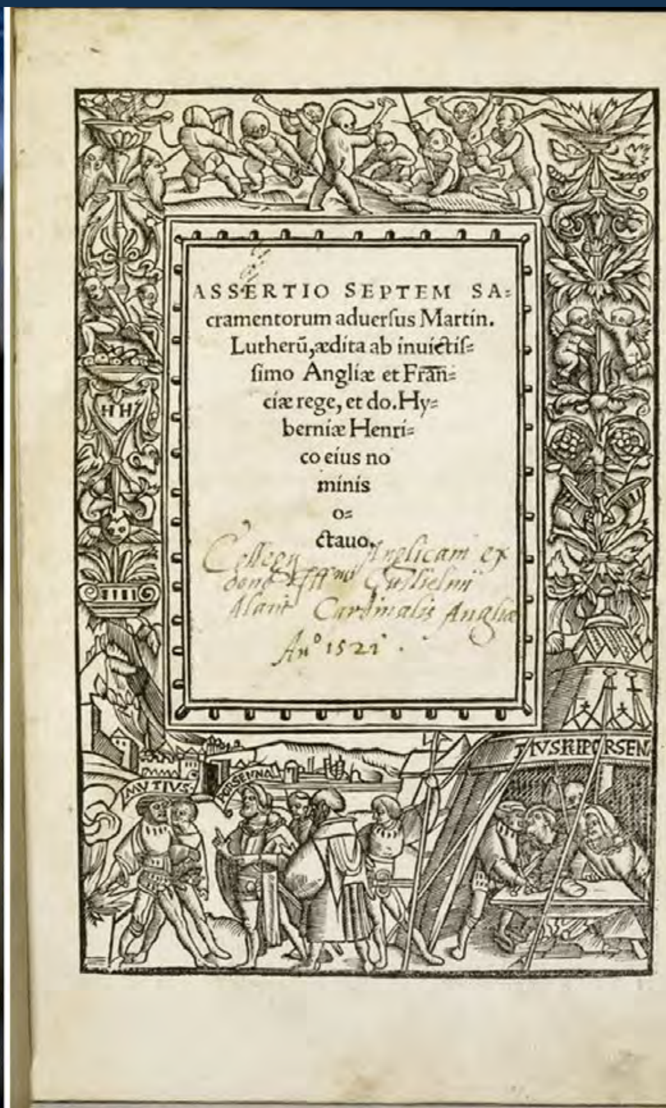
LONDON MATHEMATICAL SOCIETY

EST. 1865

NEWSLETTER

Issue: 506 - May 2023



PRACTICAL MATHS & LATIN ELOQUENCE

THE MATHS BEHIND THE CASIMIR EFFECT

MATHEMATICS NEWS FLASH

EDITORIAL OFFICE

London Mathematical Society
De Morgan House
57–58 Russell Square
London WC1B 4HS
newsletter@lms.ac.uk

Charity registration number: 252660

COVER IMAGE

Figures 1 (left) and 2 from the article 'Practical Mathematics and Latin eloquence: *De Arte Supputandi*, the First Mathematics Book Printed in England', by Alisz Reed and Deborah Kent, on page 18.

Do you have an image of mathematical interest that may be included on the front cover of a future issue? Email images@lms.ac.uk for details.

ISSN: 2516-3841 (Print)
ISSN: 2516-385X (Online)
DOI: 10.1112/NLMS

NEWSLETTER WEBSITE

The Newsletter is freely available electronically at lms.ac.uk/publications/lms-newsletter.

MEMBERSHIP

Joining the LMS is a straightforward process. For membership details see lms.ac.uk/membership.

SUBMISSIONS

The Newsletter welcomes submissions of feature content, including mathematical articles, career related articles, and microtheses from members and non-members. Submission guidelines and LaTeX templates can be found at lms.ac.uk/publications/submit-to-the-lms-newsletter.

Feature content should be submitted to the editor-in-chief at newsletter.editor@lms.ac.uk.

News items should be sent to newsletter@lms.ac.uk.

Notices of events should be prepared using the template at lms.ac.uk/publications/lms-newsletter and sent to calendar@lms.ac.uk.

For advertising rates and guidelines see lms.ac.uk/publications/advertise-in-the-lms-newsletter.

CONTENTS

NEWS	The latest from the LMS and elsewhere	4
LMS BUSINESS	Reports from the LMS	14
FEATURES	Practical Mathematics and Latin eloquence: *De Arte Supputandi*, the First Mathematics Book Printed in England	18
	The Mathematics Behind the Casimir Effect	24
	Notes of a Numerical Analyst	30
	Mathematics News Flash	31
REVIEWS	From the bookshelf	32
OBITUARIES	In memoriam	38
EVENTS	Latest announcements	40
CALENDAR	All upcoming events	44

LMS NEWS

New Chair of the Heilbronn Institute

The University of Bristol has announced the appointment of Professor Catherine Hobbs as the new Chair of the Heilbronn Institute for Mathematical Research (HIMR) and Professor of Mathematics and Mathematics Education.

Professor Hobbs will lead the institute's external research activities, which includes over 40 postdoctoral fellows and over 40 PhD students based at over 15 universities across the UK, as well as an extensive programme of conferences, workshops, summer schools and visitors. She will also be responsible for sustaining and enhancing HIMR's profile as an internationally leading research institute.

Professor Hobbs, Vice-President and member of the Board of Trustees of the LMS, has a distinguished record of leadership in her previous roles across the UK mathematics community. Currently Academic Dean in the Faculty of Engineering, Environment and Computing at Coventry University, she served previously as Associate Dean Research and Enterprise, Faculty of Environment and Technology (2018-22) and Head of Department, Engineering Design and Mathematics (2010-18) at the University of the West of England. A member of the Executive Committee of the new national Academy of Mathematical Sciences, she has also served as Chair of the national network of Heads of Departments of Mathematical Sciences, and is Honorary Education Secretary and a member of the Board of Trustees of the IMA.

As Chair of the Heilbronn Institute, Professor Hobbs will succeed Professor Geoffrey Grimmett, FRS. Alongside this role, she will take up the position of Professor of Mathematics and Mathematics Education in the School of Mathematics, where she will promote and enrich new teaching and learning initiatives. She will assume her new roles from 1 September.

LMS President Ulrike Tillmann writes: “I am absolutely delighted that one of our Vice-Presidents is taking on this important national role, and can only congratulate the Heilbronn Institute on such an excellent appointment.”

Cecil King Travel Scholarships

Fiona Torzewska (left) and Alex Rutar

The Cecil King Travel Scholarship is a prestigious award established in name of the renowned British newspaper proprietor and businessman Cecil Harmsworth King. The funding is provided by the Cecil King Memorial Foundation to support two individuals who demonstrate exceptional academic potential in the fields of mathematics. Two scholarships are awarded each year to finance a study or research visit abroad for a period of three months.

Following this year's round of applications, the LMS and the Cecil King Memorial Foundation are delighted to announce that two Cecil King Travel Scholarships 2023 were awarded to Alex Rutar and Dr Fiona Torzewska.

Alex Rutar is a PhD student at the University of St Andrews who is actively working in fractal geometry, dynamical systems, and the fine geometry of sets and measures, with a particular emphasis on the intersections of these areas. This includes multifractal analysis of self-similar and self-affine sets and measures; symbolic dynamics, matrix product theory, and random substitutions; general dimension theory, dimension interpolation, and classification results. Mr Rutar also loves searching for connections to adjacent areas such as probability theory, harmonic analysis, number theory, and statistical physics, and he will be visiting the University of Oulu, Finland.

Dr Fiona Torzewska is a postdoctoral researcher at the University of Leeds who is interested in the mathematics of topological field theories and their applications to quantum computing. Recently Fiona has been working on motion groupoids and their representation theory. Dr Torzewska will follow her interest and investigate the connections between motion groupoids and other frameworks of topological field theory during her visit to the University of Vienna, Austria.

The LMS and the Cecil King Memorial Foundation believe that the scholarship programme offers early career mathematicians a variety of opportunities in the form of mentorship, professional development, networking and progress in their academic careers.

If you would like to find more about Cecil King Travel Scholarships and how to apply, please visit tinyurl.com/2p92jzpz. The application deadline for the 2024 programme is 15 November 2023. The application form will be available on the LMS website later in 2023. Shortlisted applicants will be invited to an interview during which they will be expected to make a short presentation on their proposal. Interviews will take place in January 2024.

Valeriya Kolesnykova
Accounts, Fellowships & Membership Assistant

OTHER NEWS

10ECM: Call for Bids

Outline bids to organise the 10th European Congress of Mathematics 2028 are now invited, with a deadline of 30 June 2023. For more information visit tinyurl.com/hr76zauu. The European Congress of Mathematics (ECM) is organised every four years. The first ECM was held in Paris in 1992, and since then it has been held in Budapest, Barcelona, Stockholm, Amsterdam, Kraków, Berlin and Portorož. The next congress will be held 15–19 July 2024 in Seville: ecm2024sevilla.com.

Initially only outline bids are invited, giving a clear idea of the proposal and possible sources of financial and local support. The Executive Committee of the EMS will consider the bids received and invite one or more to be set out in greater detail. The deadline for such 'worked up' bids will be 31 October 2023.

Anyone interested is welcome to discuss informally with the EMS President. For more information visit: tinyurl.com/hr76zauu.

MaRDI Consortium

The Mathematical Research Data Initiative (tinyurl.com/3wauxnm2) is dedicated to building infrastructures to make mathematical research data FAIR (Findable, Accessible, Interoperable, and Reusable). It is thus essential for the success of the project to engage as many members of the mathematics community as possible early on. You are invited to subscribe to the newsletter *Math & Data Quarterly* (tinyurl.com/yc2vcxwk) and to make contact via the community help desk (https://tinyurl.com/ywmtmbp8). The latest newsletter on 'Reusability' is just out now!

International Mathematics Competition for University Students

The 30th IMC will be held from 31 July to 6 August 2023, in Blagoevgrad, Bulgaria. It is organised by University College London and hosted by the American University in Bulgaria, Blagoevgrad. Universities are invited to send several students and one teacher as Team Leader. Individual students without Team Leaders are welcome. The competition is planned for students just completing their first, second, third, or fourth years of university education and will consist of two sessions of five hours each. Problems will be from the fields of Algebra, Analysis (Real and Complex), Geometry and Combinatorics. The maximum age of participants is normally 23 years of age at the time the IMC, although exceptions can be made. The working language will be English.

The IMC in Blagoevgrad is a residential competition, and all student participants are required to stay in the accommodation provided by the hosts. Past participants have gone on to distinguished careers in mathematics. Most notably, in 2018, Caucher Birkar (born Faraydoun Derakhshani) received mathematics' most prestigious award, the Fields Medal. In 2000 he participated in the 7th IMC that was held at UCL. In 2022 a Kyiv-born mathematician, Maryna Viazovska, was also awarded the Fields Medal. She participated in the IMC as a student four times, in 2002, 2003, 2004 and 2005.

Over the past 29 competitions the IMC has had participants from over 200 institutions from over 50 countries. For further information and online registration visit the website at www.imc-math.org.uk. Further details may be obtained from Professor John Jayne (j.jayne@ucl.ac.uk).

MATHEMATICS POLICY DIGEST

Voice of the Future 2023

Cecil King Scholarship holder Dr Prachi Sahjwani (pictured, right) asked a question on gender disparities in the mathematical sciences on behalf of the Council for Mathematical Sciences, as part of the Voice of the Future event in March 2023. The respondees were Stephen Metcalfe MP, Carol Monaghan MP and Katherine Fletcher MP. Watch the video at https://youtu.be/mckLH7pZJi4. Read more about Voice of the Future, which is organised by the Royal Society of Biology, at tinyurl.com/voiceofthefuture.

LMS Statement on the Prime Minister's vision for Maths to 18

On 17 April, the Prime Minister announced further work to deliver his Maths to 18 policy with an advisory group being formed to help guide the work. Commenting on the news, Professor Ulrike Tillmann, President of the London Mathematical Society said:

"Maths is critical to the future success of our nation's young people, and we greatly welcome the Prime Minister's commitment to strengthening the provision of maths teaching.

"The world is changing and more maths — from numeracy to mathematical proof, from algorithms to statistics — will help prepare the next generation for life and a job market that is more data-driven and requires quantitative skills at all levels.

"Naturally this means also supporting and training more maths teachers, and it is right that this is central to the government's plans."

Furthermore, Professor Catherine Hobbs, Vice-President of the London Mathematical Society added:

"Maths plays a crucial role in our understanding of all the sciences as well giving young people the skills to interpret data and technical information in an increasing digital world. We welcome the Prime Minister's commitment to improving the maths skills of our young people, so they are better prepared for the rapidly changing job market.

"The advisory group is an important step forward, recognising that improving the mathematical pipeline is not just the responsibility of mathematicians but of education leaders and those in business. As the UK's learned society for mathematics, we actively promote the benefits of mathematics and mathematical education through encouraging the public — and young people in particular — to appreciate and engage with mathematics and supporting mathematicians to engage with the public. We look forward to working with the advisory group to create a positive way forward for the future of mathematics".

Digest prepared by Katherine Wright
Society Business, Research & Communications Officer

Note: items included in the Mathematics Policy Digest are not necessarily endorsed by the Editorial Board or the LMS.

EUROPEAN MATHEMATICAL SOCIETY NEWS

EMS Young Academy Launch

The EMS is proud to announce the launch of its Young Academy (EMYA) and the election of the first cohort of EMYA members. Thirty early career European mathematicians, spanning 18 countries and representing a broad spectrum of research fields, have been elected, and an inaugural online meeting was held on 6 March 2023. The list of current members can be found on the EMYA webpage at euromathsoc.org/EMYA-list2023.

Each member is invited to serve for four years, and at full scale EMYA will have 120 members. EMYA provides a platform for the next generation of mathematicians to debate and plan for the future of European mathematics, to organise activities through the EMS, and to advise the EMS on its actions and priorities. More details about EMYA, including the procedures and timeline for future nominations and selection, are available on the EMYA webpage at euromathsoc.org/EMYA. The EMS is grateful to the new EMYA members for their willingness to serve, and is excited to start working with them in support of the wider European mathematical community.

Call for 10ECM

Outline bids to organise the 10th European Congress of Mathematics in 2028 are now invited, with a deadline of 30 June 2023. More information here: tinyurl.com/2ehj2574. See details on page 6.

Raising Public Awareness (RPA)

The RPA committee of the EMS has new members. Dr Katie Chicot began chairing the committee in January 2023. Katie is the CEO of the charity MathsWorldUK (mathsworlduk.com), which has launched MathsCity, the UK's first hands-on mathematics centre in Leeds (mathscity.co.uk). She is also a Senior Lecturer and Staff Tutor in Mathematics and Statistics at the Open University: both roles involve mathematics outreach. The RPA committee also welcomed new members from around Europe: Can Ozan Ouz (Galatasaray University), Maria de Fátima Rodrigues (FCT-UNL, Lisbon), Constanza Rojas-Molina (Université de Cergy-Pontoise), Olga Paris-Romaskevich (CNRS at Université Aix-Marseille), Sandra Lucente (Università degli Studi di Bari Aldo Moro), Arghir Zarnescu (BCAM, Bilbao).

EMS Magazine

The March 2023 edition of the EMS Magazine is available online, free to read now at euromathsoc.org/magazine/issues/127. Highlights include a message from the new EMS President Jan Philip Solovej.

EMS News prepared by David Chillingworth
LMS/EMS Correspondent

Note: items included in the European Mathematical Society News represent news from the EMS are not necessarily endorsed by the Editorial Board or the LMS.

LMS Grant Schemes

The next closing date for research grant applications (Schemes 1,2,3,4,5,6 and AMMSI) is 15 May 2023. Applications are invited for the following grants to be considered by the Research Grants Committee at its June 2023 meeting. Applicants for LMS Grants should be mathematicians based in the UK, the Isle of Man or the Channel Islands. For grants to support conferences/workshops, the event must be held in the UK, the Isle of Man or the Channel Islands:

Conferences (Scheme 1)

Grants of up to £5,500 are available to provide partial support for conferences. This includes travel, accommodation and subsistence expenses for principal speakers, UK-based research students, participants from Scheme 5 countries and Caring Costs for attendees who have dependents.

Visits to the UK (Scheme 2)

Grants of up to £1,500 are available to provide partial support for a visitor who will give lectures in at least three separate institutions. Awards are made to the host towards the travel, accommodation and subsistence costs of the visitor. Potential applicants should note that it is expected the host institutions will contribute to the costs of the visitor. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents.

