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## COVER IMAGE

The cover image is Figure 3 ('Music–Maths Sketching') from Emily Howard's article 'Torus – sphere – Antisphere' (page 23).

Do you have an image of mathematical interest that may be included on the front cover of a future issue? Email [images@lms.ac.uk](mailto:images@lms.ac.uk) for details.

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## LMS NEWS

## Update on the LMS *Newsletter*

Readers may recall that there has been a debate about continuing to print the LMS *Newsletter* for those members that wish to receive a print version. This has been prompted by environmental as well as financial concerns. The LMS is committed to operating sustainably, which militates against producing large quantities of printed material, particularly if it is not read. In addition, decreasing income to the Society from its publications as we move towards an open access publications model has forced the Society to consider its spending priorities.

A membership survey was carried out last year and the results were very supportive of continuing to print for those members who opt in, notwithstanding the desire to be more sustainable as an organisation. Members were also asked about the frequency of the *Newsletter*. Most accepted that reducing the frequency from 6 issues per year to 4 and continuing to print would be preferable to ceasing print altogether. There were also many supportive comments from members about what they enjoy about the *Newsletter* content and format.

LMS Council has taken note of the response from members and has made the decision to continue to budget for printing for the next three years, assuming overall income remains stable during this time, with a reduction in the number of issues from 6 to 4 per year. It was felt that this compromise would enable the Newsletter Board to maintain the quality of the *Newsletter*, support those members who really appreciate print, and go some way towards achieving the desired reductions in carbon footprint and costs. The Newsletter Board has taken this on board and will be moving to a 4-issue model from September this year. Issues will be published in September, December, February and May.

Many thanks to all members who responded to the survey.

Professor Cathy Hobbs  
LMS Vice-President

## *Bulletin* and *Journal* get a revamp



At its Strategy Retreat last year, Publications Committee decided to restructure the editorial board of the LMS *Bulletin* and *Journal*. Rather than remaining a single, homogeneous board, with all final acceptances made by the Managing Editors, in future the board will be organized into seven subject-based sections under Section Editors who will have the authority to accept papers.

There are many reasons for this. Most importantly, we wish to bring the decisions closer to where the specialist expertise resides. Moreover, as the journals are no longer printed, there are no longer any page constraints. This means that the quality threshold for acceptance can now be absolute as opposed to relative; where formerly choices had to be made between papers competing for space, now each paper will be judged on its qualities alone. We are also drawing back from the concept of the *Journal* and *Bulletin* as 'generalist' journals that only accept papers of general interest: we are now willing to accept more specialized papers, although we do want to insist on well-written introductions which set the research in a context that a wider community of mathematicians can understand.

Above all, we want the journals to build a reputation for making quick, fair decisions at the earliest appropriate moment, and we want as far as possible to avoid rejections at a very late stage, and excessively long periods between initial submission and final decision.

We expect that, as a result of these changes and in line with recent trends, more papers will be published. As

we move towards Open Access (already the proportion of Open Access in the LMS journals is nearly 50%), this will result in stabilization of net publishing income to the LMS — which, in line with its charitable objectives, the Society spends on mathematics, for example through a multitude of small grants to mathematicians which we know are greatly appreciated.

If all goes to plan, the switchover to the new structure will happen in the early autumn. From an author's perspective, there should be very little difference — you submit to your choice of Handling Editor, as at present. But behind the scenes we expect that the *Bulletin* and *Journal* will be better able to make good decisions quickly, and we encourage you to consider them as destinations for your research.

The size of the editorial board is likely to increase to accommodate these changes. The role of the Managing Editors will also change. Our current *Journal* editors, Mark Haskins and James Maynard FRS, have performed heroic feats in handling very large numbers of papers (while still finding time to prove Fields Medal-winning results); their successors Caroline Series FRS and Stuart White should not have to do the same. Their role, and that of the *Bulletin* editors Andrey Lazarev and Sibylle Schroll, will be to oversee and ensure quality and balance between the different sections.

Professor Niall MacKay  
LMS Publications Secretary

## Atiyah UK–Lebanon Fellowships 2023–24



Georges Habib (left) and Julia Wolf

The Atiyah UK–Lebanon Fellowships were set up in 2019 as a lasting memorial to Sir Michael Atiyah (1929–2019) in the form of a two-way visiting programme for mathematicians between the UK and the Lebanon, where Sir Michael had strong ties. The

Fellowships are a partnership between the Centre for Advanced Mathematics (CAMs) at the American University of Beirut and the LMS.

The LMS is delighted to announce that two Fellowships have been awarded for the academic year 2023–24, with one Fellow travelling in each direction. This has been made possible by generous additional funding from the ICMS in Edinburgh and from CAMs, to whom we are very grateful.

Georges Habib will be visiting the UK for a period of three months during 2024. He has been Professor of Mathematics at the Lebanese University in Beirut since 2008. His field of research is geometric analysis, in particular the study of the spectrum of several differential operators on manifolds (Laplacian, Dirac operator, etc). He will be visiting the University of Durham for three months from July to September 2024 to work with Norbert Peyerimhoff and Katie Gittins on the study of the magnetic Steklov operator on differential forms.

Julia Wolf is a Professor of Pure Mathematics in the Department of Pure Mathematics and Mathematical Statistics at her alma mater, the University of Cambridge. Her research interests are mostly discrete in nature, and lie at the intersection of combinatorics, number theory and harmonic analysis.

