fee but registration is essential.

Postgraduate Combinatorial Conference

Location: University of Birmingham
Date: 27–29 March
Website: tinyurl.com/hpckyjab

This conference is a well-established three-day event promoted by the British Combinatorial Committee. The conference is aimed at current research students in all areas of combinatorial and discrete mathematics. The main goal of the conference is to provide an opportunity for research students to discuss their research in a relaxed environment. Each student is encouraged to contribute by giving a talk. The deadline for registration is 10 March 2023. The conference is supported by an LMS Scheme 8 grant.

British Applied Mathematics Colloquium

Location: Bristol
Date: 3–5 April
Website: tinyurl.com/BAMC2023

The BAMC is the UK's largest applied mathematics conference, bringing together students, academics and industry professionals to discuss recent advances in areas including mathematical biology, continuum mechanics, mathematical modelling, numerical analysis, and network science.

Modelling in Industrial Maintenance and Reliability

Location: Nottingham
Date: 4–6 July
Website: tinyurl.com/bdcvbd68

This IMA international conference is the premier maintenance and reliability modelling conference in the UK and builds upon a very successful series of previous conferences. It is an excellent international forum for disseminating information on the state-of-the-art research, theories and practices in maintenance and reliability modelling and offers a platform for connecting researchers and practitioners from around the world.

LMS Research School

Random Structures, Applied Probability and Computation

26–30 June, University of Liverpool

Website: https://sites.google.com/view/lms-school-liverpool2023/home

The school will focus on advances in the field of random structures and is intended for PhD students and early-career researchers interested in this area. Prominent researchers have been invited to give 3 mini-courses, together with tutored problem sessions, and plenary talks. The mini-courses will be given by Søren Asmussen (Aarhus), Jean Bertoin (Zürich), and Cécile Mailler (Bath).

LMS Meeting

LMS General Meeting and Hardy Lecture

30 June, venue to be announced

Website: lms.ac.uk/events/society-meetings/general-meeting-society

The meeting will open with Society Business, during which LMS members will have the opportunity to sign the Members' Book. This will be followed by the Hardy Lecture, given by Eva Miranda (UPC and CRM-Barcelona). The accompanying speaker will be Sir Roger Penrose.

Register at the URL above. A Society Dinner will be held after the meeting, costing £45 (including drinks). If you would like to attend the dinner, please indicate this on the registration form and send payment using the details which will be given in the acknowledgement email.

These lectures are aimed at a general mathematical audience. All interested, whether LMS members or not, are welcome to attend.

LMS Meeting

LMS Invited Lecture Series 2023

17–21 July, University of Durham

Website: lms.ac.uk/events/lectures/invited-lectures

The LMS Invited Lecturer in 2023 is Filippo Santambrogio (Université Lyon 1, France). His talk will be on *Optimal transport and its applications*. The accompanying lectures will be given by David Bourne (Heriot-Watt), Sara Farinelli (Lagrange Centre), Hugo Lavenant (Bocconi University), Emanuela Radici (University of L'Aquila) and Matthew Thorpe (Manchester).

Register at the URL above. Funds are available for partial support to attend; requests for support with an estimate of expenses should be addressed to the organiser, Dr Alpár Mészáros (alpar.r.meszaros@durham.ac.uk).

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School of Mathematics

A Celebration in Honour of
Jon Keating's 60th Birthday

Random Matrices from Quantum Chaos to the Riemann Zeta Function

A three-day conference to celebrate the 60th birthday of Jon Keating. The meeting will showcase recent results in the areas of mathematics to which Jon Keating has contributed and will feature mathematicians working at the interface of quantum chaos, analytic number theory, probability and random matrix theory.

Organisers: Emma Bailey (CUNY), Tamara Grava (Bristol and SISSA), Francesco Mezzadri (Bristol), Nina Snaith (Bristol) and Brian Winn (Loughborough)

University of Bristol
5 – 7 July 2023

Speakers:

Louis-Pierre Arguin CUNY
Emma Bailey CUNY
Michael Berry Bristol
Brian Conrey AIM
Neil O'Connell UCD
Alexandra Florea UC Irvine
Yan Fyodorov KCL
Alice Guionnet Lyon/CNRS
Alexander Its IUPUI
Jens Marklof Bristol
Zeev Rudnick Tel Aviv
Peter Sarnak Princeton/IAS
Nick Simm Sussex
Kannan Soundararajan Stanford

Photo: Chrystal Cherniwchan

Public Lecture:
5 July 2023, 5-6 pm
Persi Diaconis Stanford

Please contact
heilbronn-coordinator@bristol.ac.uk
for further information, or visit heilbronn.ac.uk/events

Funding is available to support a limited number of UK-based PhD students and Early Career Researchers. Support for US-based Early Career Researchers might be available pending approval. We also welcome applications for caring costs.

Scan me to register

Society Meetings and Events

March

22 Christopher Zeeman Medal 2020 and 2022: Lectures and Ceremony, Royal Society London

27-30 LMS Midlands Regional Meeting, Warwick

30 LMS Professional Development Series for Early Career Researchers; Session 5: Applying for Jobs in Industry, online

April

4 LMS Meeting at the BMC 2023, Bath

24-28 LMS Research School Adaptive Methods and Model Reduction, Nottingham

June

9 LMS Spitalfields History of Mathematics Meeting, London

26-30 LMS Research School Random Structures, Applied Probability and Computation, Liverpool

30 LMS General Meeting & Hardy Lecture, London

July

3-7 LMS Research School Machine Learning in Mathematics and Theoretical Physics, Oxford

16-28 LMS Undergraduate Summer School, Sheffield

17-21 LMS Invited Lecture Series 2023, Durham

24-28 LMS Research School Algebraic Groups and their Representations, Birmingham

24–4 Aug LMS-Bath Mathematical Symposium Operators, Asymptotics, Waves, hosted at Bath

August

1-11 LMS-Bath Mathematical Symposium Categorical and Geometric Representation Theory, Bath

Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website (www.lms.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

March

27-29 New Perspectives in Pure Mathematics, University of Bristol (504)

27-29 Postgraduate Combinatorial Conference, University of Birmingham (505)

27-31 UK Easter Probability Meeting, Manchester (505)

29-31 Kings and Queens of Gravity, Queen Mary University of London and LMS De Morgan House (504)

April

3-5 British Applied Mathematical Colloquium, Bristol (505)

3-6 British Mathematical Colloquium, University of Bath (503)

17-21 Graph Rigidity and Applications, Lancaster University (504)

27-28 4th IMA and OR Society Conference on Mathematics of Operational Research, Birmingham (504)

May

9-10 At the Interface of Asymptotics, Conformal Methods and Analysis in General Relativity, Royal Society, London (505)

22-23 Scottish Combinatorics Meeting, University of Strathclyde, Glasgow (504)

22-24 Gregynog Welsh Mathematics Colloquium 2023, Gregynog Hall, Tregynon, Powys (504)

June

19-23 Probabilistic Group Theory CMI-HIMR Summer School, University of Bristol (504)

July

4-6 Modelling in Industrial Maintenance and Reliability, Nottingham (505)

10-14 Remembering Victor Snaith: Topology, Number Theory and Interactions, University of Bristol (504)

24-28 Algebraic Groups and their Representations LMS Research School, University of Birmingham (504)