Online Lecture Series (Scheme 3)

Grants of up to £1,000 are available per year to provide support to mathematicians, or groups of mathematicians, delivering online lecture series in mathematics. Applications for this element of the Scheme 3 grant is open to both Joint Research Groups (new and current) and to mathematicians who are not part of a Joint Research Group.

Research in Pairs (Scheme 4)

For those mathematicians inviting a collaborator, grants of up to £1,200 are available to support a visit for collaborative research either by the grant holder to another institution abroad, or by a named mathematician from abroad to the home base of the grant holder. For those mathematicians collaborating with another UK-based mathematician, grants of up to £600 are available to support a visit for collaborative research either by the grant holder to another institution or by a named mathematician to the home base of the grant holder. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents.

Research Reboot (Scheme 4)

Grants of up to £1,000 are available to provide support to mathematicians who have found themselves without the time to engage in research due to illness, caring responsibilities, increased teaching or administrative loads or any other factors. The grant offers funding to cover travel, accommodation, subsistence and caring expenses so the applicants can leave their usual environment to focus entirely on research for a period from two days to a week, in order to restart their research activity.

Collaborations with Developing Countries (Scheme 5)

For those mathematicians inviting a collaborator to the UK, grants of up to £3,000 are available to support a visit for collaborative research, by a named mathematician from a country in which mathematics could be considered to be in a disadvantaged position, to the home base of the grant holder. For those mathematicians going to their collaborator's institution, grants of up to £2,000 are available to support a visit for collaborative research by the grant holder to a country in which mathematics could be considered to be in a disadvantaged position. Applicants will be expected to explain in their application why the proposed country fits the circumstances considered eligible for Scheme 5 funding. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents. Contact the Grants team if you are unsure whether the proposed country is eligible, or check the IMU's Commission for Developing Countries definition of developing countries (tinyurl.com/y9dw364o).

Research Workshop Grants (Scheme 6)

Grants of up to £10,000 are available to provide support for Research Workshops. Research Workshops should be an opportunity for a small group of active researchers to work together for a concentrated period on a specialised topic. Applications for Research Workshop Grants can be made at any time but should normally be submitted at least six months before the proposed workshop.

LMS Research Schools and Research School in Knowledge Exchange 2024: Call for Proposals

Extended deadline: 1 June 2023

Grants of up to £15,000 are available to support up to three LMS Research Schools in 2024, including a Research School in Knowledge Exchange, which provide training for research students in all contemporary areas of mathematics. The LMS Research Schools support participation of research students from both the UK and abroad. The lecturers are expected to be international leaders in their field. The LMS Research Schools are often partially funded by the Heilbronn Institute for Mathematical Research (heilbronn.ac.uk).

Information about the submission of proposals can be found at: lms.ac.uk/events/lms-research-schools and lms.ac.uk/events/lms-research-schools-ke along with a list of previously supported Research Schools. Applicants are strongly encouraged to discuss their ideas for Research Schools with the Chair of the Early Career Research Committee, Professor Chris Parker (research.schools@lms.ac.uk) before submitting proposals.

Proposals should be submitted to Lucy Covington (research.schools@lms.ac.uk) by 1 June 2023.

Clay Mathematics Institute Enhancement and Partnership Program

To extend the international reach of the Research School, prospective organisers may also wish to consider applying to the Clay Mathematics Institute (CMI) for additional funding under the CMI's Enhancement and Partnership Program. Further information about this program can be found at: tinyurl.com/y72byonb.

Prospective organisers are advised to discuss applications to this program as early as possible by contacting the CMI President, Nick Woodhouse (president@claymath.org). There is no need to wait for a decision from the LMS on your Research School application before contacting the CMI about funding through this programme.

LMS Early Career Fellowships 2022–23

The LMS Early Career Fellowship is a funding scheme supported jointly by the Heilbronn Institute for Mathematical Research (HIMR) and UKRI/EPSRC, which provides financial support for talented UK mathematicians who have recently completed their PhD and not yet secured their next postdoctoral position. During this transition period the Society, together with its sponsors HIMR and UKRI, offers a stipend of £1,475 per month to support a research visit to a chosen mathematics department of between three and six months, along with additional relocation funding of £800.

These fellowships provide financial support for research and career development and help to foster collaboration and promote the advancement of mathematics as a discipline.

In 2022–23, the Early Career Research Committee considered 30 applications and awarded 8 fellowships. These awards will support selected Early Career Researchers during their collaborations and projects at universities both in the UK and overseas. A full list of current Fellows is available on the Society's website at tinyurl.com/47y8cysk.

Visit our website tinyurl.com/2s399vc3 to read the glowing testimonials from previous grant holders and learn about the impact of this support.

For the next round, which will be also supported by HIMR and UKRI, applications will open later in 2023 with a deadline of 14 January 2024 to support fellowships starting from April 2024. For full information about the scheme and how to apply, visit the Society website (tinyurl.com/3ar7h38e) or direct your enquiries to fellowships@lms.ac.uk.

LMS Invited Lectures Series 2024: Call for Proposals

Deadline: 15 May 2023

Proposals are invited from members and their departments to host the next LMS Invited Lectures Series in 2024. This lecture series consists of meetings held in the UK at which a single speaker gives a course of about ten expository lectures, examining some subject in depth, over a five-day period (Monday to Friday) during a University vacation. The meetings are residential and open to all interested. Funding of up to £6,000 is offered to the host department to support the Invited Lecturer's costs and participants' attendance at the lectures.

Proposals for the Invited Lectures 2024

Any member who would like to suggest a topic and lecturer and be prepared to organise the meeting at their own institution or a suitable conference centre can submit a proposal. For further details, please visit the Society's website: lms.ac.uk/events/lectures/invited-lectures#IL%20Call.

The deadline for proposals is 15 May 2023.

LMS Invited Lecturer 2023

The LMS Invited Lecture Series 2023 on Optimal Transport and its Applications will be given by Professor Filippo Santambrogio (Université Lyon 1) at Durham University from 17-21 July 2023. Further details available here: lms.ac.uk/events/lectures/invited-lectures#IL%20Call.

Recent previous Invited Lecturers:

- 2022 (postponed from 2021) Olga Kharlampovich (CUNY Graduate Center and Hunter College) *Equations in Groups and Complexity,* Newcastle University from 18–22 July 2022.
- 2022: Gitta Kutyniok (Munich) *The Mathematics of Deep Learning,* Cambridge University from 28 February to 4 March 2022.
- 2020: Yulia Mishura (University of Kyiv) *Fractional Calculus and Fractional Stochastic Calculus, including Rough-Paths, with Applications,* Zoom via Brunel University, 15–19 June 2020.
- 2019: Søren Asmussen (Aarhus University), ICMS in Edinburgh, 20–24 May 2019.

Enquiries about the Invited Lectures may be addressed to the Chair of the Society Lectures and Meetings Committee; Brita Nucinkis (lmsmeetings@lms.ac.uk).

Call for Proposals for LMS–Bath Mathematical Symposia in 2025

Deadline: 15 December 2023

Proposals are invited for the LMS-Bath Mathematical Symposia to be held at the University of Bath in 2025. Two symposia will be held, funded by either the Isaac Newton Institute or the International Centre for Mathematical Sciences. Core funding at approximately £40,000.00 is available to support each symposium.

Founded in 1974, the LMS Mathematical Symposia is an established series of international research meetings, which provides excellent opportunities to explore an area of research in depth, learn of new developments, and instigate links between different branches of mathematics.

The format is designed to allow substantial time for interaction and research. The meetings are by invitation only and will be held in July/August, with up to 50 participants, roughly half of whom will come from the UK. A novel element of the symposia is that they will be complemented by a summer school to prepare young researchers such as PhD students, or a 'research incubator', where problems related to the topic of the conference is studied in groups. These entire events, summer school/incubator and workshop, will typically last around two weeks.

Prospective organisers should send a formal proposal to the LMS Grants Team (grants@lms.ac.uk) by 15 December 2023. Proposals are approved by the Society's Research Grants Committee after consideration of referees' reports.

Proposals should include:

- A full list of proposed participants, divided into specific categories:
  - Category A – Scientific Organisers
  - Category B – Key Overseas Participants
  - Category C – Key UK-based Participants
  - Category D – Important Overseas Participants
  - Category E – Important UK-based Participants
- Proposers are encouraged to actively seek to include women speakers and speakers from ethnic minorities or explain why this is not possible or appropriate.
- A detailed scientific case for the symposium, which shows the topic is active and gives reasons why UK mathematics would benefit from a symposium on the proposed dates.
- Details of additional support from other funding bodies, or proposed avenues of available funding.
- Indicative plans for the summer school or research incubator.

- Where appropriate, prospective organisers should consider the possibility of an 'industry day'.

For further details about the LMS Mathematical Symposia, visit the Society's website: www.lms.ac.uk/events/mathematical-symposia or the LMS-Bath symposia's website: bathsymposium.ac.uk

Before submitting, organisers are welcome to discuss informally their ideas with the Chair of the Research Grants Committee, Professor Andrew Dancer: (grants@lms.ac.uk).

Call for Institutions to Host the LMS Mathematical Symposia from 2026 to 2030

Deadline: 1 December 2023

Proposals from UK-based institutions or consortia are invited to host the LMS Mathematical Symposia from 2026 to 2030.

The LMS Mathematical Symposia were historically held at the University of Durham, and since 2020 have been held at the University of Bath. To ensure that all UK institutions have the opportunity to benefit from hosting, the Society encourages applications from universities or consortia of universities that have not traditionally hosted the events.

Responsibilities of Host Institutions

Host institutions will be responsible for:

- Hosting the LMS Mathematical Symposia for a period of five years, with the first event to be held in July/August 2026. Traditionally, two to three meetings are held each July/August.
- Providing the infrastructure to host the Symposia i.e. lecture theatres, provision for online attendance, accommodation, catering and support for those with caring responsibilities.
- Producing the format of the Symposia so that each last two weeks to allow substantial time for interaction and research. Previous models are listed below but prospective hosts are welcome to suggest new models (e.g. a partnership between a place with experience of hosting mathematical events and a place with the infrastructure could help to expand mathematics in a particular area):
  - Meetings by invitation only and held in July/August each year, lasting 1-2 weeks, with 50-70 participants, roughly half of whom will come from the UK.
  - Meetings by invitation only and held in July/August, usually lasting for one week, with up to 50 participants, roughly half of whom will come from the UK and complemented
    - Either by a summer school, to prepare young researchers such as PhD students,
    - or a "research incubator", where problems related to the topic of the conference are studied in groups.
    - These events can take up to an additional week.
- Budgeting and securing funding to host the Symposia; the events typically cost c £40,000 each and a small amount of funding (£3–5,000) can be applied for from the LMS via the Scheme 6 (Workshops) Grant Scheme by the organisers of each individual Symposium held at the Host Institution.
- Working with the LMS Research Grants Committee and any external funders to solicit and assess proposals to organise LMS Mathematical Symposia at the Host Institution; this process usually begins up to 18-24 months prior to the first event being held. Typically, this would involve a member of staff from the Host Institution joining the LMS Research Grants Committee as the Symposia Rep from 2024 to 2030 and attending their meetings up to four times per year.

Proposals

Proposals should include:

(1) A demonstration of the following:

- A strong mathematical case for the Symposia to be held at the applicants' department, which highlights current expertise and/or how the department would use it to build human capital, that is, how the Symposia would be transformative for the department.
- A commitment to host at least two LMS Mathematical Symposia per year from 2026-2030 and to work with the Research Grants Committee from 2024 to solicit and assess proposals from prospective organisers.

- A commitment to equality, diversity, inclusivity and accessibility across the community; i.e. amongst speakers, participants, and organisers.
- A commitment to sustainability goals, e.g. provision for hybrid meetings including how an online component would be supported.

(2) An outline of the proposed formats for the Symposia events.

(3) An outline of a typical budget per Symposium and plans for securing funding to run at least two Symposia per year over a five-year period, including likely sources and timescales required for applications.

(4) The names and short CVs (max three pages each) of the team at the Host Institution who will be supporting Organisers in running the LMS Mathematical Symposia at the Host Institution. The CVs should include examples of other events they have run. One team member should be nominated to join the LMS Research Grants Committee as the Symposia Rep in 2024. (Ideally, the same Symposia Rep would remain with the Committee from 2024-2030 but the Society understands that teams can change and a change of Symposia Rep in this period would be fine).

(5) A description of the proposed location and infrastructure i.e. lecture theatres, provision for online attendance, accommodation, catering and support for those with caring responsibilities.

(6) The details of the administrative arrangements, including arrangements for dealing with any relevant visa or work-permit issues.

For further details about the LMS Mathematical Symposia in its current iteration, please visit the Society's website: tinyurl.com/mathsymposia.

Proposals should be submitted by 1 December 2023 to grants@lms.ac.uk.

Before submitting, organisers are welcome to discuss informally their ideas with the Chair of the Research Grants Committee, or the LMS Grants and Membership Administrator (grants@lms.ac.uk).

LMS Council Diary — A Personal View

In an effort to contribute to a greener environment, the LMS Council decided last year to hold every other Council meeting exclusively online. This was the case on 3 February 2023. Most Council members were in 'remote' attendance and were joined by the LMS President-Elect Jens Marklof, Professor of Mathematical Physics and Dean of the Faculty of Science at the University of Bristol.

The meeting opened as usual with a verbal report by the President on the activities she participated in since the previous Council meeting in November 2022. One highlight was her participation in the 11th Abel Committee Symposium on 12 January in Cambridge, held in the presence of the Ambassador of Norway. Talks were delivered by James Maynard (Oxford), Sylvia Serfaty (Courant Institute of Mathematical Sciences), Parimala Raman (Emory College) and Dennis Sullivan (Stony Brook). Another highlight was the LMS South-West and South Wales Regional meeting in Southampton.

Several items on the Agenda deserve notice. It is pleasing to report that Council supports the application of the Mongolian Mathematical Society to become a full member of the IMU. The LMS was invited by the IMU to show support as Adhering Organisation. On another front, Council was briefed by Vice-President Iain Gordon on progress made by the working group he chairs, and which is tasked to monitor and respond to the developing work of the proto-Academy for the Mathematical Sciences, while also considering the LMS role and interests as the Academy develops. Council members were presented with the first quarterly update prepared for the Council of Mathematical Sciences (CMS) by the Chair of the Executive Committee of the Academy and delivered in December 2022. Three points to note are that the proto-Academy now has a live website at acadmathsci.org.uk, that a consultation with the Mathematical Sciences community is planned in April/May 2023 and that there is intention to seek a Go/no-Go decision from the CMS by October 2024.

The Society's Executive Secretary, Simon Edwards, updated Council on the Levelling Up programme for A-level students from under-represented backgrounds. The scheme is sponsored by Dr Tony Hill and is transitioning from the pilot phase to a maturity phase, broadening the spectrum of subjects tutored by undergraduate students and bringing the tutorials online. The LMS will focus on mathematics in the future and monitor how the material we have developed is used.

Other items discussed feature elsewhere in recent LMS Newsletters, such as the open letter to the Vice-Chancellor of Birkbeck, University of London, expressing serious concern about proposed cuts to the Mathematics and Statistics department. The letter was co-signed by the presidents of the LMS, Royal Statistical Society, Institute of Mathematics and its Applications and Edinburgh Mathematical Society.

Council received a report from the LMS Scrutineers, Professors Charles Goldie and Chris Lance, on the 2022 LMS elections to Council and Nominating Committee. Chris Lance has now concluded his term as a Scrutineer and Council joined Professor Goldie in thanking Chris for his longstanding service. There was also an opportunity to thank Professor Brita Nucinkis and Professor Anne-Christine Davis who stepped down as Members-at-Large of Council at the Annual General Meeting on 18 November 2022. Their work on LMS Council was praised.

Other business included various committee reports and memberships.

The meeting concluded in good time, with a reminder that the next Council meeting would take place in person on 21 April, within the framework of Council's Strategic Retreat 21–22 April, where the focus will be on the Society's ambitions and budget.

Anne Taormina
Member-at-Large

Maximising your LMS Membership: Books Discount

The LMS has published world-class mathematical texts since its founding in 1865. The Society now publishes 12 peer-reviewed journals, two book series, and a number of individual book titles in collaboration with different partners. All journal publications include high quality, well-written articles which appeal to a broad audience. The Society's members can receive a 25% discount on books in the following two series when they are purchased directly from LMS publishing partners:

- The LMS Lecture Notes series was founded in 1968 and has become an established and valuable source of information for mathematicians and research professionals. Most of the volumes are short monographs written in an informal style, which present an overview of current knowledge in an advanced topic and provide a convenient path to understanding recent developments across a wide range of mathematical topics. The series also covers conference proceedings and similar collective works that meet its general objectives.
- The LMS Student Texts series was introduced in 1983 to complement the LMS Lecture Notes and is designed for undergraduates or beginner graduate students. The textbooks cover the full range of pure mathematics, as well as topics in applied mathematics and mathematical physics involving substantial use of modern mathematical methods. The series allows non-specialists and students with some background and knowledge in specific topics to get to grips with the subject.