She plans to make two visits to Lebanon in 2024 of around 4 weeks each, hosted by the Centre for Advanced Mathematics at the American University of Beirut, where she will teach a sequence of short graduate courses in discrete mathematics, broadly conceived. She also hopes to be able to interact with female students of mathematics at all levels, and to participate in local outreach activities.

The 2022–23 Fellowship was awarded to Dr Rémi Mokdad Mokdad, from Lebanon but currently a postdoc at the University of Burgundy, Dijon, France, to visit Dr Juan Valiente Kroon at QMUL.

An account by the 2020–21 Fellow Professor Maciej Dunajski of his delayed visit to CAMs and impressions of Lebanon is on page 21.

For further information about the Fellowships and information on how to apply, please visit [lms.ac.uk/grants/atiyah-uk-lebanon-fellowships](https://lms.ac.uk/grants/atiyah-uk-lebanon-fellowships).

Applications for Fellowships to be held the academic year 2024–25 will open in early September 2023, with a closing date of 31 January 2024.

Professor Caroline Series FRS  
Chair of the Atiyah UK–Lebanon Fellowships Panel

## *Moduli* — A New Mathematics Journal

As the Managing Editors, we are pleased to introduce you to *Moduli*, a new open access journal owned by the Foundation Compositio Mathematica and published in collaboration with the LMS and Cambridge University Press.

Why a journal on moduli theory? The term ‘moduli’ first appeared in Riemann’s work to describe the quantities which parameterize (in modern terms) complex structures on a two-dimensional surface. Moduli spaces and their theory are now a fundamental topic that shows up in different forms across much of modern mathematics. Researchers approach the study of moduli problems using a vast array of techniques from algebraic, differential and arithmetic geometry, combinatorics, dynamical systems, gauge theory, geometric analysis, geometric group theory, mathematical physics, representation theory, and topology. Interactions across these different areas of mathematics are vital and the different perspectives add to the richness of the topic.

Our goal with *Moduli* is to create a common forum, to bring researchers from different corners of mathematics together and build a community around the common theme of moduli theory. The journal will facilitate communications across disciplines in a way which subject specific journals might miss. To promote this community-building, the managing editors of *Moduli* also plan to organize a series of events loosely affiliated with the journal. The first such event will be the three-week Summer Workshop in Mathematics taking place on 1–19 July 2024 at the Simons Center for Geometry and Physics (Stony Brook, NY), which will be dedicated to *Moduli*.

The question “There really is no journal dedicated to moduli theory — why don’t we create one?” was

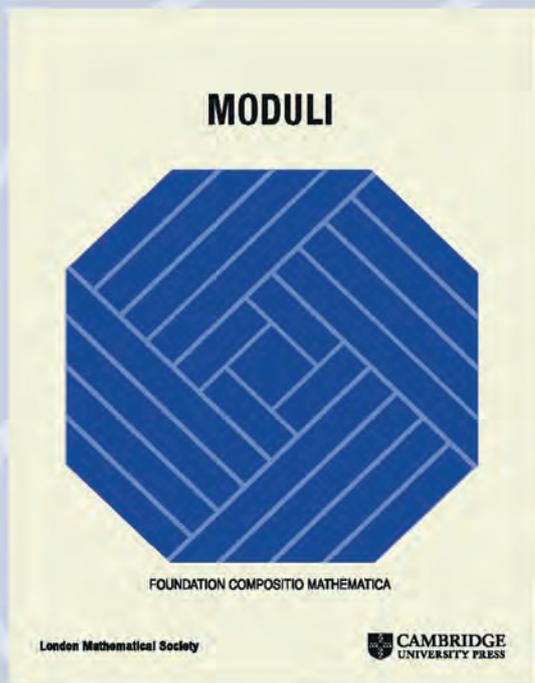
posed by one of us (Jørgen) to the steering committee for the Vector Bundles on Algebraic Curves (VBAC) group as we were discussing how to cope with retirement of the group’s founder and long-time leader, Peter Newstead. Created in the early 1990s, this group has met for an annual workshop every year with just one small gap in 1995/1996 and one COVID hiatus in 2021. These annual gatherings anchored and inspired more than one generation of researchers in a broad range of topics. After guiding the group for over thirty years, Peter retired from his active leadership in 2021. While we discussed Peter’s legacy, and life in VBAC without him at the helm, we reflected on how valuable it has been to have a community built around this corner of moduli theory. That is when we realized the potential for an even wider, interdisciplinary community, united around moduli theory more broadly.

Four members of the VBAC steering committee were joined by three other researchers in moduli theory to form the founding managing editorial board. We pitched the idea to the Foundation Compositio Mathematica, and *Moduli* was born. The new journal welcomes original research contributions of the highest quality on all aspects of moduli, and emphasises clarity of exposition to enable all papers to bridge as many boundaries as possible within the target audience.

We encourage you and the entire mathematical community to participate in this exciting new venture by submitting high-quality papers in the broad area of moduli spaces and their theory to *Moduli*. We look forward to your submissions (see our website at [moduli.nl](http://moduli.nl) for more information).

Jørgen Ellegaard Andersen, Steve Bradlow, Dan Halpern-Leistner, Vicky Hoskins, Frances Kirwan, Margarida Melo, and Anna Wienhard

# NEW FROM COMPOSITIO MATHEMATICA



## MODULI

### A new open access journal

Supported by an Editorial Board of world-renowned researchers, *Moduli* provides a new forum for significant new results on all aspects of moduli theory or related mathematics.