All publications from these book series are available to individual members at a discounted price when ordered directly from the Cambridge University Press website. Visit bit.ly/LMSmember for more information.

In addition, if your institution has purchased electronic editions of titles in the series, you can get free online access to it. This will be indicated by the word 'Access' (in green) and a tick mark next to these titles on the Cambridge Core platform.

The discount codes and other exciting news about the Society's activities and events are sent to all members in our monthly eUpdates. Please visit our website lms.ac.uk/publications for more information about our publications or contact us on membership@lms.ac.uk if you have any other queries.

Elizabeth Fisher
Membership & Grants Manager

Upcoming LMS Early Career Events

LMS Research Schools 2023

Adaptive Methods and Model Reduction

The Jubilee Campus Hotel and Conferences, Nottingham, 15–19 May 2023. The lecturers and plenary speakers are:

- Olga Mula (Université Paris-Dauphine)
- Simona Perotto (Politecnico di Milano)
- Serge Prudhomme (École Polytechnique de Montréal)
- Gianluigi Rozza (Scuola Internazionale Superiore di Studi Avanzati) (SISSA)
- Zhiqiang Cat (Purdue University)
- Barbara Wohlmuth (Technische Universität München)

Random Structures, Applied Probability and Computation

Liverpool, 26–30 June 2023. The three main lecture course topics are:

- Pólya urns and other reinforced processes (Cécile Mailler, University of Bath)
- Computational methods for Lévy models (Søren Asmussen, Aarhus Universitet)
- Self-similar Markov trees (Jean Bertoin, Universität Zürich)

Machine Learning in Mathematics and Theoretical Physics

Oxford, 17–21 July 2023. The three main lecture course topics are:

- Introduction to Machine Learning (Fabian Ruehle, Northeastern University & Magdalena Larfors, Uppsala University)

- Discrete Optimisation Methods (Steve Abel, Durham University & Andre Lukas, University of Oxford)
- Machine Learning in Pure Mathematics (Alexander Kasprzyk, Nottingham University & Lara Anderson, Virginia Tech)

Algebraic Groups and their Representations
Birmingham, 24–28 July 2023. The three main lecture course topics are:

- Categorical approaches to representation theory (Jonathan Brundan, University of Oregon)
- Geometry arising from algebraic groups (Martina Lanini, Università degli Studi Roma Tor Vergata)
- Representations of reductive algebraic groups (Simon Riche, Université Clermont Auvergne)

For more information and links to the events visit: lms.ac.uk/events/lms-research-schools.

LMS Undergraduate Summer School 2023
University of Sheffield, 16–28 July 2023
Course Lecturers:

- Amanda Turner (University of Leeds)
- Carmen Molina-Paris (University of Leeds)
- Lewis Combes (Dr Haluk Sengun support — University of Sheffield)
- Alex Schenkel (University of Nottingham)
- Milena Hering (University of Edinburgh)
- Oliver Johnson (University of Bristol)
- Nic Freeman (University of Sheffield)

Colloquium Speakers:

- Vandita Patel (University of Manchester)
- Tyler Kelly (University of Birmingham)
- Jill Johnson (University of Sheffield)
- Corina Constantinescu (University of Liverpool)
- Silke Weinfurtner (University of Nottingham)

The Summer Schools are co-funded with Heilbronn and UKRI. For more information visit: lms.ac.uk/events/lms-summer-schools.

Membership of the London Mathematical Society

The standing and usefulness of the Society depends upon the support of a strong membership, to provide the resources, expertise and participation in the running of the Society to support its many activities in publishing, grant-giving, conferences, public policy, influencing government, and mathematics education in schools. The Society's Council therefore hopes that all mathematicians on the staff of UK universities and other similar institutions will support mathematical research by joining the Society. It also very much encourages applications from mathematicians of comparable standing who are working or have worked in other occupations.

Benefits of LMS membership include access to the Verblunsky Members' Room, free online subscription to the Society's three main journals and complimentary use of the Society's Library at UCL, among other LMS member benefits (lms.ac.uk/membership/member-benefits).

If current members know of friends or colleagues who would like to join the Society, please do encourage them to complete the online application form (lms.ac.uk/membership/online-application).

Contact membership@lms.ac.uk for advice on becoming an LMS member.

LEVELLING UP

This column includes the latest updates about the Levelling Up: Maths scheme being developed by the LMS, made possible by a generous donation from Dr Tony Hill and Mr Simon Goodwin. The scheme seeks to widen participation of those who are under-represented in mathematics. It is part of a broader Levelling Up: STEM project which also covers Physics and Chemistry.

Starting from this year, four universities will be delivering Levelling Up: Maths (LU:M) for Black Heritage Students. They are the University of Kent, Queen Mary University London, and Warwick and Coventry Universities working together to jointly deliver LU:M.

Other universities working in partnership on LU:M include the University of Salford, Manchester Metropolitan University and the University of Manchester. These three universities have co-ordinated joint training for university mathematics students who want to be LU:M tutors. One of the aims of LU:M is to provide mathematics mentoring by university students who have similar backgrounds to those of the LU:M cohort.

A new Levelling Up: Maths for Computer Science is about to be launched by the British Computer Society (BCS). The BCS will use some of the existing Levelling Up: Maths content in conjunction with their own computer science educational materials.

The LMS and IMA brought together all the current LU:M universities, as well as universities interested in joining the different mathematics streams, for a sharing and learning session on 15 March. The community session was attended by 13 universities and by Julia Goedecke who designed the Moodle learning materials. The session provided an opportunity to hear from universities such as Durham, which is in its third year of running the scheme, and QMUL, where the scheme has recently started.

Jennifer Gunn
LMS Head of Society Business

Records of Proceedings at LMS meetings
Ordinary Meeting: Midlands Regional Meeting 2023

held on 27 March 2023 at the Lecture theatre MS.01, Warwick Mathematics Institute, University of Warwick, as part of the Midlands Regional Meeting & Workshop on *Ergodic Theory of Group Extensions*. Over 40 members and guests were present for all or part of the meeting.

The meeting began at 1.15pm with Vice-President Professor Cathy Hobbs, in the Chair.

There were 41 members elected to Membership at this Society Meeting.

Seven members signed the Members' Book and were admitted to the Society.

Professor Richard Sharp, University of Warwick, introduced the first lecture given by Professor Viveka Erlandsson (University of Bristol) on *Counting geodesics on surfaces*.

After a break, Professor Richard Sharp introduced the second lecture by Professor Mark Pollicott (University of Warwick) on *Counting geodesic arcs on surfaces with no conjugate points*.

After tea, Professor Richard Sharp introduced the third lecture by Professor Michael Magee (Durham University) on *Spectral properties of hyperbolic surfaces*.

Professor Hobbs thanked the speakers for their excellent lectures and then expressed the thanks of the Society to the organiser, Professor Richard Sharp (University of Warwick), for a wonderful meeting and workshop.

Afterwards, a wine reception was held at Mathematics Institute. The Society dinner was held at the Scarman Conference Centre.

Practical Mathematics and Latin eloquence: *De Arte Supputandi*, the First Mathematics Book Printed in England

ALISZ REED AND DEBORAH KENT

Now half-a-millennium old, Cuthbert Tunstall's 1522 work *De arte supputandi libri quattuor* (Four books on the art of reckoning) is the first mathematical book printed in England. Augustus De Morgan described this guide to mercantile mathematics as "the most classical which was ever written on the subject in Latin".

The year 2022 marked an impressive five centuries of mathematical book printing in England. *De arte supputandi libri quattuor* (Four books on the art of reckoning) was first published in London in 1522 by Robert Pynson, a leading printer at the time, who was appointed King's Printer to both Henry VII and Henry VIII.

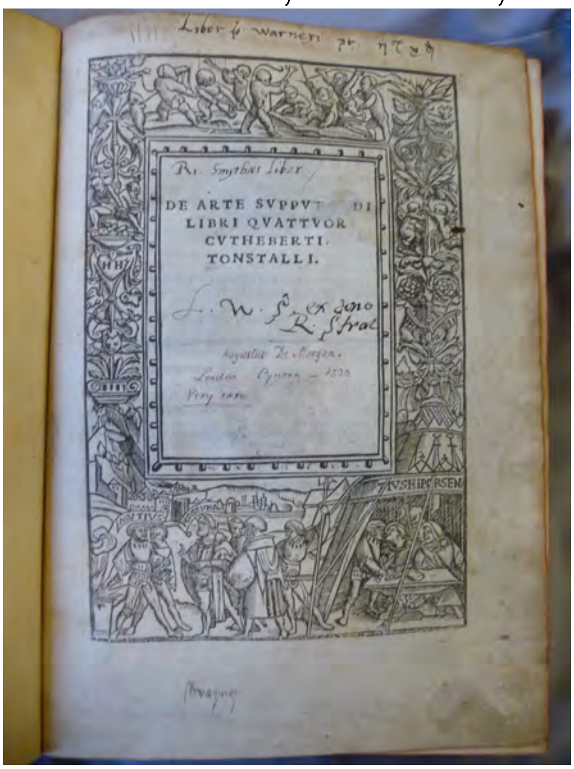


Figure 1: The frontispiece of *De arte supputandi libri quattuor* features a woodcut by Hans Holbein. This copy belonged to Augustus DeMorgan. It is currently held at Senate House Library and reproduced with permission.

In fact, as was common practice, Pynson recycled a frontispiece for *De arte supputandi*. The wood-block border depicting Scaevola and Porsenna is an English copy of a design Hans Holbein originally made for the printer Froben in Basel. The perhaps unlikely author of this mathematical text was Cuthbert Tunstall. Tunstall was born in Yorkshire in 1474. As a young man, he spent some time in the 1490s at both Oxford and Cambridge, although left each without taking a degree. Tunstall studied Canon

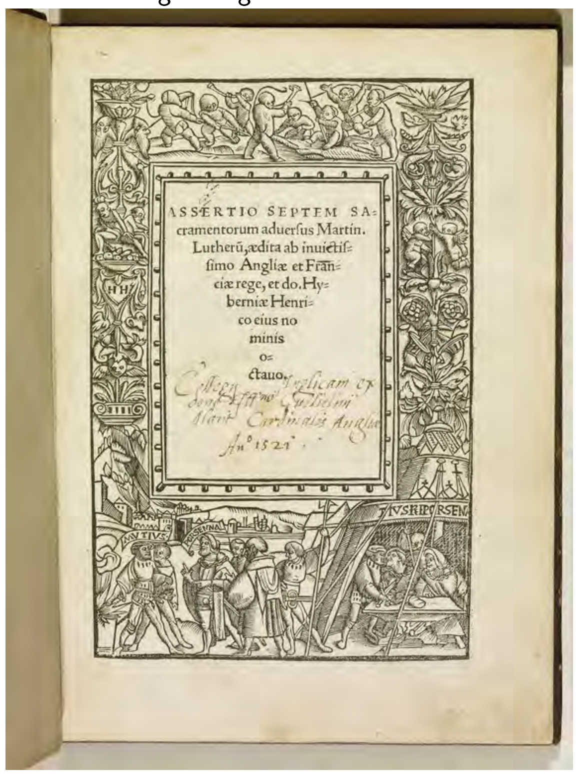


Figure 2: The frontispiece for Henry VIII's *Assertio septem sacramentorum* (or *Defence of the seven sacraments*), which was also printed by Richard Pynson in 1521 and 1522. Courtesy of the Royal Collection Trust.

and Roman law from 1499-1505 at the University of Padua, where he earned two degrees and a reputation as an excellent scholar in Greek and Latin. He then returned to England and served as rector in various parishes and eventually was ordained a priest in 1511. Tunstall became Archdeacon of Chester in 1515. That same year, he participated in a delegation to negotiate treaties with the Duke of Burgandy, which bolstered his reputation as a skillful diplomat. He served as an ambassador to the court of Charles V, attending his coronation in Germany in 1520 and the next year accompanied the monarch to the Diet of Worms. In 1522, Tunstall not only became the Bishop of London, but also wrote *De arte supputandi*, the first printed work devoted exclusively to mathematics that was published in England.

Tunstall went on to become Prince-Bishop of Durham in 1530 and officiated at the coronation of Edward VI in February 1547. As a conservative in the time of Reformation, he was charged with treason and imprisoned in 1550, first under house arrest and later in the Tower of London. Tunstall was convicted of felony and stripped of his bishopric in 1552, then reinstated by Mary Tudor in 1553, but it was not to last. He refused to swear the oath of supremacy under Elizabeth and was re-deprived of the bishopric in 1559.

The text itself is a slim book, just over 200 pages in length, split into four sections. The first of these discusses the basic operations – addition,

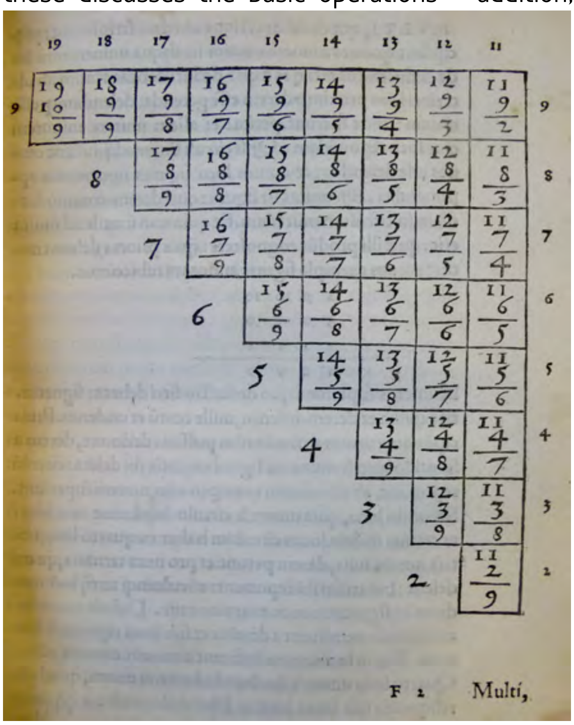


Figure 3: A table of subtraction from *De arte supputandi libri quattuor*.

subtraction, multiplication and division – as applied to integers.

The second section revisits these operations when working with fractions. Practical exercises and word problems set in mercantile contexts make up the third section and finally the fourth section explores ratios. The book presupposes little prior knowledge, beginning with an introduction of the ten primary numbers and their Latin names and symbols.

However, the text moves rapidly on from this starting point. Tunstall claims that addition is a skill a reader can become proficient at "in one hour, if only he is willing to pay attention" (Book 1, On Addition). By the end of the first section, only about 50 pages after introducing the integers, Tunstall is already tackling how to find the cube root of large numbers.

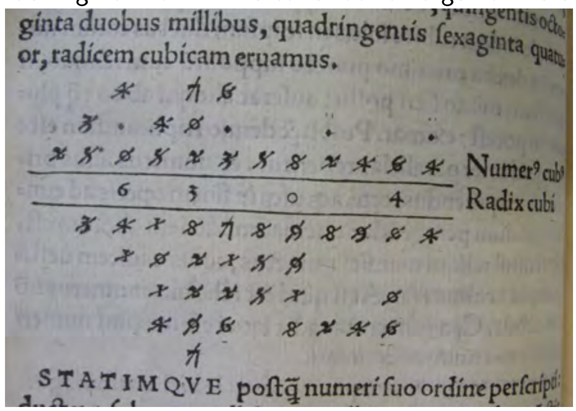


Figure 4: A tableau demonstrating the manual extraction of 6304 as the cube root of 250523582464.

A letter to Thomas More that forms part of the preface of *De arte supputandi libri quattuor* explains that aversion to the trickery of money changers is what motivated Tunstall's mathematical learning. Only the letter has been translated [3] (other translations in this text are by the authors). The letter makes explicit Tunstall's claims that the book's aim is for merchants, students and "the Everyman" to have the "art of reckoning sufficiently at [their] command to be too wide-awake to be tricked by any and every swindler"[3]. As such, it draws on previous work to provide a comprehensive guide and the exercises needed to become practiced in the arithmetic of money-based scenarios. Tunstall further insists that for "those who devote themselves either to business or to leisure, nothing will be more necessary than to keep ready the art of counting" (Preface).