#### MANAGING EDITORS:

Jørgen Ellegaard Andersen, Steve Bradlow, Daniel Halpern-Leistner, Victoria Hoskins, Frances Kirwan, Margarida Melo, Anna Wienhard

[cambridge.org/mod](https://cambridge.org/mod)

#### EXPLORE

The full Editorial Board



#### DISCOVER

Our aims and the full scope



## COMPOSITIO MATHEMATICA

*Moduli* follows the success of *Compositio Mathematica* – a prestigious, well-established journal publishing first-class research papers that traditionally focus on the mainstream of pure mathematics.



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## *Sbornik, Izvestiya and Russian Mathematical Surveys*

It fills me with sadness to report that the London Mathematical Society, IOP Publishing and the Russian Academy of Sciences (RAS) have jointly terminated the agreements to publish the journals *Sbornik:Mathematics*, *Izvestiya:Mathematics* and *Russian Mathematical Surveys* in English translation.

For over 60 years and through the dark times of the cold war, the LMS has collaborated with the RAS to publish these journals. It is thus deeply regrettable that this long-standing partnership with fellow mathematicians has needed to come to an end.

In March 2022, the LMS published a statement to condemn the invasion of Ukraine and established a working group to coordinate the Society's response. UK financial sanctions frustrated operations, and publication of the journals was suspended in

July 2022. In late 2022, a decision was taken to explore the formal termination of the publishing agreements due to the continuation of the sanctions and the closure of the IOP Publishing Moscow office, which effectively brought a permanent halt to the publishing arrangements.

It has been agreed to transfer the publication of the English editions to the Steklov Mathematical Institute of the RAS in order to ensure that publication can resume as quickly as possible. These journals serve an important role for the mathematical community in disseminating the outputs of Russian mathematical research, coupled with their international standing and proud tradition (since 1866) that parallels that of the Society's own journals.

An updated summary of actions the Society has been taking since the invasion of Ukraine can be found at [lms.ac.uk/policy/ukraine-response](https://lms.ac.uk/policy/ukraine-response).

Professor Ulrike Tillmann  
LMS President

## OTHER NEWS

### Wolf Prize in Mathematics 2023

The 2023 Wolf Prize in Mathematics has been awarded to Ingrid Daubechies (Duke University USA), for work in wavelet theory and applied harmonic analysis. Ingrid is an LMS Honorary Member, elected in 2007.

Ingrid Daubechies is a Belgian mathematician and physicist at Duke University in Durham, North Carolina. She earned her bachelor's degree in physics from the Free University of Brussels in 1975. She then continued her research at the same university, earning her doctorate in physics with a thesis on the Representation of quantum mechanical operators by kernels on Hilbert spaces of analytic functions.

Read more at [bit.ly/wolf-prize-2023](https://bit.ly/wolf-prize-2023).

### Academy for the Mathematical Sciences: Policy Specialist Roles

The Academy for Mathematical Sciences is establishing a Policy Unit and has two vacancies for roles they wish to fill: Policy Manager and Policy Analyst.

These roles will provide the opportunity to proactively contribute to national discussions on policy topics that require mathematical sciences input, from artificial intelligence and quantum computing to net zero initiatives, circular economy, biodiversity, and ethics, in addition to playing a vital role in shaping policy reports, papers, and communications on issues concerning the mathematical sciences including education, funding and recruitment. The closing date for applications for both roles is Thursday 22 June. See more information on the Academy for Mathematical Sciences website: [bit.ly/acadmthsci-jobs](https://bit.ly/acadmthsci-jobs).

### Royal Society Fellows 2023

Eighty outstanding researchers, innovators and communicators from around the world have been elected as the newest Fellows of the Royal Society, the UK's national academy of sciences and the oldest science academy in continuous existence. The following have LMS connections:

- Professor Sourav Chatterjee FRS (Current Member of the Editorial Board of the Proceedings of the LMS) Professor of Statistics, Department of Statistics,

and Professor of Mathematics, Department of Mathematics, Stanford University, United States

- Professor Shafi Goldwasser, ForMemRS (LMS Honorary Member) Director, Simons Institute for the Theory of Computing, University of California, Berkeley, United States
- Professor James Maynard FRS (Managing Editor of the Journal of the LMS) Professor of Number Theory, Mathematical Institute, University of Oxford
- Professor Ivan Smith FRS (Managing Editor of the LMS Journal of Topology) Professor of Geometry, Centre for Mathematical Sciences, University of Cambridge
- Professor Karen Uhlenbeck, ForMemRS (LMS Honorary Member) Distinguished Visiting Professor, Institute for Advanced Study, United States

## STEM for BRITAIN 2023



Left to right: Chris Hickey, Arkady Wey, Jenny Power.  
Credit: The Parliamentary & Scientific Committee and John Deehan Photography

Congratulations to Arkady Wey (Oxford) for winning the gold award for Mathematics in the 2023 STEM for BRITAIN awards, which took place in the Houses of Parliament on Monday 6th March, during British Science Week.

This annual event sees early career researchers in science, engineering and mathematics awarded for presenting posters which best communicated their high-level research in their respective discipline to a lay audience. A total of 120 posters are chosen across all subjects for a poster exhibition in parliament, and a panel of expert judges chooses three medallists for each discipline.

“I’m surprised and thrilled,” Wey said on receiving the gold award. “I’m very near to the end of my PhD, and I applied for this competition as a way of

rounding off my studies and presenting my research. My main feeling is one of pride that my work has been recognised on this platform. I’m less interested in maths for maths’ sake, and I want to produce something with real world, social impact. Today feels like a great success.”