Intriguingly, despite this stated audience and practical focus, *De arte supputandi* clings tightly to the tradition of academic Latin – intending to be as aesthetically pleasing as it is utilitarian. Whether the text was successful in its aims is far less definite, with the reception differing widely between England and continental Europe. A closer look at its marginalia and the context surrounding its publication gives insight into the many facets and the successes and failures of this ambitious text.

While the text's focus on mercantile mathematics may seem mundane to the modern reader, the topic was surprisingly unusual for 16th century England. Trade had long been an important part of the economy and had involved monetary exchanges of gold coinage since its introduction in 1343 [4]. However, unlike Italy, with its long-standing tradition of *scuola d'abbaci* – schools dedicated to teaching commercial arithmetic to aspiring merchants, accountants and clerks – the education in England in this regard was far more rudimentary [4]. In part, this may have been the case because of a slower transition from Roman to Arabic numerals. Alternatively, it may have simply been that the roles of mint masters and those dealing with the more complicated arithmetic were dominated, even in England, by Italians, and English merchants did not see the need for extensive mathematical education for the comparably simple transactions they were conducting. It was not until the end of the 15th century that individual instances of English merchants educating themselves in Italian-commercial arithmetic started to crop up [4].

De arte supputandi emerged in the context of this growing engagement with mercantile mathematics and much of its content draws on Italian and other continental writing. The text never claims to contain original work, rather, as Tunstall explains in the Preface, "from many things in many books [he] selected a number" [3]. Particularly influential in that regard was Luca Pacioli's 1494 *Summa de arithmetica*. In *De arte supputandi*, Tunstall lauds Pacioli as a mathematician "whose name is celebrated in the art not without reason" (Book 2, On Finding Proper Roots in Numbers neither Squares nor Cubes). Tunstall also takes inspiration from French sources. Here, Guillaume Bude's 1515 *De asse et partibus eius*, or *The whole and its fractions*, his own copy of which he heavily annotated, seems to have been a significant source, along with its abbreviated accompanying text the *Brevarium* [1]. This is seen in part in similar phrasing between the works, as well as a direct reference to the text, complemented by a description of Bude as "one of the finest learned men of our age" (Book 1, On Numbering).

Different "first" editions

Even though copies of *De arte supputandi* dated 1522 are often referred to as **the** first edition, there are at least three different type settings evident so far in individual copies of *De arte supputandi*. Below are two different presentations of Question 21, each in a 1522 printing of the book. The compositor, likely collaborating with Tunstall, worked within the constraint of two typeset pages to reformat the problem and solution. Careful study of these pages using bibliographic methods may provide insight into the development of early mathematical typesetting.

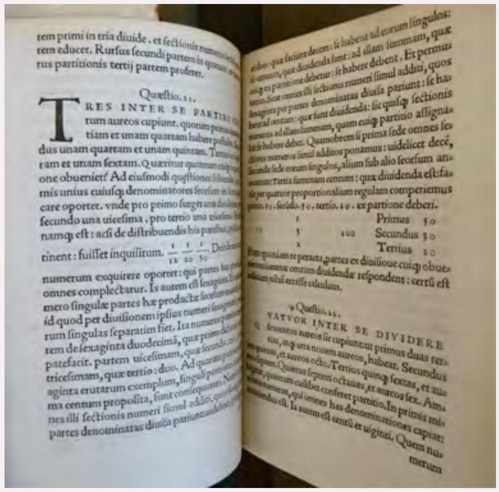


Figure 5: A 1522 copy at Jesus College, Oxford.

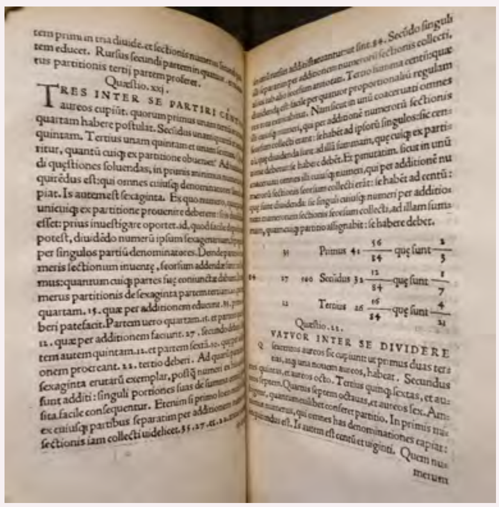


Figure 6: A 1522 copy at The British Library.

Considering this information, we can view *De arte supputandi* as an attempt to fill the previously discussed void of knowledge by amalgamating and condensing the rich mathematical knowledge of Europe in a way that was accessible to the English public, and especially the English merchant.

This book then has a curiously humanitarian aim, and the importance of this mission is perhaps indicated by the previously discussed unexpected nature of its author. Cuthbert Tunstall was not by specialty a mathematician, although a respected scholar, he was and remains better known as the Bishop of London and later of Durham. Indeed, the preface suggests that he often found the work laborious and difficult, at times "overcome by weariness", as well as "hopeless of being able to carry out what [he] had proposed", and yet he finished the project out of a sense of duty to its goal [3].

While facilitating mathematical education remains an ambitious objective in itself, a secondary motivation makes the text even more fascinating. Practicality on its own was not sufficient for Tunstall, rather he sought elegance, a motivating factor in his choice to write the text in Latin, even as this practice was rapidly falling out of favour. In the preface he describes other languages as "barbaric" and laments the difficulty of eloquently expressing mathematics, as though he is trying to make an unsophisticated subject simultaneously accessible and refined. Travis Williams, a literary scholar at the University of Rhode Island, posits that Tunstall thus questions the very elements that make a mathematical text valuable, engaging with mathematical reading as an aesthetic experience that extends beyond the useful and exploring the idea that a maths book can simply be a "good read" [5].

With this context and these lofty goals in mind, the question remains whether the text was indeed a revolutionary piece of work that both tastefully and accessibly filled the English knowledge-gap in mercantile mathematics, or whether it was an overly ambitious project by a man outside his field, rightfully lost to the crevices of history.

Despite the elegance of much of the language, the book retains a distinctly practical flavour, with worked examples, as well as exercises for the reader. Mostly the exercises deal with various transactions of gold coins in a number of scenarios.

The content then certainly shows promise of being a useful educational tool. However, the book was never

Working out of Question 21

Figure 7: Annotations including work for Question 21 from the copy owned by Augustus De Morgan, held by Senate House Library. This copy has XXX marks on the printed text, perhaps indicating requested corrections to the typesetting.

They want to share three hundred gold coins with each other such that the first person has one-third plus one-fourth of the coins. The second person has one-fourth plus one-fifth. The third person has one-fifth plus one-sixth. How many coins does each person get?

1. Adjust the fractions so they have the same denominator of 60 and add each persons fractions together.

$$\frac{1}{3}+\frac{1}{4}=\frac{20}{60}+\frac{15}{60}=\frac{35}{60}$$
$$\frac{1}{4}+\frac{1}{5}=\frac{15}{60}+\frac{12}{60}=\frac{27}{60}$$
$$\frac{1}{5}+\frac{1}{6}=\frac{12}{60}+\frac{10}{60}=\frac{22}{60}$$

2. Add the numerators together.

$$35+27+22=84$$

3. Adjust the ratio so it is out of 100 instead of 84.

$$35\div 84\times 100=41\tfrac{2}{3}$$
$$27\div 84\times 100=32\tfrac{1}{7}$$
$$22\div 84\times 100=26\tfrac{4}{21}$$

4. Multiply by 3 to get the correct proportions for 300 coins

$$41\tfrac{2}{3}\times 3=125$$
$$32\tfrac{1}{7}\times 3=96\tfrac{3}{7}$$
$$26\tfrac{4}{21}\times 3=78\tfrac{4}{7}$$

reprinted in England and there is no conclusive evidence that the text had any significant effect on improving the mathematical knowledge of English merchants or the more general population. We do know that it became a recommended text for Cambridge in 1549 [4].

Additionally, it formed part of the libraries of a number of Oxbridge graduates. Notably including Sir Walter Mildway, chancellor to Queen Elizabeth I, whose use of the text is evidenced by the annotations in his own copy [4]. Furthermore, the scholar and diplomat Sir Thomas Smith found academic use for it around 1562 while researching the wages paid to Roman soldiers. The mathematician John Dee is also known to have owned the book. Many years later,

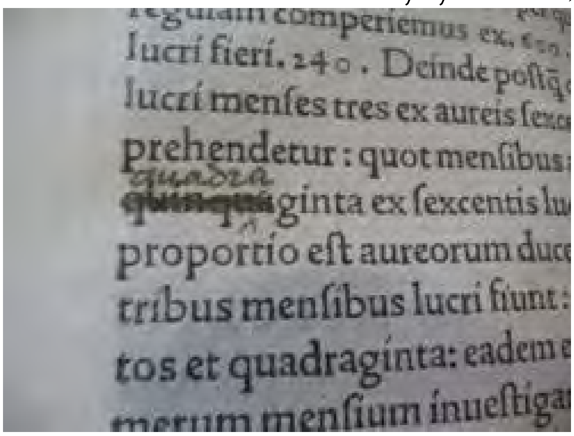


Figure 8: An example of a reader's correction of the text.

in 1847, Augustus De Morgan would describe it as "the most classical which was ever written on the subject in Latin, both in purity of style and goodness of matter [...] For plain common sense, well expressed, Tunstall's book has been rarely surpassed, and never in the subject of which it treats" [2]. What is notable about these names, is that they are not merchants, but rather well-educated men, who almost certainly, like De Morgan, had a greater appreciation for the Latin language of the text. A case can be made that the text was too academic for the average merchant and that the language presented a barrier to "the Everyman". Alternatively, it could be argued that these are simply the names that are remembered because books from their libraries are more likely to have been preserved than a merchant's copy might have been.

The traces of many other readers remain in the fascinating marginalia of the copies we have today. There has not yet been a complete census of known copies of the first print run of *De arte supputandi libri quattuor*. To date, the authors have looked at 15 first-editions around the UK. These texts feature a range of manuscript commentary, including minor edits and corrections to the lengthy working out of problems, crammed around the borders of the pages. Many other mysteries remain tied into the marginalia. For example, one book contains a beautifully drawn multiplication triangle whose connection to the text remains uncertain.

Questions of provenance are difficult to answer, so the authors of these marginal comments remain almost entirely unknown. They style of the handwriting in many cases suggests that they were contemporaries of Tunstall. One copy held at the Bodelian Library likely belonged to John Wallis, Savilian Professor of Geometry at Oxford from 1649-1703.

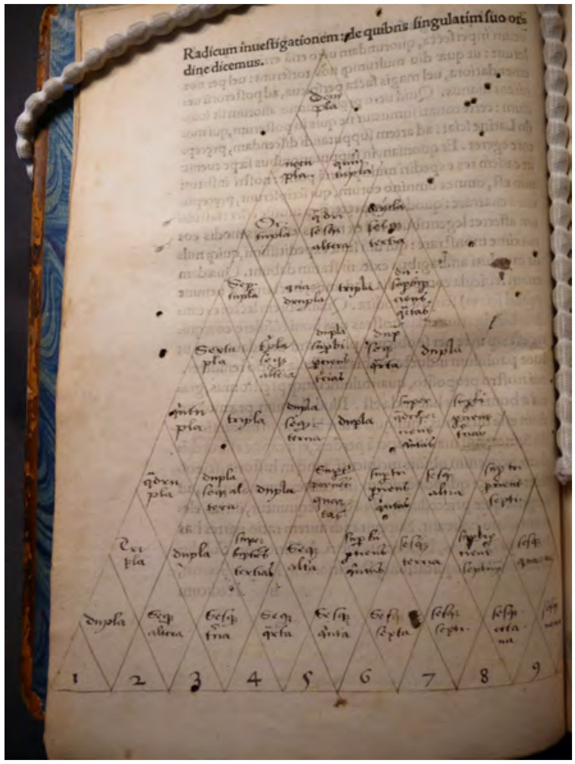


Figure 9: A curious scaling triangle in a 1522 copy of *De Arte Supputandi* at The British Library.

While more in-depth research and palaeography is needed to determine more information about them, the wide range of annotations suggests that readers were engaging with the book by working out its exercises, possibly for an either practical or aesthetic problem-solving experience.

While today we celebrate the text's significance to English mathematical book printing, during the 16th century *De arte supputandi libri quattuor* seems to have had far wider acclaim on the continent, where other similar books already existed. Well-established printer Robert Estienne published three separate editions of Tunstall's book in Paris between the years of 1529 and 1538. Although more research is needed to understand sixteenth-century readership and usage for Tunstall's text, insight into the extent of the text's popularity in France is perhaps given by the fact that three years after its initial publication in Paris, a reference to Tunstall made it into Francois Rabelais' famous Gargantua and Pantagruel pentalogy. Here, a person's mathematical prowess is illustrated by drawing comparison to Tunstall. A year later, in Basel, Erasmus dedicated an edition of Euclid's *Elements* to Tunstall in recognition of his significance to the mathematical world.

These facts suggest a lively market and wide renown for Tunstall's work in Europe. A further four editions were printed in Strasbourg between 1543 and 1551, underscoring ongoing interest.

Curiously, the very feature that may have made it less popular in England – the Latin language of the text – may have been a contributing factor to its continental popularity. So, for example, John Sturm, the editor of the Strasbourg edition, acknowledges that while the content of the book is not unique, Tunstall "instructs us in learned, clear Latin, which the rest do not" [3]. Although this text went through multiple printings and enjoyed some sixteenth-century popularity, it has not received much scholarly attention since. The recent 500th anniversary has generated a resurgence of interest in Tunstall's mathematical work that has generated rich questions for further investigation.

The history of this book as an object, for example, raises many queries about the circulation and transmission of the text, as well as about discrepancies in typesetting. Additionally, the extensive marginalia invites investigation of reader engagement with *De arte supputandi*. Further, the practical purpose of the book – to protect its readers against swindling – is called into question not only by the value placed on its Latin language, but also by the often impractical quantities and situations that underlie the facade of real-world problems in the book. Is this then perhaps a work of recreational reading? Amidst these open questions there emerges an enterprising text that was valued in a multitude of ways by a variety of readers both in England and on the continent. A book whose unique amalgamation of features and open mysteries make it a fascinating topic for further research.

FURTHER READING

[1] A. Burnett, *Bude's "Breviarium": Authorship, date and purpose*, Journal of the Warburg and Courtauld Institutes, 80 (2017), 101–126.
[2] A. De Morgan, *Arithmetical books from the invention of printing to the present time: being brief notices of a large number of works drawn up from actual inspection*, Taylor and Walton, London, 1847.
[3] C. Sturge, *Cuthbert Tunstall, Churchman, Scholar, Statesman, Administrator*. Longman's, Green, and Company, New York, 1938.
[4] J. Williams, *Mathematics and the alloying of coinage 1202–1700: Part ii*, Annals of Science, 52 (3) (1995), 235–263.
[5] T.D. Williams, "Procrustean marxism and subjective rigor: Early modern arithmetic and its readers", in: *Raw Data Is an Oxymoron*. MIT Press, Cambridge, 2013, 41–59.

Alisz Reed

Alisz is a recent graduate of the University of St Andrews in English and mathematics. Her research interests include graph theory, the history of maths, and mathematics in literature. When she is not lost in an anthology of mathematical love poetry, she's probably trying to prove that spinning at a ceilidh is immune to the speed limits of the universe.

Deborah Kent

Deborah is a senior lecturer in history of mathematics at the University of St Andrews. While her usual research focuses on mathematical sciences in the 19th and early 20th centuries, occasionally questions outwith become irresistible. She is also becoming accustomed to Scottish prepositions.

The Mathematics Behind the Casimir Effect

ALEXANDER STROHMAIER

I give an overview of recent mathematical achievements that clarify computations in quantum field theory and physics around the vacuum energy. This area of physics is intimitely linked to spectral geometry and obstacle scattering theory and I will try to point out the particular areas of overlap. Very concretely though, the Casimir energy, a subject of great inspiration and speculation, can be written as the trace of a self-adjoint pseudo-differential operator given in terms of fractional Laplacians.

The spectrum of a domain

If $\Omega \subset \mathbb{R}^d, d \geq 2$ is a smooth bounded domain then the fundamental frequencies λ_j of the domain are the eigenvalues of the operator $-\Delta$ with Dirichlet boundary conditions. They are characterised by the existence of an orthonormal basis (ϕ_j) in the space $L^2(\Omega)$ of square integrable functions, consisting of smooth functions on the closed domain $\overline{\Omega}$ with the property

$$-\Delta\phi_j = \lambda_j^2\phi_j, \quad \phi_j|_{\partial\Omega} = 0,$$

where $\partial\Omega$ denotes the boundary of Ω. The set of frequencies is called the spectrum of the (root of the) Dirichlet-Laplace operator.

For numerous reasons this spectrum and the properties of the corresponding eigenfunctions have been the subject of intensive investigation over the past hundred years. Starting with Fourier's work on the heat equation it was realised that the spectrum of the Laplace operator describes heat propagation, the time evolution of quantum mechanical particles, sound waves, and Brownian motion, all at the same time. Many interesting questions remain open; for example, it is still unknown whether the domain is determined by the sequence of fundamental frequencies (λ_j). The attempt to answer these has uncovered surprising links to dynamical systems of billiards and number theory. For example, the Gauss circle problem and the Riemann hypothesis can be formulated in terms of spectral properties of Laplace operators.

Whereas these spectra can only be computed explicitly in a few very symmetric examples, a lot is known about these fundamental frequencies in general domains. In dimension two and three fast and reliable numerical methods exist for the computation and approximation of eigenfunctions and eigenvalues. The high frequency behaviour λ_j is described by Weyl's law. In order to study the dependence of the spectrum on the geometry of the domain various methods have been developed. They can be roughly divided into low and high frequency methods. Low frequency methods tend to make use of min-max principles, or the maximum principle or other properties of elliptic equations. The high frequency, $\lambda_j \to \infty$, behaviour can be analysed using partial differential equations that encode some kind of dynamics and allow the use of phase space analysis.

Perhaps the simplest and most classical method is that of the heat trace. The initial value problem for the heat equation with Dirichlet boundary conditions

$$\partial_t u(t,x) = \Delta u(t,x), \quad u(0,\cdot) = f, u(t,\cdot)|_{\partial\Omega} = 0$$

has a unique solution which can be written as

$$u(t,x) = \sum_{j=1}^{\infty} e^{-\lambda_j^2 t}\langle f, \phi_j\rangle_{L^2(\Omega)}\phi_j(x).$$

For any $t > 0$ the solution operator $K_t : L^2(\Omega) \to L^2(\Omega), f \mapsto u(t,\cdot)$ is an integral operator

$$(K_t f)(x) = u(t,x) = \int_\Omega k_t(x,y)f(y)\mathrm{d}y,$$

with integral kernel

$$k_t(x,y) = \sum_{j=1}^{\infty} e^{-\lambda_j^2 t}\phi_j(x)\overline{\phi_j}(y).$$

The sum converges uniformly in $x, y \in \overline{\Omega}$ together with all derivatives in these variables. The kernel $k_t(x,y)$ is therefore a smooth function of x and y for any $t > 0$.

This operator is a trace-class operator, and its trace can therefore be computed in two ways: as a sum over the eigenvalues, or as an integral over the diagonal of its integral kernel $k_t(x,y)$. Namely,

$$\mathrm{tr}(K_t) = \sum_{j=1}^{\infty} e^{-\lambda_j^2 t} = \int_\Omega k_t(x,x)\mathrm{d}x.$$

Trace-class operators

An operator $T : \mathcal{H} \to \mathcal{H}$ on a Hilbert space is called a trace-class operator if it is compact and the sum of its singular values is summable: By the spectral theorem for compact self-adjoint operators and the polar decomposition any such operator admits a singular value decomposition in the form of a norm convergent representation

$$T = \sum_j \mu_j \langle \cdot, v_j \rangle w_j,$$

for some orthonormal sets v_j and w_j that span the range of T and the orthogonal complement of the kernel of T. The singular values μ_j are the non-negative square roots of the eigenvalues of T^*T. An operator is therefore trace-class iff $\sum_{j=1}^{\infty} \mu_j < \infty$. If T is trace-class the absolutely convergent sum $\mathrm{tr}(T) = \sum_\alpha \langle e_\alpha T, e_\alpha \rangle$ does not depend on the choice of an orthonormal basis (e_α). This number is called the trace of T.

Mercer's theorem

If Ω is a smooth bounded domain and $K : L^2(\Omega) \to L^2(\Omega)$ has a smooth integral kernel $k \in C^\infty(\overline{\Omega} \times \overline{\Omega})$, so that $(Kf)(x) = \int_\Omega k(x,y)f(y)\mathrm{d}y$. Then K is trace-class and its trace is given by

$$\mathrm{tr}(K) = \int_\Omega k(x,x)\mathrm{d}x.$$

The asymptotic behaviour of λ_j as $j \to \infty$ is therefore encoded in the small t behaviour of the heat kernel $k_t(x,x)$. One finds

$$k_t(x,x) \sim a_0(x) t^{-\frac{d}{2}}$$

as $t \to 0_+$. Now the coefficient $a_0(x)$ of this small t expansion of $k_t(x,x)$ contains only local information around the point x. To understand why that is on an intuitive level it is helpful to recall the physical interpretation of the kernel in terms of heat propagation. We start with an initial temperature distribution given by a Dirac δ-distribution localised at the point x. Next we wait time t and let the heat spread out. The value of the kernel $k_t(x,x)$ is then the temperature at the point x at that time t. For very short times this temperature will depend only on the structure of the equation near that point x as there is not enough time for heat to propagate from anywhere else. In more general settings the expansion coefficients will then only depend on the coefficients of the equation near the point x. In our case the coefficients are constant, and therefore $a_0(x) = a_0$ is constant. From the explicit formula for the heat kernel in $\mathbb{R}^d$ one finds $a_0 = (4\pi)^{-\frac{d}{2}}$. Integrating in x over Ω this gives

$$\mathrm{tr}(K_t) \sim a_0 \mathrm{Vol}(\Omega) t^{-\frac{d}{2}}$$

as $t \to 0_+$. A subtle point in this analysis is that the error of the asymptotic is not uniform in x, the above interchange of asymptotic and integration therefore requires a small justification, which I have skipped here. The Weyl law,

$$N(\lambda) = \#\{\lambda_j < \lambda\} \sim \frac{1}{(4\pi)^{\frac{d}{2}}} \frac{1}{\Gamma(\frac{d}{2}+1)} \mathrm{Vol}(\Omega)\lambda^d$$

on the number of fundamental frequencies smaller than λ can be deduced from this formula. In particular, the above argument shows how the volume enters. Indeed, a quick comparison shows that the Weyl asymptotic of the form $N(\lambda) \sim b\lambda^d$ implies

$$\begin{aligned}\mathrm{tr}(K_t) &= \int_0^\infty e^{-\lambda^2 t}\mathrm{d}N(\lambda) = 2\int_0^\infty \lambda t e^{-\lambda^2 t} N(\lambda)\mathrm{d}\lambda \\ &\sim b\,\Gamma(\frac{d}{2}+1) t^{-\frac{d}{2}}.\end{aligned}$$

This computation yields the constant as claimed. The Karamata Tauberian theorem is basically the statement that the converse conclusion holds, making this heuristic computation rigorous. These Tauberian arguments typically use the fact that the counting function is increasing.

The above idea, relating the spectrum to the short time behaviour of the solutions of a partial differential equation, is at the heart of modern proofs of the Weyl formula. More information is in fact obtained by studying the solutions of the wave equation rather than the heat equation.

Approximate solutions of the wave equations, so called parametrices, allow then to extract precise information about the growth of the counting function.

The spectrum of the exterior of a domain

The situation changes drastically if instead of the interior of a domain we want to understand the spectral representation of the Dirichlet Laplace operator on an exterior domain $M = \mathrm{int}(\Omega^c)$, i.e. on the interior of the complement Ω^c of Ω. For notational simplicity we assume that M is connected. In this case instead of a discrete family of eigenfunctions and discrete eigenvalues we will have a continuum of generalised eigenfunctions. The spectrum is $[0,\infty)$, and the characteristic frequencies will need to be replaced by scattering data.

To the eyes of the pure mathematician in the absence of discrete data this seems much less of an appealing structure. There is however much to be said about the importance of this problem. Of course the physical motivation and applications remain the same in this non-compact situation. But there is a new feature, namely dispersion. Dispersion is the decay of solutions of the wave equation for large time and due to the escape of waves to infinity, something which cannot happen in a compact setting. Dispersive estimates of solutions of the linear equations are one of the main starting points of the analysis of non-linear partial differential equations. Another appealing structure is the appearance of another type of discrete structure, that of the scattering resonances in the complex plane. I will now describe the scattering structure in a bit more mathematical detail.

It is a consequence of the Rellich uniqueness theorem that the Laplacian on M cannot have a square integrable eigenfunction. This is in analogy with the Laplace operator on $\mathbb{R}^d$ without boundary conditions, i.e. the free Laplace Δ_0. The Fourier transform provides a unitary map $\mathcal{F} : L^2(\mathbb{R}^d) \to L^2(\mathbb{R}^d)$ that turns the free Laplace operator into a multiplication operator. The plane waves $e^{\mathrm{i}\xi x}$ provide for every $\xi \in \mathbb{R}^d, \|\xi\|^2 = \lambda^2$ a generalised eigenfunction with eigenvalue λ^2. By the spectral theorem for unbounded self-adjoint operators the exterior Dirichlet Laplace operator on Ω^c can also be diagonalised in that sense. In the same way as for the free Laplace operator the diagonalisation is achieved by a family of generalised eigenfunctions, which we will denote by $E_\lambda(\Phi)$. Thanks to stationary mathematical scattering theory the form of the generalised eigenfunctions and the spectral decomposition is known quite precisely.

The generalised eigenfunctions $E_\lambda(\Phi) \in C^\infty(\overline{M})$ are indexed by $\lambda > 0$ and $\Phi \in C^\infty(\mathbb{S}^d)$. They are uniquely determined by the following properties

(1) $(-\Delta - \lambda^2)E_\lambda(\Phi) = 0$,

(2) $E_\lambda(\Phi)|_{\partial M} = 0$,

(3) The asymptotic expansion

$$E_\lambda(\Phi) = \frac{\mathrm{e}^{-\mathrm{i}\lambda r}\mathrm{e}^{\frac{\mathrm{i}\pi(d-1)}{4}}}{r^{\frac{d-1}{2}}}\Phi + \frac{\mathrm{e}^{\mathrm{i}\lambda r}\mathrm{e}^{-\frac{i\pi(d-1)}{4}}}{r^{\frac{d-1}{2}}}\Psi_\lambda + O\left(\frac{1}{r^{\frac{d+1}{2}}}\right),$$

as $r \to \infty$ holds uniformly in the spherical coordinate for some $\Psi_\lambda \in C^\infty(\mathbb{S}^d)$.

As a result Ψ_λ is uniquely determined and implicitly defines a linear mapping

$$\mathrm{S}_\lambda : C^\infty(\mathbb{S}^d) \to C^\infty(\mathbb{S}^d), \quad \Phi \mapsto \tau\Psi_\lambda,$$

where $\tau : C^\infty(\mathbb{S}^d) \to C^\infty(\mathbb{S}^d)$ is the pull-back of the antipodal map. The map $\mathrm{S}_\lambda : C^\infty(\mathbb{S}^d) \to C^\infty(\mathbb{S}^d)$ is called the scattering matrix, and $\mathrm{A}_\lambda = \mathrm{S}_\lambda - \mathrm{id}$ is called the scattering amplitude. The scattering matrix extends to a unitary operator $\mathrm{S}_\lambda : L^2(\mathbb{S}^d) \to L^2(\mathbb{S}^d)$ for $\lambda > 0$ and is a real analytic function on $(0,\infty)$.

These generalised eigenfunctions are the main objects of stationary scattering theory. The first term in the above expansion describes an incoming spherical wave, and the function Φ determines the asymptotic behaviour of this incoming spherical wave. The scattering matrix describes what happens to this incoming spherical wave after being scattered by the object Ω, as the second term in the expansion corresponds to the outgoing spherical wave. The $E_\lambda(\Phi)$ diagonalise the Dirichlet Laplace operator in the following sense. If h is a bounded Borel function then for any compactly supported smooth functions $f, g \in C_0^\infty(M)$ we have the representation

$$\langle h(-\Delta)f, g\rangle = \frac{1}{2\pi}\sum_\nu \int_0^\infty h(\lambda^2)\langle f, E_\lambda(\Phi_\nu)\rangle\langle E_\lambda(\Phi_\nu), g\rangle \,\mathrm{d}\lambda.$$

Here the sum is over an orthonormal basis of spherical harmonics (Φ_ν) on $\mathbb{S}^d$. The sum converges absolutely. As in the interior case this gives a formula for the heat kernel,

$$K_t = \frac{1}{2\pi}\sum_\nu \int_0^\infty e^{-\lambda^2 t}\langle\cdot, E_\lambda(\Phi_\nu)\rangle E_\lambda(\Phi_\nu)\,\mathrm{d}\lambda.$$

We have an integral representation instead of a sum, which reflects the fact that the spectrum is absolutely continuous. Another notable difference is that this operator is no longer trace class, due to the continuity of the spectrum.

One can however consider the difference

$$e^{t\Delta} - e^{t\Delta_0} = K_t - K_{0,t}$$

to the free heat kernel. It turns out this is a trace-class operator upon restriction to $L^2(M)$ and its trace is given by the Birman-Krein formula

$$\mathrm{tr}(e^{t\Delta} - p_M e^{t\Delta_0} p_M) = \frac{1}{2\pi \mathrm{i}}\int_0^\infty e^{-\lambda^2 t}\log\det S_\lambda\,\mathrm{d}\lambda,$$

where $p_M : L^2(\mathbb{R}^d) \to L^2(M)$ is the orthogonal projection, i.e. the multiplication operator by the indicator function of $M \subset \mathbb{R}^d$. This formula needs some explanation. The determinant $\det S_\lambda$ is the Fredholm determinant of the scattering matrix S_λ, and the branch of the logarithm is chosen to depend continuously on λ and fixed by the requirement that $\log\det S_\lambda$ vanishes at zero.

The Fredholm determinant

If T is a trace-class operator then the operator $1 + T$ admits a well-defined determinant, which is called the Fredholm determinant. If T is normal, i.e. $T^*T = TT^*$, and trace-class this implies that the eigenvalues σ_j of T are absolutely summable. Hence, the Fredholm determinant of $1+T$ can in this case simply be defined simply by $\det(1+T) = \prod_j (1+\sigma_j)$.

The function $\frac{1}{2\pi \mathrm{i}}\log\det S_\lambda$ is sometimes called the *scattering phase*. It, in many ways, replaces the counting function. For example, it satisfies a Weyl law

$$\frac{1}{2\pi\mathrm{i}}\log\det S_\lambda \sim -\frac{1}{(4\pi)^{\frac{d}{2}}}\frac{1}{\Gamma(\frac{d}{2}+1)}\mathrm{Vol}(\Omega)\lambda^d.$$

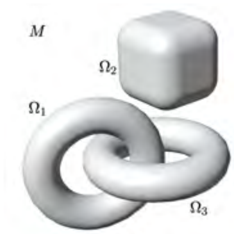
The Birman-Krein formula can be used for functions h that decay faster than $\lambda^{-\frac{d}{2}}$ at infinity. For example, we also have for $s > \frac{d}{2}$ the formula

$$\mathrm{tr}((-\Delta+1)^{-s} - p_M(-\Delta+1)^{-s}p_M)$$
$$= \frac{1}{2\pi\mathrm{i}}\int_0^\infty (\lambda^2+1)^{-s}\log\det S_\lambda\,\mathrm{d}\lambda.$$

It is however important to note that $(-\Delta+1)^{-s} - p_M(-\Delta+1)^{-s}p_M$ is not longer a trace-class operator when s is smaller than $\frac{d}{2} - 1$. In case $s < 0$ the individual operators are no longer bounded. We will see in the next section that we can still associate a trace-class operator to a configuration in that case.

The relative trace

In the following we consider the case where Ω has several connected components Ω_j. As before $M = \mathrm{int}(\Omega^c)$ is assumed to be connected. With this configuration there are a variety of natural operators one can consider. For the following discussion it is useful to consider the Laplace operator Δ with Dirichlet boundary condition on $\partial\Omega$ as an unbounded operator in $L^2(\mathbb{R}^d)$. This means here Δ is the direct sum of the interior Dirichlet Laplace operator on $L^2(\Omega)$ and the exterior Dirichlet Laplace operator on $L^2(M)$ under the orthogonal decomposition $L^2(\mathbb{R}^d) = L^2(\Omega) \oplus L^2(M)$. In particular Δ has absolutely continuous and discrete spectrum. We can also introduce the Dirichlet Laplacian for each object individually, i.e. Δ_j will be the Laplace operator with Dirichlet boundary conditions imposed on $\partial\Omega_j$. This operator does not see the other objects, but knows only about the presence of Ω_j. Finally, we also have the free Laplacian Δ_0.