Arkady’s poster was titled ‘A Multiscale Mathematical Model For Filtration’ and was followed by Jenny Power (Bath) who garnered the silver award with ‘Making Cancer Treatments Safer With Mathematics’, while Chris Hickey (Arup) won the bronze award for ‘Seismic Simulations: Implementing And Improving A New Dynamic Structural Analysis’.

STEM for BRITAIN is organised by the Parliamentary & Scientific Committee and has been held in Parliament since 1997. Chaired by Stephen Metcalfe MP, its aim is to give members of the Houses of Parliament an insight into the outstanding research work being undertaken in UK universities by early career researchers. It is supported by the Council for the Mathematical Sciences, the Clay Mathematics Institute, the Heilbronn Institute, United Kingdom Research and Innovation alongside other important scientific, engineering and mathematics institutions and organisations.

Kieran O’Connor  
LMS Events Co-ordinator

## AIMS News

In January this year AIMS South Africa launched an experimental new applications portal for their AI for Science stream with the aim of assessing students’ mathematical, problem-solving and coding abilities. Candidates were given a mix of questions to answer, and the results yielded valuable data on how mathematical challenges are approached across Africa, and which areas of mathematics and the computational sciences require more targeted investment across the continent. Here is one of the shorter questions, which readers may like to contemplate. The solution will be published in their Newsletter.

*40 identical balls are rolling along a straight line. They all have speed equal to  $v$ , but some of them might move in opposite directions. When two of them collide they immediately switch their direction and keep the speed  $v$ . What is the maximal number of collisions that can happen?*

For information about AIMS South Africa see [aims.ac.za](http://aims.ac.za).

## MATHEMATICS POLICY DIGEST

## Maths to 18 Advisory Group

A Maths to 18 Expert Advisory Group has been formed to provide advice to the Prime Minister and Secretary of State for Education on how to implement the mission for all young people to study maths to age 18.

The advisory group will collate evidence and provide advice on the essential maths knowledge and skills young people need to study and how this should be reflected in a post-16 education system.

The membership of the group is as follows:

- Dr Maggie Aderin-Pocock MBE (Chancellor, University of Leicester)
- Peter Cooper (Executive Principal and CEO, Heart of Mercia Multi-Academy Trust)
- Lucy-Marie Hagues (CEO, Capital One UK)
- Professor Jeremy Hodgen (Professor of Mathematics Education, University College London)
- Simon Lebus (Non-Executive Chairman, Sparx)
- Tim Oates CBE (Group Director of Assessment Research and Development, Cambridge University Press and Assessment)
- Charlie Stripp MBE (CEO, MEI and Director, National Centre for Excellence in Teaching of Mathematics)
- Fionnuala Swann (Assistant Principal (Academic), Nelson and Colne College Group).

For further details on the Advisory's Group aims, visit the website: [bit.ly/mathsto18-advisory](https://bit.ly/mathsto18-advisory)

## 'Broken promise' on Maths Puts Science Plans in Peril

On 27 March, Tom Whipple, Science Editor for *The Times*, covered the 'broken promise' made to those in the mathematics community on page 7 of the day's printed edition of the newspaper.

The report covers the £300 million promised additional funding for the mathematical sciences, made by the former Chancellor in January 2020. The UK Government announced that it would be allocating the

additional funding to "fund experimental and imaginative mathematical sciences research by the very best global talent over the next five years". However, more than half of this funding is yet to materialise.

Sir Roger Penrose, Nobel Laureate and Emeritus Professor of Mathematics at the University of Oxford, commented that:

"The government keeps saying the right things about making the UK a STEM [science, technology, engineering and maths] superpower. It is maths that will underpin that goal because it's the mathematical sciences that drive all the innovations that the government likes to reference, such as AI and quantum computing. What we need to see now is action and funding."

Sir Roger also highlighted the impending financial cuts to mathematics departments across the country, carrying the potential to create swathes of 'maths deserts', areas with no opportunities to study mathematics beyond A-Level, across the country. He said: "We are seeing university maths departments at many institutions, including my own alma mater at Birkbeck, threatened, trimmed and even closed down."

Last year, James Maynard, the 2022 Fields Medalist, noted the mixed messages coming from the government on the place of mathematics within STEM:

"It sometimes feels like the government forgets about the M in STEM. To be a science superpower and to lead the world in AI and new technologies [...] we must foster and support the mathematical sciences."

When asked for comment on the piece, The Department for Science, Innovation and Technology said:

"We are already providing an additional £124 million funding for mathematical sciences, on top of up to £30 million a year for grants, fellowships and to support maths students." This further suggests that the mathematical sciences will continue to be the victim of broken promises and mixed messages, placing the discipline at ever-increasing risk.

## Horizon Europe Update

The Department for Science, Innovation and Technology has announced a three-month extension to provide support for UK Horizon Europe applicants until the end of September 2023. Successful applicants will receive

the full value of their funding at their UK host institution for the lifetime of the grant. Successful awardees do not need to leave the UK to receive this funding. The Government is still in negotiations over the UK's involvement in research programmes.

## Protect Pure Maths: Latest Activities

*Update provided by Connect PA*

Protect Pure Maths (PPM) continues to work with colleagues across the mathematical sciences community to influence policy and decision making at the highest level.

### Influencing No 10

In the late summer of 2022, PPM launched a 'maths manifesto' ahead of the election of the new Conservative leader.

The Prime Minister personally committed to this agenda in his New Year's speech, when he announced his policy for all students to study some form of mathematics until the age of 18. PPM spokespeople were featured across a number of media outlets including Sky News, BBC News, LBC and the Spectator and LBC.

The Prime Minister returned to this subject in April, and PPM's links with No 10 secured a quote from Professor Ulrike Tillmann in the Government press release, a sign that Government is starting to listen. The campaign's profile was again boosted by appearances on radio and TV by Professor Tillmann, Professor Nira Chamberlain, Dr Tom Crawford and Zubin Siganporia.

### Maths in and around Parliament

Protect Pure Maths continues to actively contribute to Parliamentary and Government business and has submitted evidence to the Government consultation into a new scheme for R&D tax credits and to Select Committee inquiries into the governance of artificial intelligence and into Teacher recruitment, training, and retention. Members of the Steering Group met with Robin Walker MP, Chair of the Education Committee to discuss in further detail.

Professor Ulrike Tillmann was directly quoted in the Science and Technology Select Committee's report into Diversity and Inclusion in STEM to which she gave oral evidence earlier in 2022. Following meetings with the Parliamentary Office for Science and Technology (POST), we are exploring ways to raise the visibility of maths in policymaking.

PPM is contributing to debates, working with Sir Stephen Timms on the first ever parliamentary debate dedicated to mathematics, and Bob Blackman MP to mark National Numeracy Day.

We have continued to build cross-party relationships and have secured endorsements from Lord Ravensdale, The Rt Hon Lord Mandelson, Kirsty Blackman MP, and Anthony Browne MP.

### Looking ahead

With a General Election likely in 2024, PPM is working to influence manifesto development across the political parties, including submitting evidence to the Labour Party's National Policy Forum and meeting with Matt Western MP, Shadow Minister for Higher Education.

### Women in Maths Day

To celebrate Women in Maths Day, PPM worked with the Piscopia Initiative and the Press and Journal to publish an article on Women in Maths Day: Outdated attitudes still hold too many back. We have continued to build links with the wider mathematical sciences community, including bringing on Caoimhe Rooney, a NASA mathematician and member of the Mathematigals as a supporter of the campaign, and working with Dr Eugenia Cheng to promote her work on 'Mathematical thinking for a new gender approach' to mark International Women's Day.

### University Cuts

We are working with the learned societies to support those academics in mathematics departments that may be at risk of cuts. Most recently, this has focused on Birkbeck and Brighton universities. This has included direct engagement with Vice-Chancellors, and encouraging local politicians to press the Government to intervene. We are in regular contact with the responsible Ministers and are using the media to build pressure, for example Professor Tillmann's article for the Times Higher Education on why the UK cannot afford to cut the 'M' from STEM.

We are in active conversation with those affected about the next steps and more broadly, PPM is exploring a future campaign which focuses on the impact of the trend towards cuts at lower tariff universities and the possible creation of 'maths deserts'.

Digest prepared by Kieran O'Connor  
LMS Events Co-ordinator

*Note: items included in the Mathematics Policy Digest are not necessarily endorsed by the Editorial Board or the LMS.*

## OPPORTUNITIES

## LMS Grant Schemes

The next closing date for research grant applications (Schemes 1, 2, 4, 5, and 6) is 15 September 2023. Applications are invited for the following grants to be considered by the Research Grants Committee at its October 2023 meeting. Applicants for LMS grants should be mathematicians based in the UK, the Isle of Man or the Channel Islands. For grants to support conferences/workshops, the event must be held in the UK, the Isle of Man or the Channel Islands:

### Conferences (Scheme 1)

Grants of up to £5,500 are available to provide partial support for conferences. This includes travel, accommodation and subsistence expenses for principal speakers, UK-based research students, participants from Scheme 5 countries and caring costs for attendees who have dependents.

### Visits to the UK (Scheme 2)

Grants of up to £1,500 are available to provide partial support for a visitor who will give lectures in at least three separate institutions. Awards are made to the host towards the travel, accommodation and subsistence costs of the visitor. Potential applicants should note that it is expected the host institutions will contribute to the costs of the visitor. In addition, the Society allows a further amount (of up to £200) to cover caring costs for those who have dependents.

### Joint Research Groups in the UK (Scheme 3) — Application deadline 30 September 2023

Grants of up to £1,500 are available to support joint research meetings held by mathematicians who have a common research interest and who wish to engage in collaborative activities, working in at least three different locations (of which at least two must be in the UK).

### Research in Pairs (Scheme 4)

For those mathematicians inviting a collaborator, grants of up to £1,200 are available to support a visit for collaborative research either by the grant holder to another institution abroad, or by a named mathematician from abroad to the home base of the grant holder. For those mathematicians collaborating with another UK-based mathematician, grants of

up to £600 are available to support a visit for collaborative research either by the grant holder to another institution or by a named mathematician to the home base of the grant holder. In addition, the Society allows a further amount (of up to £200) to cover caring costs for those who have dependents.

### Research Reboot (Scheme 4)

Grants of up to £500 for accommodation, subsistence and travel plus an additional £500 for caring costs are available to assist UK mathematicians who may have found themselves with very little time for research due to illness, caring responsibilities, increased teaching or administrative loads, or other factors. This scheme offers funding so that they can leave their usual environment to focus entirely on research for a period from two days to a week. For applications submitted by the next deadline (15 September 2023), the reboot retreats should take place between 01 November 2023 and 30 January 2024.