Three objects in space

In what follows we will assume for simplicity that Ω has two connected components Ω_1 and Ω_2. Consider the subspace $C_0^\infty(M) + C_0^\infty(\Omega)$ of smooth functions that vanish near the boundary $\partial\Omega$. Then this subspace is dense in $L^2(\mathbb{R}^d)$ and for any $s > 0$ each operator Δ_j^s has this subspace in its domain. This means we can form linear combinations of the four operators $(-\Delta)^s, (-\Delta_1)^s, (-\Delta_2)^s, (-\Delta_0)^s$. In case s is an integer these operators all coincide on this subspace, and this case is clearly not very interesting. The situation is completely different when $s = \frac{1}{2}$. In this case it can be shown the operators $(-\Delta)^{\frac{1}{2}}, (-\Delta_1)^{\frac{1}{2}}, (-\Delta_2)^{\frac{1}{2}}, (-\Delta_0)^{\frac{1}{2}}$ span a four dimensional space of unbounded operators. The surprise is however, that there exists a one dimensional subspace of trace-class operators, spanned by $(-\Delta)^{\frac{1}{2}} - (-\Delta_1)^{\frac{1}{2}} - (-\Delta_2)^{\frac{1}{2}} + (-\Delta_0)^{\frac{1}{2}}$. More generally, the relative operator

$$D_s = (-\Delta + m^2)^s - (-\Delta_1 + m^2)^s - (-\Delta_2 + m^2)^s + (-\Delta_0 + m^2)^s$$

is trace-class for any positive s and $m \geq 0$. In order to define this operator it is important that all the operators are defined on the same Hilbert space.

In fact, more can be said. One can compute the trace D_s from a universal function Ξ that does not depend on m or s as follows

$$\mathrm{tr}(D_s) = \frac{2s}{\pi}\sin(\pi s)\int_m^\infty \lambda^{2s-1}\frac{\lambda}{\sqrt{\lambda^2 - m^2}}\Xi(\lambda)\mathrm{d}\lambda.$$

The function Ξ can be expressed in terms of single layer boundary operators, again as a Fredholm determinant. The above relations were discovered and proved in [1].

The single layer operator

The single layer operator $\mathcal{S}_\lambda : L^2(\partial\Omega) \to L^2(\partial\Omega)$ acts on functions on the boundary. It is explicitely given in terms of the integral kernel $k_\lambda(x, y)$ of the free Green's function. The single layer operator is then defined as

$$(\mathcal{S}_\lambda f)(x) = \int_{\partial\Omega} k_\lambda(x, y) f(y)\mathrm{d}y.$$

Some care is needed here as the integral is singular as $x \in \partial\Omega$, but this is simply defined as a principal value. The single layer operator played an important role historically in the solution of the Dirichlet and Neumann problems for domains, but it is still very well known in numerical analysis as it provides access to boundary element methods. The single layer operator is invertible for all $\lambda \geq 0$.

We have $\Xi(\lambda) = \mathrm{logdet}\left(\mathcal{S}_\lambda \circ (\mathcal{S}_{1,\lambda}^{-1} \oplus \mathcal{S}_{2,\lambda})^{-1}\right)$, where $\mathcal{S}_{j,\lambda}$ is the single layer operator of the object Ω_j, acting on $L^2(\partial\Omega_j)$. In case $d = 3$ we have

$$k_\lambda(x, y) = \frac{1}{4\pi}\frac{\mathrm{e}^{-\lambda|x-y|}}{|x - y|},$$

and in other dimensions there are explicit formulae in terms of spherical Hankel functions. This makes the single layer operator and the function Ξ accessible numerically.

Casimir effect of the scalar field

The physical significance of the relative trace is that it represents to vacuum energy of the scalar field of mass $m \geq 0$ with Dirichlet boundary conditions as

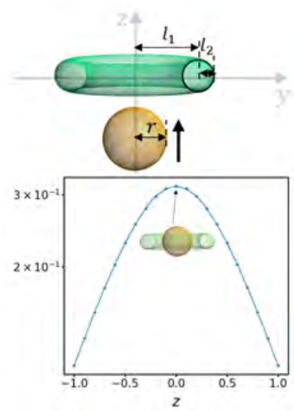
$$E_{\mathrm{Cas}} = \frac{1}{2}\mathrm{tr}(D_{\frac{1}{2}}).$$

This formula is not standard and needs a justification. The consequence of the determinant formula is a formula for the Casimir energy in terms of the single layer operators

$$E_{\mathrm{Cas}} = \frac{1}{\pi}\int_0^\infty \Xi(\lambda)\mathrm{d}\lambda.$$

This formula appeared in the physics literature around 2005 by various authors (Kenneth & Klich, Emig, Graha, Jaffe and Kardar, Johnson just to mention a few names) to compute Casimir forces. Its derivation in the physics literature is not based on the relative trace, as suggested above, but on a path integral derivation of the contribution of surface current fluctuations, i.e. a slightly different mechanism for the Casimir effect. It has been key to the recent progress in Casimir energy computations. The function $\Xi(\lambda)$ is directly accessible numerically and since the computation is effectively reduced to the boundary this reduces the complexity of the problem significantly.

A precise meaning in terms of single layer operators together with the existing boundary element software allows one to efficiently compute the Casimir energy of more complex configurations. I include here an example obtained in collaboration with Timo Betcke and Xiaoshu Sun from UCL. We use modern numerical



Negative Casimir energy as function of vertical displacement of a sphere inside a torus ($l_1 = 2$, $l_2 = 0.5$ and $r = 1$).

methods to speed up the computations and utilise the software Bempp.

This energy measures the effect the two objects have upon each other. It can be shown that this finite and well defined energy is indeed the correct notion, as it reproduces the forces computed from the regularised stress energy tensor when one of the objects is moved rigidly. This was proved only recently in [2].

Photons and Maxwell's equations

I will now briefly explain, without going into details, how this has to be modified to incorporate the theory of photons, i.e. the theory that describes the actual Casimir effect when ideal metals are placed in a vacuum and interact accordingly with the quantum photon field. In this case one can again define a trace-class operator, and a corresponding formula for its trace in terms of a single layer operator. The details and precise definitions can be found in [4] where the result was recently proved. In this case the formula

$$E_{\mathrm{Cas}} = \frac{1}{\pi}\int_0^\infty \Xi(\lambda)\mathrm{d}\lambda.$$

remains with Ξ as before but the single layer operator must be replaced by the Maxwell single layer operator $\mathscr{L}_\lambda$. One has

$$\Xi(\lambda) = \log\det\left(\mathscr{L}_\lambda \circ (\mathscr{L}_{1,\lambda} \oplus \mathscr{L}_{2,\lambda})^{-1}\right).$$

The Maxwell single layer operator

The Maxwell single layer operator

$$\mathscr{L}_\lambda : L^2(\partial\Omega, T\partial\Omega) \to L^2(\partial\Omega, T\partial\Omega)$$

acts on tangential vector-fields on the boundary. It is defined by the integral kernel

$$\gamma_t \circ \mathrm{curl}_x \mathrm{curl}_x k_\lambda(x,y).$$

Here the scalar free Green's kernel k_λ acts component-wise on the tangential vector-fields. Restriction to the tangential vector-fields is achieved by taking the limit in the x-variable to the boundary $\partial\Omega$ and taking the exterior product with the exterior unit normal vector field ν. This tangential restriction operator is denoted by γ_t.

The Maxwell single layer operator is a standard operator in computational electrodynamics and is available numerically in computer packages such as Bempp. For the background on Maxwell theory and boundary layer theory I refer to the excellent monograph [3].

Acknowledgements

The research leading to the mathematical description of the Casimir effect has kindly been funded by the Leverhulme Trust grant RPG-2017-329.

FURTHER READING

[1] Hanisch, F., Strohmaier, A. & Waters, A. A relative trace formula for obstacle scattering. *Duke Math. J.*. **171**, 2233-2274 (2022).
[2] Fang, Y. & Strohmaier, A. A mathematical analysis of Casimir interactions I: The scalar field. *Ann. Henri Poincaré*. **23**, 1399-1449 (2022).
[3] Kirsch, A. & Hettlich, F. The mathematical theory of time-harmonic Maxwell's equations. (Springer, Cham, 2015), Expansion-, integral-, and variational methods
[4] Strohmaier, A. & Waters A., The relative trace formula in electromagnetic scattering and boundary layer operators, *arxiv.org/abs/2111.15331.*

Alexander Strohmaier

Alexander is a Professor of mathematics at the University of Leeds. His main area of research is global analysis of PDEs on manifolds, but he has strong interests in mathematical physics of quantum theory. In his free time he likes to play badminton, program microcontrollers, and solder electronic circuits.

Notes of a Numerical Analyst

Dribbling a Ball

NICK TREFETHEN FRS

During a recent visit of mine to the University of Delaware, Toby Driscoll told me of a computational problem he'd assigned his students. Solve the slightly damped linear oscillator equation

$$y'' + 0.04y' + y = 0.1\sin(\omega t) \tag{1}$$

with initial data $u = u' = 0$ and measure the amplitude of the resulting oscillation as a function of the forcing frequency ω. You'll see a nice amplitude peak at the resonant frequency $\omega_0 = 1$. Now do the same for the nonlinear pendulum

$$y'' + 0.04y' + \sin(y) = 0.1\sin(\omega t). \tag{2}$$

What happens to the response curve?

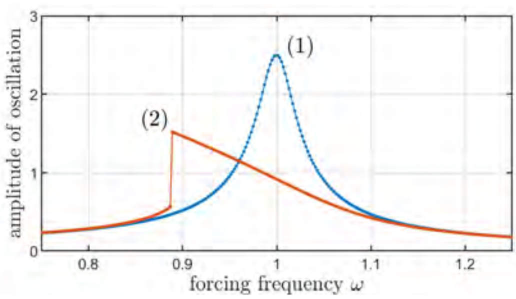


Figure 1. The linear oscillator (1) has a simple response with a peak at $\omega_0 \approx 1$. What's going on with the nonlinear pendulum (2)?

This is an easy calculation with your favourite ODE solver (I used Chebfun). The effect that turns up is striking, shown in Figure 1. The response for (2) suddenly jumps at around $\omega = 0.89$, more than doubling in amplitude.

With further numerical explorations, not shown, it's not hard to figure out what's going on. The linear oscillator (1) has natural frequency $\omega_0 \approx 1$, independent of amplitude. For $\omega \ll \omega_0$, the amplitude is small, so $\sin(y) \approx y$ and (2) has about the same behaviour. As ω increases toward ω_0, however, the amplitude increases and the natural frequency of (2) decreases, bringing ω and ω_0 closer together and enhancing the resonance. Around $\omega = 0.89$ the process becomes unstable and takes off, with the amplitude jumping up and ω_0 plunging down to a new value less than ω. Exercises 8.4.9–8.4.10 of [2] analyze such an event as a *cusp catastrophe* for the Duffing equation, a familiar effect to experts in nonlinear oscillations.

Figure 2. Toby Driscoll with a bouncy ball. As the amplitude grows, the natural frequency decreases, so to get the ball off the ground, you have to dribble fast at first and then slower.

As Toby and I chatted about (2), we were playing with a big green ball. Suddenly we noticed, this was a variation on the same nonlinear theme! You can't get the ball off the ground by patting it at a fixed frequency, but the sky's the limit if you start patting fast and then slow down as the bounce gets higher and the natural frequency falls. We are sure LeBron James knows this.

FURTHER READING

[1] A. H. Nayfeh and D. T. Mook, *Nonlinear Oscillations*, Wiley, 2008.
[2] S. H. Strogatz, *Nonlinear Dynamics and Chaos*, CRC Press, 2018.
[3] L. N. Trefethen, Á. Birkisson and T. A. Driscoll, *Exploring ODEs*, SIAM, 2018.

Nick Trefethen
Trefethen is Professor of Numerical Analysis and head of the Numerical Analysis Group at the University of Oxford.

Mathematics News Flash

Jonathan Fraser reports on some recent breakthroughs in mathematics.

Large Banach spaces with no infinite equilateral sets

AUTHORS: Piotr Koszmider and Hugh Wark
ACCESS: https://arxiv.org/abs/2104.05330

A subset of a Banach space (or a metric space) is called *equilateral* if all distances realised between pairs of distinct points are the same. For example, the vertices of an equilateral triangle in the plane form an equilateral set of size 3. By results of Brass and Dekster, for all integers k, all d-dimensional Banach spaces will contain an equilateral set of size k provided d is large enough in terms of k. It is an open conjecture that in fact d can be chosen as small as $k-1$ for $k \geq 2$.

The discussion above ensures that infinite dimensional Banach spaces necessarily contain arbitrarily large equilateral sets and it would be natural to expect them to also contain infinite equilateral sets, but this is not necessarily the case. This paper, published in *Bulletin of the London Mathematical Society* in 2022, proves that there even exist non-separable Banach spaces with no infinite equilateral sets. Recall that a Banach space is *separable* if it contains a countable dense subset and so the absence of infinite equilateral sets is perhaps more surprising in the non-separable case because these spaces are “even bigger”.

Tiling the integers by translation

AUTHORS: Izabella Łaba and Itay Londner
ACCESS: https://arxiv.org/abs/2106.14044

Consider a finite set of integers $A \subseteq \mathbb{Z}$. We say A *tiles the integers (by translation)* if there exists a set of integers T with the property that

$$\mathbb{Z} = \bigcup_{t \in T} (A+t)$$

with the union disjoint. It turns out that if this is the case the set T (and the tiling itself) must be periodic. For example, $\{0,1,5\}$ tiles the integers with $T = 3\mathbb{Z} = \{\cdots -6,-3,0,3,6,\dots\}$. In this case T is periodic with period 3. On the other hand, $\{0,1,3\}$ does not tile the integers. Can you see why?

In the late 1990s Coven and Meyerowitz gave a sufficient condition, which they called “(T2)”, which ensured a finite set tiled the integers. They also proved (T2) is necessary when the cardinality (size) of A has at most two distinct prime factors.

This substantial paper of more than 100 pages, published in *Inventiones* in 2023, proves that (T2) holds for all tilings with period $(p_1p_2p_3)^2$ for distinct odd primes p_1, p_2, p_3.

Black holes that aren’t spheres

AUTHORS: Marcus Khuri and Jordan Rainone
ACCESS: https://arxiv.org/abs/2212.06762

In the 1970s Steven Hawking proved that the surface of a black hole must be a 2-dimensional sphere (at any given time point). This falls in line with our intuition about the universe, where planets and moons are (approximately) spheres, formed out of dust clouds drawn together towards a centre of mass. This paper investigates the possible ‘shapes’ of black holes when the number of ambient spatial dimensions is increased. This is a physically relevant problem, especially given various models for the structure of the universe which use more than our familiar 3 spatial dimensions. They prove that in dimensions 5 and higher there are, in fact, an infinite number of different possibilities! Although purely theoretical, this type of result opens up the possibility of providing evidence in support of high dimensional models for the universe; all you have to do is go out and observe a black hole with a strange shape.

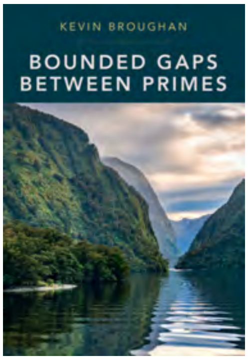
Jonathan Fraser is a Professor at the University of St Andrews and an Editor of this Newsletter. He likes fractal geometry but is pictured here with Dylan during a practical exploration of the concept of ‘drag’.

Bounded gaps between primes

by Kevin Broughan, Cambridge University Press, 2021, £40,

ISBN 978-1108799201

Review by Sam Chow



Prime numbers, the building blocks of the integers. We know so much but yet so little about them. By the prime number theorem, the average gap between consecutive primes up to x is roughly $\log x$. This motivates the *Cramér random model*, that a positive integer n has an independent probability $1/(\log n)$ of being prime. Based on this, one might guess that

$$\#\{p_1, p_2 \le x \text{ primes} : p_1 - p_2 = 2\} \approx \frac{x}{(\log x)^2}.$$

Hardy and Littlewood refined this heuristic by considering divisibility by small primes, and empirical data support their conjecture that

$$\#\{p_1, p_2 \le x \text{ primes} : p_1 - p_2 = 2\} \sim 2C_2 \frac{x}{(\log x)^2},$$

where

$$C_2 = \prod_{p \neq 2 \text{ prime}} \left(1 - \frac{1}{(p-1)^2}\right) \approx 0.66$$

is the twin prime constant. Even before that, de Polignac had predicted that there are infinitely many twin primes, that is, pairs of primes differing by two—the twin prime conjecture. The twin prime conjecture is one of Landau's four problems presented at the 1912 International Congress of Mathematicians, and is one of the most coveted open problems in number theory.