### Collaborations with Developing Countries (Scheme 5)

For those mathematicians inviting a collaborator to the UK, grants of up to £3,000 are available to support a visit for collaborative research, by a named mathematician from a country in which mathematics could be considered to be in a disadvantaged position, to the home base of the grant holder. For those mathematicians going to their collaborator's institution, grants of up to £2,000 are available to support a visit for collaborative research by the grant holder to a country in which mathematics could be considered to be in a disadvantaged position. Applicants will be expected to explain in their application why the proposed country fits the circumstances considered eligible for Scheme 5 funding. In addition, the Society allows a further amount (of up to £200) to cover caring costs for those who have dependents. Contact the Grants Team if you are unsure whether the proposed country is eligible or check the IMU's Commission for Developing Countries definition of developing countries ([tinyurl.com/y9dw364o](https://tinyurl.com/y9dw364o)).

### Research Workshop Grants (Scheme 6)

Grants of up to £10,000 are available to provide support for Research Workshops. Research Workshops should be an opportunity for a small group of active researchers to work together for a concentrated period on a specialised topic. Applications for Research Workshop Grants can be made at any time but should normally be submitted at least six months before the proposed workshop.

The next closing date for early career research grant applications (Schemes 8, 9 and ECR Travel Grants) is 15 October 2023. Applications are invited for the following grants to be considered by the Early Career Research Committee at its November 2023 meeting:

### Postgraduate Research Conferences (Scheme 8)

Grants of up to £2,500 are available to provide partial support for conferences, which are organised by and are for postgraduate research students. The grant award will be used to cover the costs of participants. In addition, the Society allows the use of the grant to cover to cover caring costs for those who have dependents.

### Celebrating New Appointments (Scheme 9)

Grants of between £400 and £500 are available to provide partial support for meetings to celebrate the new appointment of a lecturer at a university. Potential applicants should note that it is expected that the grant holder will be one of the speakers at the conference. In addition, the Society allows the use of the grant to cover to cover caring costs for those who have dependents.

### ECR Travel Grants

Grants of up to £500 are available to provide partial travel and/or accommodation support for UK-based Early Career Researchers to attend conferences or undertake research visits either in the UK or overseas.

For full details of these grant schemes, and to find information on how to submit application forms, visit the LMS website: [lms.ac.uk/content/research-grants](https://lms.ac.uk/content/research-grants). Queries regarding applications can be addressed to the Grants Administrator Lucy Covington (020 7927 0807, [grants@lms.ac.uk](mailto:grants@lms.ac.uk)), who will be pleased to discuss proposals informally with potential applicants and give advice on the submission of an application.



# Call for Proposals

## RIMS Joint Research Activities 2023-2024

**Application deadline : August 31, 2023, 23:59 (JST)**

### Types of Joint Research Activities

- \*RIMS **Satellite** seminars 2024
- \*RIMS **Review** seminars 2024
- \*RIMS **Workshops Type C** 2024
- \*RIMS **Research Project** 2025

**More Information :** RIMS Int.JU/RC Website  
<https://www.kurims.kyoto-u.ac.jp/kyoten/en/>



京都大学  
KYOTO UNIVERSITY



Research Institute for  
Mathematical Sciences

## IMA–LMS Christopher Zeeman Medal Lectures

Article originally published in *Mathematics Today*, June 2023 issue.



Figure 3: Matt Parker and Simon Singh, centre, with their medals, and the two Presidents

Christopher Zeeman Medal to Dr Simon Singh MBE Hon FIMA, who gave his fascinating talk, *From Fermat's Last Theorem to The Simpsons to Tutoring Online*. Simon's article based on his talk is also included below.

### Every Interesting Bit of Maths Ever

Matt Parker described his journey from teacher to professional mathematics communicator, using interesting examples of mathematics throughout.

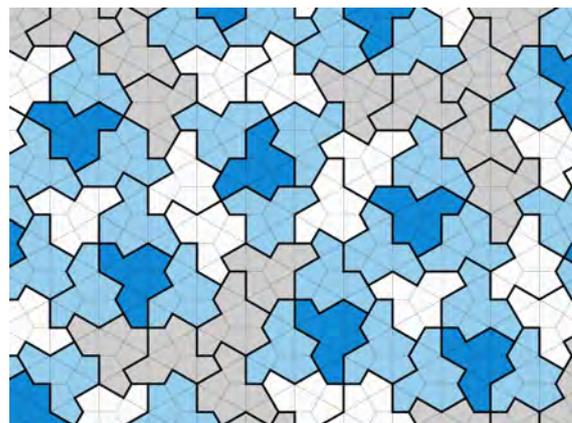
He started by giving the audience some up-to-date news! In the few days before the event, an aperiodic monotile had been published; this is an elegantly simple shape that tiles the plane but never periodically. Matt described how this was the culmination of 60 years of research, and rather excitingly the team have actually found an uncountably infinite family of tiles with this property.

He went on to describe his own tiling experiences, and his attempts to design his bathroom tiling patterns in ways that would require infinitely many tiles. For some reason the tilers did not seem keen on his plan which is a surprise as the job would have presumably proved rather lucrative! Sadly the bathroom was tiled with rectangles.

Matt and his wife, the physicist Professor Lucie Green who was in the audience, attempted a more mathematical approach in their garden paving. Matt has kindly provided us the photo on page 15 to prove it, but has begged the author to point out that it was taken before it was finished!

In 1978, Professor Sir Christopher Zeeman FRS became the first mathematician to deliver the Royal Institution Christmas Lectures. Then, in 2008, the IMA founded a medal in Sir Christopher's honour to recognise the contributions of mathematicians involved in promoting mathematics to the public and engaging with the public in mathematics in the UK. The medal is awarded by the Institute and the LMS. In March, the medals for 2020 and 2022 were awarded in a joint (and packed) ceremony at the Royal Society in London.

Matt Parker was awarded the 2020 Christopher Zeeman Medal by IMA President Professor Paul Glendinning FRSE FIMA and then gave an entertaining talk on *Every Interesting Bit of Maths Ever*. A report on Matt's talk can be found below. LMS President Professor Ulrike Tillmann FRS FIMA awarded the 2022



The first true aperiodic monotile defined by shape alone [cs.uwaterloo.ca/~csk/hat/](https://cs.uwaterloo.ca/~csk/hat/), CC BY4.0

Matt then introduced the topic of football stadia signage. He has previously campaigned against the image of a football made entirely of hexagons which is used on UK road signs.

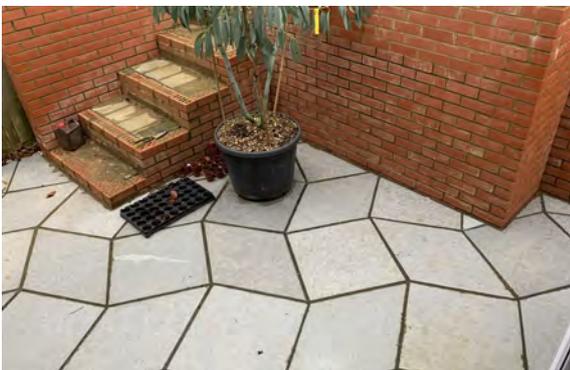
Following this Matt introduced his two books, focussing on his second, *Humble Pi: A Comedy of Maths Errors*, in which he described some of the (apparently) trivial maths errors that have led to major issues. One of the most memorable was a

story of a promotional offer by Pepsi, which implied you could collect tokens and gain a Harrier Jump Jet. This led to a court case where the definition of 'a joke' was argued over, and has led to a successful TV series describing the event. The main observation being: They didn't do the maths...

Matt described his work in the media, focussing on his description of how big a trillion is. He described how a typical human thinks logarithmically, so to most people a trillion is about as far from a billion, as a billion is from a million and so on. He used a time based approach to better illustrate these differences:

- A million ( $10^6$ ) seconds takes around about a week and a half.
- A billion ( $10^9$ ) seconds is around 32 years.
- A trillion ( $10^{12}$ ) seconds took us (at the time of the lecture) to Thursday 17 December 33711.

As well as using interesting and engaging mathematical stories, Matt described how important it is to use the media of our times to encourage the next generation. For example, he and his collaborators have embraced platforms like YouTube to great effect. Matt notes that YouTube has algorithms that are there to promote good content, so if mathematicians create good content and upload it, then the YouTube algorithms will really act in our favour by promoting that content.



Mathematical garden paving

One of the projects I suspect Matt really enjoyed, was creating geometric mirror balls for a festival, but the major topic in the final section of his talk was, perhaps surprisingly, almost a word game.

Matt has been working on a challenge to find five, five letter words, with no letters in common. His preferred coding

medium is Python, and using this code he developed a program that took about a month to solve the problem. The only set of five words that 'worked' were:

FJORD GUCKS NYMPH VIBEX WALTZ



Matt Parker talks on tiling

In case you are wondering, a guck is the hybrid offspring of a goose and duck, and a vibex is a species of sea snail or a large purple spot appearing under the skin.

In the month or so that Matt's Python code was running, other mathematical types got in on the act, and developed faster algorithms, some using graph theory, to get the same answer. In fact, in what feels like a 'race to the bottom' in a good way, Matt's fellow programmers managed to get the same answer in under a second, in fact in under a millisecond!

Matt thanked all of the amazingly talented people he has worked with, and acknowledged the support he has had from his wife, who was pointed at by the people around her every time Matt mentioned her!

There followed a question and answer session, and then the company adjourned for a lot of networking, especially amongst the maths communication community who had turned out to support Matt and Simon. Matt spent a long time talking to audience members after the event; in fact I left about an hour later and I do not think he had left the auditorium by that point!

The evening combining both talks was in the author's view amazingly successful, and the two speakers complemented each other (and indeed complimented each other) beautifully. We wait with bated breath for the next winner in 2024.

Edward Rothead CMath CSci FIMA  
Defence Science and Technology Laboratory

The views and opinions expressed in this article are not necessarily those of Dstl.

### From Fermat's Last Theorem to The Simpsons to Tutoring Online

I am not a great fan of books. I don't think I read a popular science or maths book before going to university and I don't think I have read one in the last decade. I earned a B grade on my O-level English Language paper, and I only ever read a novel if I was trying to impress someone. Probably a girl.

Despite my reluctance to read books, somehow I have a knack for writing them. In particular, I seem to be able to write so-called narrative non-fiction, intertwining human stories with mathematical and scientific explanations.

I suspect that my writing skills owe more to television than literature. I loved watching TV as a kid, everything from *TISWAS* to *Steptoe and Son*. And then I spent six years at the BBC Science Department, in particular working on *Tomorrow's World*, where I learned about writing scripts from some of the best in the business. If you can figure out how to explain the latest breakthrough in astronomy to an audience of five million, aged 8 to 80, in four minutes, then writing a story about Fermat's last theorem to dedicated, curious readers over the course of three hundred pages is a piece of cake.

I am proud of all my books and my TV work. Indeed, nothing pleases me more than meeting someone who says that they decided to study mathematics at university because they read one of my books at school or they watched my BBC documentary about Fermat's last theorem. The film is almost three decades old, so I now meet professors who tell me they were inspired to study mathematics as teenagers when they stared at their telly, learned about the notorious note of Pierre de Fermat and witnessed the passion of Sir Andrew Wiles.