I first encountered James Maynard at a graduate student conference in Bristol, in May 2013. In his presentation, he described his attempts to prove *bounded gaps between primes*, i.e., that

$$\liminf_{n \to \infty} (p_{n+1} - p_n) < \infty,$$

where $p_1 < p_2 < \ldots$ are the primes. As we were aware, the media had reported just days earlier that unheralded mathematician Yitang Zhang had proved this [4], specifically that

$$\liminf_{n \to \infty} (p_{n+1} - p_n) \le 70\,000\,000.$$

Maynard was not discouraged, however, and later that year released proofs [2] that

$$\liminf_{n \to \infty} (p_{n+1} - p_n) \le 600$$

and

$$\liminf_{n \to \infty} (p_{n+m} - p_n) < \infty \qquad (m \in \mathbb{N}).$$

The mathematical community was naturally curious as to the extent to which these bounds could be sharpened using essentially the same methods. The Polymath8 project, led by Terence Tao, was set up for this purpose, with a second goal of understanding and clarifying Zhang's argument. Polymath8a reduced Zhang's bound to 4 680. Polymath8b improved Maynard's bound from 600 to 246, where it remains to this date [3].

Kevin Broughan is an Emeritus Professor at the University of Waikito, who has researched in analytic number theory and is the author of the two-volume work *Equivalents of the Riemann Hypothesis*. In the extensive book under review, he chronicles the full story behind these developments. After an introduction and some background on sieve theory, the book presents early work, the groundbreaking method of Goldston, Pintz and Yildirim [1] upon which subsequent papers build, the aforementioned breakthroughs of Zhang and Maynard, and the meticulous refinements of Polymath8. Computational inputs are also discussed at length, as are related topics.

As well as supplying the mathematical details, Broughan embraces the human aspects of the saga. It is, after all, a wonderful tale of how two lesser-known mathematicians worked extremely hard to solve an intriguing, long-standing open problem that so many leading experts could not. The author draws from many sources, and writes with unbridled passion. He has an unusual but nonetheless effective way of writing proofs, breaking the arguments up into numbered steps, each of which is fairly short. I must caution that the book itself is very long, so the reader would not necessarily want to read every chapter or every proof.

The material is presented at a serious level and is not intended for the general public. *Bounded Gaps Between Primes* is suitable for graduate students in analytic number theory, but others may also find it interesting and informative.

FURTHER READING

[1] D. A. Goldston, J. Pintz and C. Y. Yildirim, Primes in tuples I, Ann. of Math. (2) 170 (2009), 819–862.
[2] J. Maynard, Small gaps between primes, Ann. of Math. (2) 181 (2015), 383–413.
[3] D. J. H. Polymath, The "Bounded Gaps between Primes" Polymath Project: A Retrospective Analysis, EMS Newsletter, December 2014, 13–23.
[4] Y. Zhang, Bounded gaps between primes, Ann. of Math. (2) 179 (2014), 1121–1174.

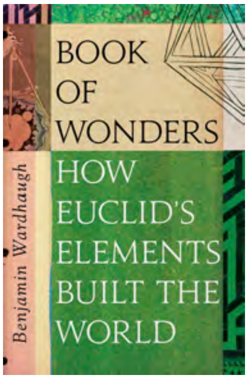
Sam Chow

Sam Chow is a lecturer of mathematics at the University of Warwick. He is a number theorist working on diophantine approximation and diophantine equations. He enjoys playing chess and other games.

The Book of Wonders: The Many Lives of Euclid's *Elements*

by Benjamin Wardhaugh, William Collins, 2020, £25, ISBN: 978-0008299903

Review by Vicky Neale



There aren't many books in history that merit a biography, but then there aren't many books in history that have the longevity, reach and impact of Euclid's *Elements*. I think that 'biography' really is the best description of Benjamin Wardhaugh's new book. It tells the story of the *Elements* through around 2300 years of history, and back and forth across the world. A book like this might seem daunting (especially to a mathematician, rather than historian of mathematics, like me): it's hardback, and quite thick, and scholarly. But in reality it is not daunting at all: it is readable, digestible, and enjoyable. The chapters are quite short, and each feels like a self-contained satisfying read in itself, while part of the sweeping narrative, and that makes it easy to read in sections.

The book starts by taking us from Euclid himself through to the 17th century. It turns out that this is the first of four sections, but because I wasn't concentrating when I read the contents page, I was disconcerted to find that section two jumps right back to Plato (before Euclid), before heading forwards through time again. I'm not sure that I can clearly articulate the difference between these two sections; I'm sure there is one, but both felt to me like tracking the book through time. I was less surprised when section three arrived, and I think this one was more about the practical uses to which the ideas of the *Elements* have been put. The final, fourth, section, addresses the limitations of the *Elements*, and the ways in which societies have fallen out of love with Euclid. My slight confusion about the different sections didn't detract from my engagement with the book, but did leave me wondering whether it might be helpful to have a timeline (or to have created my own) showing the sequence of events addressed in overlapping chronologies.

I enjoyed the broad sweep through time and between cultures. But I also, at least as much, enjoyed the individual chapters, each feeling like a snapshot of a moment in time and space. There were many new insights and historical details here for me. I was intrigued by the characterisation that "Greek geometry was essentially a performance, consisting of drawing a diagram and talking about it, to oneself or to an audience". The pandemic-imposed need to move to lecture videos in recent months has prompted me to reconsider many aspects of my own teaching practice, including the 'performance' angle, which feels to me different but just as important at my kitchen table as at a whiteboard in a lecture theatre. Probably my absolute favourite nugget in the book was about the first attempts to typeset the *Elements* for printing, in the late 15th century. Remember that this was a world of physical blocks for characters, which had to be arrayed on a frame before being inked for printing. Geometrical diagrams, with disproportionately many references to points A, B, C, D, meant that compositors would literally run out of certain characters!

Alongside the relatively mainstream narrative about the *Elements*, I enjoyed the cameo appearances by numerous people I'm sorry to say I hadn't heard of before, or where I was unaware of their interest in Euclid. No doubt many readers of the *LMS Newsletter* will know of the work of Hroswitha of Gandersheim, but I didn't. She was a Benedictine canoness in Saxony in the late 10th century, very well educated, and an author of numerous works, including a play called *Wisdom*, which included some number theory. Another example: Anne Lister's extraordinary life was recently brought to public attention when her diaries inspired the BBC television drama series *Gentleman Jack*. As Wardhaugh tells us, Lister chose in her twenties to resume her study of the *Elements*, and got further "frankly, than most university graduates".

Wardhaugh understandably places much emphasis on tracing the progress of the *Elements* as it moves between cultures and indeed between languages, with each new editor/author/translator giving their own new take on it. In a chapter named after Xu Guangqi, he explores the difficulties of translating words such as 'definition', 'proof' and 'axiom' into Chinese in a way that would capture their significance in the Greek tradition. I found this thought-provoking: having been raised in a certain tradition, seeing another culture's viewpoint can prompt us to reassess the meaning of ideas we otherwise take as self-evident.

This biography explores the impact of Euclid's great work on mathematics, of course, but also on other areas. The chapter on Piero della Francesco describes the influence of the *Elements* on the style of his pioneering writing on the theory of perspective. An earlier chapter on Muhammad Abu al-Wafa al-Buzjani has a fascinating discussion of the creation of a certain number of squares from the dissection and rearrangement of smaller squares — a question with interesting geometrical significance, but also a practical one for artisans creating mosaic designs. A chapter late in the book is named after two fictional characters, Maggie and Tom Tulliver, from George Eliot's *The Mill on the Floss*. Marian Evans (the real name of Eliot) elected to study geometry in later life, but her creation Tom Tulliver was no fan of Euclid after his bad experiences at school, and Wardhaugh uses this to exemplify "the failure of Euclidean education in the nineteenth century: or at least of the failure of that education once it had become detached from humanity, and indeed from common sense about the needs of different students and their differences from one another". A sobering reminder for us all.

I could keep going with tasty morsels from the book, but frankly Wardhaugh tells the stories better than I do. So instead, let me consider: who will want to read this book? I am sure that historians of mathematics will find much of value in this book, helped by the extensive notes and references at the end. But it deserves to have a wider readership too, amongst mathematicians and aspiring mathematicians, who will enjoy and learn from dipping in or indeed reading from cover to cover, and getting another glimpse into the development of our subject, and the understanding and insight to be gained through the exchange, refinement and revision of ideas as they pass between cultures. A book of wonders indeed.

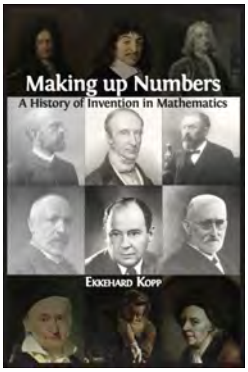
Vicky Neale

Vicky Neale is the Whitehead Lecturer at the Mathematical Institute and Balliol College, University of Oxford. Her job is to be enthusiastic about maths with undergraduates, school students and the wider public. She enjoys mathematical knitting and crochet, with the 'help' of her two cats.

Making up Numbers: A History of Invention in Mathematics

by Ekkehard Kopp, Open Book Publishers, 2020, ISBN 978-1-80064-097-9

Review by Adrian Rice



God created the natural numbers, all else is the work of man. — attributed to Leopold Kronecker.

Is mathematics invented or discovered? Do mathematical concepts spring into existence only when first imagined by a human mind or are they eternal? The famous quotation above makes Kronecker's opinion on the matter crystal clear, encapsulating his constructivist philosophy that, aside from the positive integers, the entirety of mathematics is a human invention. In its strongest form, this view maintains that nothing in mathematics can really be said to exist unless a human has seen it, thought of it, or written it down. This has always seemed to me to be a little extreme: for example, even if no one else in the history of humanity has had any reason to write down the complex number $57.49544982368932873252537 + 0.00027265238742335340712662i$ before me, my claim to have invented it is somewhat dubious. On the other hand, the case for invention in mathematics is still strong: for instance, nothing like the concept of a Galois group can be said to have existed in any form before the 19th century—it is purely a creation of the human mind. In fact, as the history of mathematics reveals, the subject's development has been a combination of both invention *and* discovery: concepts and techniques are invented to solve certain problems, the solution of which often leads to the discovery of new ideas, that themselves pose new questions, often requiring the creation of new concepts and techniques, and the process continues.

Making up Numbers is written from the standpoint of mathematics as a human invention. More specifically, since the notion of number has been essential to human civilization for millennia, as well as possibly the oldest concept in mathematics, this book presents a broadly chronological treatment of the history of the use of numbers in pure mathematics from antiquity to the 20th century. From Mesopotamia and the ancient Greek world to more recent developments, the author presents an engaging account of how the concept of number was gradually extended to include negatives, irrationals and complex numbers. The first four chapters concentrate on these extensions and the motivations behind them, from the Pythagorean obsession with positive integers to Gauss's proof of the fundamental theorem of algebra. We then come to a two-chapter interlude on issues arising from the concepts of infinity and infinitesimals, obviously concerning the creation of calculus and analysis. This leads inevitably to questions concerning continuity, with the final four chapters concerning the rigorisation of the real number system, the growth of axiomatization in the 19th century, Cantor's transfinite numbers, and the infamous paradoxes of the early 20th century. In this way, readers are guided through a variety of themes, from solving the earliest known equations to the role of axioms in the foundations of mathematics.

It's a well-known story and one that is told very well here. Those for whom this book is their first encounter with the history of mathematics will find it a reliable and informative introduction. But the prospective reader should perhaps also be aware that this book is fundamentally a history of real numbers. This is by no means a criticism—and perhaps says more about the contrasting mathematical tastes of author and reviewer—but it might be helpful to briefly highlight what this book does not contain. While there is certainly plenty on complex numbers, the discussion of further extensions in this direction is halted with just a passing reference to Hamilton's creation of quaternions. This is a pity, not only because the birth of quaternions and subsequent systems of hypercomplex numbers revolutionized the whole notion of what a 'number' could be, but also because the introduction of new imaginary quantities such that $i^2 = j^2 = k^2 = ijk = -1$ is one of the most remarkable instances of invention in the history of mathematics. (Even the first LMS President, Augustus De Morgan, was taken aback by the idea of '*imagining* imaginaries', as he put it.)

There is a good discussion of primes up to the fundamental theorem of arithmetic, and an introduction to algebraic numbers later on; but there are no allusions to further possibilities, such as Gaussian primes, or other generalizations of the idea of a prime number. Staying within the real domain, mention could have been made of Mersenne primes, along with their intriguing connection to perfect numbers (another concept not mentioned). A reference to Germain primes might also have given the author a chance to highlight the work of a female mathematician—difficult to do in a work of this kind. Another figure not mentioned, but whose results alone would be sure to astonish any reader, is Ramanujan, who produced some stunning rational approximations of irrational numbers, and of whom Littlewood famously remarked 'every positive integer was one of his personal friends'. Perhaps also surprising is that, although the final chapter deals with a host of foundational issues and contributors, including Cantor, Hilbert, Russell, Brouwer and Gödel, one contribution is not included, namely that of Alan Turing, whose monumental LMS paper of 1936, as well as answering Hilbert's Entscheidungsproblem via the introduction of Turing machines, also invented a new theoretical concept of relevance to a history of real numbers: the computable number.

None of this should, however, detract from the book's significant strengths. While not so much concerned with algebra and number theory, it is nevertheless crammed with some very interesting mathematics, particularly in the realms of analysis and set theory. It does not profess to be a learned volume on the history of mathematics, since its intended audience is A-level and undergraduate students and their teachers; but it was clearly written by someone with great knowledge and appreciation of the discipline. More importantly, the author clearly conveys a deep love and understanding of the mathematics he discusses, combining lively readable prose with a clear presentation of the material to give just enough technical detail without overwhelming the relative beginner. The lay reader will definitely come away with an idea of what it is to *do* mathematics and above all, with the knowledge that mathematics is far from being a static discipline, but an ever-evolving creative process. In short, the author seeks to convince the reader that mathematics is fundamentally a human endeavour—and in that, this book succeeds admirably.

Adrian Rice

Adrian Rice is the Dorothy and Muscoe Garnett Professor of Mathematics at Randolph-Macon College in Virginia, USA, where his research focuses on 19th- and early 20th-century British mathematics. He is nostalgic for the days when his photograph made him look younger than he is.

Obituaries of Members

John W. Rutter: 1935 – 2023

Dr John W. Rutter, who was elected a member of the London Mathematical Society on 21 December 1961, died on 19 January 2023, aged 88.

Peter Giblin writes: John Westhead Rutter was born in Bolton, Lancashire in 1935 and attended Farnworth Grammar School, where he met his future wife Eileen, whom he married in 1959. He attended Oriel College Oxford on a scholarship and graduated BA in 1958. He embarked on doctoral studies in topology with Michael Barratt and undertook teaching duties at Queen's College and Magdalen College from 1958 to 1961, at which time he was appointed lecturer at Liverpool University, where he spent the whole of his career. He was awarded DPhil in 1963 for a thesis Some Problems in Algebraic Topology. He spent the academic year 1964-5 as visiting lecturer at Stanford University, and was a British Council and Great Britain Sasakawa Foundation visiting lecturer in Japan in 1988 and a visiting scholar at Wolfson College, Cambridge in 1989. John retired from the University of Liverpool in 1999.

John's mathematical interest was in homotopy theory. MathSciNet lists 36 articles in journals and conference proceedings as well as two books: Spaces of Homotopy Self-Equivalences - A Survey in 1997 and, based on a second year course at the University of Liverpool, Geometry of Curves in 2000. This book is slightly unusual in that it uses complex number methods to study real plane curves. He co-edited and contributed two review articles and a bibliography to the proceedings of a workshop held at the Gargnano Institute of the University of Milan in 1999 on groups of homotopy self-equivalences and related topics.

John had four daughters, Catherine, Sasha, Frances and Tanya with Eileen. He and Eileen divorced in 1976 and John formed a long term relationship with Jean Shearer; she died in 2021. John's interests outside mathematics were wide-ranging: he loved travelling, rambling and mountain walking in several continents, classical music (especially Baroque music), dancing and spending time with his family. He learned ballroom dancing and was a frequent visitor at the Blackpool Tower Ballroom with Jean; moreover on suitable occasions such as New Year parties he was also happy to dance to the music of the Bee Gees and Abba! He was a patron of the Early Music Festival in York, the London Festival of Baroque Music and The Bath Bach Society.

On a personal note, John was a close friend and visitor to my family home, and my wife and I were entertained by John and Jean to dinner several times. It was John's vegetarianism which persuaded us, through cooking appropriately for him, to adopt almost the same diet as his. John was a 'jolly uncle' to my children when he came for dinner around Christmas time for many years: he could be relied on to amuse them with his stories, antics and general ebullient good humour.

John is survived by Eileen, their four daughters, seven grandsons and one great-grandson.

Rex Dark: 1942 – 2022

Dr Rex Dark, who was elected a member of the London Mathematical Society on 16 June 1978, died on 5 December 2022, aged 79.

Arnold Feldman, John McDermott, and Martin Newell write: Rex was born in Huddersfield and raised in Leatherhead. From Charterhouse School, Surrey, Rex went to Magdalene College, Cambridge, earning a double First. After his 1968 PhD at Cambridge supervised by J.E. Roseblade he taught in Cambridge, and then moved to University College, Galway (later NUIG, now University of Galway) in 1973 where he remained until taking early retirement in 2003.

Rex's contributions to the UCG Mathematics department — and to the wider mathematical community — were substantial and very much appreciated. Former colleagues and collaborators re-marked in particular on his courtesy, helpfulness, quiet but wry sense of humour, and of course his brilliant mind. Former students also remember him with affection, recalling his patient mentoring and generosity with his time, as well as his enthusiastic and inspiring lectures. His research talks were equally well-organised, clear, and insightful. He undertook his full share of administrative chores and was one of the founders of the Irish Mathematical Society and the series of annual Groups in Galway meetings, remaining a staunch supporter of both throughout his life.

He was a master of the example and counterexample. His examples were beautifully constructed using his vast knowledge of techniques, including those of his own invention. Underlying them was his broad and deep understanding of groups coupled with extraordinary attention to detail, often involving intricate computations, generally done by hand. He was endlessly patient, work-ing to perfect any project, producing draft after draft of increasingly comprehensive, impeccably expressed, carefully referenced results.

His 1972 paper *Some Examples in the Theory Of Injectors of Finite Soluble Groups* so greatly changed the way group theorists looked at fitting classes that doerk and hawkes, in their comprehensive 1992 volume 'Finite Soluble Groups', devoted more than 45 pages to what they termed 'Dark's construction' and its variations. His remarkable abilities never diminished: his latest works were published in 2022, and he left nearly completed manuscripts that will be submitted for publication by his collaborators.

Rex was also very committed to many non-mathematical interests. In college he was an active union member and was shop steward for a while; he was a long-time member of the UCG Mountaineering Club and was deeply involved with the Galway branch of Alliance Française and the Galway Mountain Rescue Team. Outside UCG, his primary loyalty was perhaps to Saint Nicholas' Collegiate Church, where he was treasurer for six years. He was a chorister and also took to the stage with a church drama group. He continued his service to the Church in Westport in the last years of his retirement.

A familiar figure on his cycling commute from Moycullen to UCG, Rex was also an inveterate teller. In addition to regular journeys to his home in France he attended conferences in many places, with collaborators in Germany, Italy, Spain and the United States. He also made many trips to see his only sibling, Michael, when the latter worked abroad. Although he eventually acquired a debit card, he travelled without ever having a credit card, a feat that became more extraordinary with every year that passed.

Throughout his life Rex was very close to Michael and his family. In the beautiful eulogy given at his funeral, his niece caught the essence of the man when she concluded that he remained the same always — kind, modest, brilliant, and completely original.

Death Notices

We regret to announce the following death:

- Professor Ronnie Becker, who was a Senior Resident Researcher at AIMS South Africa and Emeritus Professor at the University of Cape Town, and who was an LMS member from 1990 to 2000, died on 26 March 2023, aged 85.

LMS Hardy Lecture Tour 2023

G.H. Hardy, LMS President
1926–1928 and 1939–1941
Photo Courtesy of Master and Fellows of
Trinity College Cambridge

Eva Miranda (UPC-Barcelona)
Hardy Lecturer 2023

The Hardy Lectureship was founded in 1967 in memory of G.H. Hardy in recognition of outstanding contribution to both mathematics and to the Society. The Hardy Lectureship is a lecture tour of the UK by a mathematician with a high reputation in research. The 2023 LMS Hardy Fellow is Professor Eva Miranda (UPC-Barcelona).

Professor Miranda will visit the UK in May, June, July and September 2023 and she will give talks at:

Cambridge 30 May;
Counting periodic orbits
Organiser: Maciej Dunajski

Royal Institute, London 1 June;
From Alan Turing to contact geometry: towards a "Fluid computer"
Organiser: Saksham Sharma

Birmingham 26 June;
Desingularizing singular symplectic structures
Organiser: Marta Mazzocco

Warwick 28 June;
Euler flows as universal models for dynamical systems
Organiser: José Rodrigo

Mary Ward House, London 30 June;
From Alan Turing to fluid computers: Explored and unexplored paths
Organiser: London Mathematical Society

Oxford 4 July;
Singular Hamiltonian and Reeb Dynamics: First steps
Organisers: Andrew Dancer and Vivat Nanda

Loughborough 6 July;
Action-angle coordinates and toric actions on singular symplectic manifolds
Organisers: Sasha Veselov and Alexey Bolsinov

Edinburgh 19 September;
From Symplectic to Poisson manifolds and back
Organiser: José Figueroa-O'Farrill

Glasgow 21 September;
Quantizing via Polytope counting: Old and new
Organiser: Ian Strachan

For further information on attending each lecture, please visit the LMS website here:
lms.ac.uk/events/lectures/hardy-lectureship#Hardy%20Current

For general enquiries about the Hardy Lectures, please contact (lmsmeetings@lms.ac.uk).

Colloquium in Combinatorics

Location: QMUL and LSE, London
Date: 10–11 May 2023
Website: tiny.cc/2dayCC

This Colloquium covers a wide range of topics of interest to all working in combinatorics or related fields. Speakers: Thomas Bloom (Oxford), Carla Groenland (Utrecht), Mihyun Kang (Graz), Cécile Mailer (Bath), Torsten Mütze (Warwick), Louis Theran (St Andrews), Martin Anthony (LSE), Annika Heckel (Uppsala), Nina Kamev (Zagreb), Guus Regts (Amsterdam), Paul Seymour (Princeton), Robin Wilson (Open University). We will also celebrate the contributions of Norman Biggs to combinatorics in the UK, in view of his recent 80th birthday. This Colloquium is supported by an LMS Conference grant.

OpenLB Lattice Boltzmann Spring School

Location: University of Greenwich
Date: 5–9 June 2023
Website: tinyurl.com/4kjcazde

The spring school introduces mathematicians, scientists, and engineers to the theory and application of the lattice Boltzmann method. It features two days of technical lectures followed by three days of tutorials and mentoring on the OpenLB software for studying flow. Participants will gain a deeper insight into the mathematical underpinnings of the lattice Boltzmann method and also experience of solving problems of practical importance. This spring school is supported by an LMS Conference grant.

Brunel Bioinformatics Workshop — Mathematical Concepts in Bioinformatics

Location: Brunel University
Date: 8–9 June 2023
Website: bbw2023.co.uk

The Computational Biology Group will be organising a 2-day hybrid workshop. The aim is to bring together well renowned speakers who will discuss mathematical solutions for problems within the field of Computational Biology. For more information and to register visit the website. This workshop is supported by an LMS Conference grant.

One-Day Meeting on Combinatorics

Location: University of Oxford
Date: 24 May 2023
Website: tinyurl.com/3bsktfjf

The meeting will take place in the Mathematical Institute, with talks from 10.45am and coffee available beforehand from 10.15am. This year's speakers are Marthe Bonamy (Bordeaux), János Pach (Rényi Institute, Hungary and IST Austria), Julian Sahasrabudhe (Cambridge), Mehtaab Sawhney (Cambridge/MIT), Mathias Schacht (Hamburg) and Maya Stein (University of Chile). Further details can be found on the website. Some funds are available to support research students to attend the meeting. The meeting is supported by an LMS Scheme 1 grant and the British Combinatorial Committee.

Workshop and School on Complex Lagrangians, Integrable Systems, and Quantization

Location: Oxford University
Date: 5–10 June 2023
Website: bit.ly/FRG-Oxford

This event aims to bring together leading experts and rising stars from different areas of mathematics and physics to explore the interrelations between complex Lagrangians, integrable systems, and quantization. The carefully curated programme and layout aims to foster a sense of unity and interaction among all participants and speakers. Funding available for students and post-docs. The event is supported by an LMS Conference Grant.

AGGITatE 2023: Algebras, Groups, Geometry, Invariants and Related Topics at Essex

Location: University of Essex
Date: 19–21 June 2023
Website: tinyurl.com/aggitate2023

This workshop aims to bring together researchers in algebraic groups and geometric invariant theory. This year's theme is *Algebraic Groups and the Cremona Group*. Speakers will include M. Bate, I. Cheltsov, A. Fanelli, A. Lonjou, P. Lins, M. Stanojkovski, D. Stewart and S. Zimmermann. Deadline for registration with funding: 26 May. Jointly funded by the Compositio Foundation and an LMS Scheme 1 grant.

LMS Meeting

LMS Invited Lecture Series 2023

17–21 July 2023, University of Durham

Website: bit.ly/3MOX9ZG

The Invited Lecturer in 2023 is Filippo Santambrogio (Université Lyon 1, France), whose talk will be titled *Optimal Transport and its Applications*. There will be accompanying lectures by:

(1) David Bourne (Heriot-Watt University)
(2) Sara Farinelli (Lagrange Centre, Paris, France)
(3) Hugo Lavenant (Bocconi University, Milan, Italy)
(4) Emanuela Radici (University of L'Aquila, Italy)
(5) Matthew Thorpe (University of Manchester)

Funds are available for partial support to attend. Requests with an estimate of expenses should be addressed to the organiser, Dr Alpár Mészáros (alpar.r.meszaros@durham.ac.uk).

The annual Invited Lecturers Series aim to bring a distinguished overseas mathematician to the UK to present a small course of about ten lectures spread over a week. Each course of Invited Lectures is on a major field of current mathematical research, and is instructional in nature, being directed both at graduate students beginning research and at established mathematicians who wish to learn about a field outside their own research specialism.

London/Oxford/Warwick Financial Mathematics Conference

Location: Bush House, London
Date: 22–23 June 2023
Website: tinyurl.com/2ujv6p2r

This conference is one of an ongoing series of events to bring together mathematicians from three centres of UK financial mathematics research with the aim of developing innovative collaborations between researchers in the UK, as well as building a relationship between financial mathematicians and potential industry partners, and to provide a platform for junior academics to present their work. Please register on the event website. The conference is supported by an LMS Scheme 1 grant.

Queer and Trans Mathematicians in Combinatorics

Location: Queen Mary University of London
Date: 6-7 July 2023
Website: queertransmath.com

QTMC is a research conference in combinatorics, broadly construed, meant to build and bring visibility to the queer and trans community in combinatorics. We will have invited talks by Bethany Marsh (Leeds) and Günther Ziegler (FU Berlin), several contributed talks, panel discussion, and more. Deadlines: 5 May to submit an abstract or apply for funding; 30 June to register. The conference is supported by an LMS Conference Grant.

Numerical Methods for Mean Field Games and Related PDE Workshop

Location: University College London
Date: 6–7 July 2023
Website: tinyurl.com/UCLMFG

This workshop will gather leading UK and international researchers on the analysis, numerical approximation and applications of Mean Field Games (MFG) and related Partial Differential Equations (PDE) such as Hamilton-Jacobi-Bellman equations. It features 10 invited speakers with topics including, but not limited to, numerical analysis and PDE analysis of MFG and nonlinear PDE, high-dimensional problems, optimal control and stochastic processes, scientific computing, as well as applications in financial mathematics and engineering. This event is supported by an LMS Conference grant.

European School of Information Theory 2023

Location: University of Bristol
Date: 17–21 July 2023
Website: tinyurl.com/5n985ypb

This five day summer school features multiple long-form tutorials and shorter seminars on topics of current interest in information theory, theoretical computer science, and machine learning. This is an annual educational event organised by the Information Theory Society, and is supported by an LMS Conference Grant, the Heilbronn Institute for Mathematical Research, the Information Theory Society, Huawei, and the School of Mathematics at the University of Bristol.

LMS Northern Regional Meeting & Workshop 2023

Location: University of York
Date: 4–6 September 2023
Website: bit.ly/3N7x9cv

This meeting forms part of the Northern Regional Workshop on Modular Lie Theory on 5-6 September 2023. The meeting will open with Society Business, during which members can sign the Members' Book, followed by talks from Anne Schilling (UC Davis; *The Mystery of Plethym*), Lewis Topley (Bath; *What the W?*) and Beth Romano (KCL; *Graded Lie Algebras and Applications to Number Theory*).

Stability and Dynamics in Fluid Mechanics and Kinetic Theory

Location: Imperial College London
Date: 10–14 July 2023
Website: tinyurl.com/5n8ttx7m

This summer school/workshop provides a unique learning opportunity for PhD students and postdocs with research interests at the interface of fluid mechanics and kinetic theory. Led by L. Saint-Raymond (IHES) and T. Drivas (Stony Brook), the mini-courses delve into specific topics within the fields, complemented by seven seminars that offer additional insights into advanced concepts. This workshop is a must-attend event for anyone interested in the topic. The workshop is supported by an LMS Conference Grant.

Classical, Un-classical and Semi-Classical Problems in Operator Theory

Location: Cardiff University
Date: 26–27 July 2023
Website: tinyurl.com/4k9f7htn

A meeting in memory of Brian Malcolm Brown (1946–2022), who was a prolific organiser of meetings and conferences: these included an Oberwolfach workshop, the AGA (2007) and INV (2011) programmes at the Isaac Newton Institute and the Gregynog meetings on spectral theory. He brought together researchers from across the mathematical sciences to work on pure and applied problems which could benefit from rigorous analytical and novel computational approaches, and was in particular a strong supporter of early career researchers. This meeting is supported by an LMS Conference Grant.

7th IMA Conference on Mathematics in Defence and Security

Location: Imperial College, London
Date: 7 September 2023
Website: tinyurl.com/4j9h4j87

This conference aims to bring together a wide variety of participants and topics applying a variety of mathematical methods with defence and security applications. It is aimed towards mathematicians, scientists and engineers from both industry and academia, in addition to government and military personnel who have an interest in how mathematics can be applied to defence and security problems.

Society Meetings and Events

June

6-7 Waves by the Thames

26-30 LMS Research School Random Structures, Applied Probability and Computation, Liverpool

22-23 Diophantine Approximation, Dynamics, and Fractals

30 LMS General Meeting & Hardy Lecture, London

July

11-14 Early Career Researchers in Mathematics (ECRM) 2023

16-28 LMS Undergraduate Summer School, Sheffield

17-21 LMS Research School Machine Learning in Mathematics and Theoretical Physics, Oxford

17-21 LMS Invited Lecture Series 2023, Durham

24-28 LMS Research School Algebraic Groups and their Representations, Birmingham

24–4 Aug LMS-Bath Mathematical Symposium Operators, Asymptotics, Waves, hosted at Bath

August

1-11 LMS-Bath Mathematical Symposium Categorical and Geometric Representation Theory, Bath

Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website (www.lms.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

May

9-10 At the Interface of Asymptotics, Conformal Methods and Analysis in General Relativity, Royal Society, London (505)

10-11 Colloquium in Combinatorics, QMUL and LSE, London (506)

22-23 Scottish Combinatorics Meeting, University of Strathclyde, Glasgow (504)

22-24 Gregynog Welsh Mathematics Colloquium 2023, Gregynog Hall, Tregynon, Powys (504)

June

5-9 OpenLB Lattice Boltzmann Spring School, University of Greenwich, London (506)

5-10 Workshop and School on Complex Lagrangians, Integrable Systems, and Quantization, Oxford (506)

8-9 Brunel Bioinformatics Workshop – Mathematical Concepts in Bioinformatics, Brunel University (506)

29-21 AGGITatE 2023: Algebras, Groups, Geometry, Invariants and Related Topics at Essex

19-23 Probabilistic Group Theory CMI-HIMR Summer School, University of Bristol (504)

4-6 Modelling in Industrial Maintenance and Reliability, Nottingham (505)

10-14 Remembering Victor Snaith: Topology, Number Theory and Interactions, University of Bristol (504)

22-23 London/Oxford/Warwick Financial Mathematics Conference, Bush House, London (506)

24-28 Algebraic Groups and their Representations LMS Research School, University of Birmingham (504)