If we go back a generation, then there were teenagers who watched Sir Christopher Zeeman's Royal Institution Christmas Lectures and who then decided to build their lives on a foundation of mathematics. Indeed, I recall watching those lectures — much better watching something on telly than reading a boring book — but something went slightly askew and I ended up studying physics, rather than maths.

While kids were watching Sir Christopher's Christmas Lectures or my documentary, we have to remember that 99% of students were receiving 99% of their mathematics in classrooms. In short, school teachers all over the country, day in day out, have taken responsibility for teaching basic numeracy, developing confidence with numbers and nurturing their keenest students towards A-level and beyond.

Now, let's dash forward to 2023. How has the mathematical landscape for teenagers changed nearly half a century after Christopher Zeeman's Christmas Lectures and more than a quarter of a century after my book about Fermat's last theorem?

Of course, kids still go to school, and we still owe a huge debt of gratitude to the nation's teachers. And kids can still watch mathematics on TV, with great ambassadors such as Professor Sir David Spiegelhalter and Bobby Seagull, and there are many new (and old) books available to read, by terrific authors such as Alex Bellos and Professor Hannah Fry.



Simon Singh on maths resources for all pupils

However, nowadays, beyond the classroom and traditional media, we have an incredible treasure trove of mathematics that students can access. The blogosphere has all sorts of material, from short blogs to long form articles. Indeed, the web gives students access to articles on virtually every single mathematical topic.

There are comic strips, such as XKCD, 'a webcomic of romance, sarcasm, math, and language', and Twitter (love it or hate it) points students to a ton of material, covering everything from mathematical games to the latest breakthroughs, from maths history to number puzzles. Websites such as NRich and Mathigon offer almost endless avenues for exploring mathematics.

And that is just the tip of the iceberg. For example, YouTube hosts hundreds of mathematically themed channels, including Numberphile, which offers a vast variety of 10-minute videos that are watched by an audience that ranges from 10-year-old kids to maths graduates. These brilliant videos have been watched roughly half a billion times! On its own, my Numberphile video about Fermat's last theorem has been watched 2.2 million times. For comparison, my BBC documentary about Fermat's last theorem was watched by 1.9 million people when it was first broadcast on BBC2. Numberphile's founder, director, editor, producer and

interviewer is one man, namely Brady Haran. In my opinion, Brady is the unsung hero of maths popularisation and deserves a knighthood at the very least.

In short, the mathematics professors of tomorrow are more likely to be inspired by watching Matt Parker on Numberphile, as opposed to tuning into BBC2 or reading a book.

But, whatever the avenue of inspiration, teenagers first need to have an appetite for mathematics. With Fortnite, TikTok and Minecraft just a click away, why on Earth would a teenager waste time exploring Numberphile?

I have neither written a book nor made a radio or TV programme for a decade, because I reckon that a better use of my time is to figure out a way to help students with genuine mathematical potential to excel by engaging with all the incredible mathematical resources that are freely available online. In particular, I want to bridge the gap between those students who are already making the most of these resources and those who have no idea of the adventure that awaits them.

Having spoken at hundreds of schools over the last 25 years, it seems that keen young mathematicians at some independent schools and many grammar schools routinely explore online maths resources in order to challenge themselves and learn new concepts. By contrast, the overwhelming majority of non-selective state schools do not seem to have a culture that encourages keen students to be curious about mathematics beyond the classroom.

That sounds like a damning claim, but it is probably not a surprise for those who spend time in classrooms. Non-selective state schools are focussed on exams, they are short of fully qualified (and experienced) teachers and they have 1000 other students to worry about, not just the half-dozen kids who are acing every test. Encouraging students to watch Numberphile videos is understandably low down on their list of priorities.

It seems a tragedy that we have more material online than could ever have been imagined a generation ago — material that is creative, challenging and stimulating — and yet most students have never even take a glimpse at it. This freely available material should be closing the attainment gap, but instead it is increasing it.

Part of the problem is that teachers do not have the time to push students beyond the curriculum, but also students see no reason to push themselves. Potentially strong students assume that they are God's gift to number theory if they scrape a gold in the UKMT Maths Challenge and are the top mathematician in their class

or year. What these students fail to realise — and it is not at all their fault — is that pupils in other schools are racing ahead, looking at tougher problems and exploring more exciting concepts.

Too many students assume that they have all the skills they need to be on track for a STEM degree at a top university, but they fail to realise that they are falling further and further behind their rivals who are making the most of the online universe of maths.

Moreover, potentially strong students who are not stretched are cursed with a lack of resilience. Because they always score well in tests, and because they always understand the material covered in lessons, they are unprepared for more challenging concepts. They spend the whole of Key Stages 3 and 4 within their comfort zone, which is the worst possible way to prepare for A-level Maths, Further Maths and beyond.



Four-Colour Map Theorem for level 4 on [parallel.org.uk](http://parallel.org.uk)

The challenge is to find cost-effective ways to support every potentially strong student, so that each one of them is able to fully engage with maths online and explore ideas beyond the curriculum, which will enhance their mathematical skills and stretch their ambitions. Ideally, these students should become independent learners, who understand there are limitless opportunities to explore ideas beyond the classroom.

So far, my efforts have had limited success, but each pilot programme or experiment has taught me a valuable lesson or added something to my skillset. And, now a new pilot is underway. As Samuel Beckett wrote in *Worstward Ho* (factorial not included): 'Ever tried. Ever failed. No matter. Try again. Fail again. Fail better.'

Simon Singh MBE Hon FIMA

*Simon Singh's latest efforts to introduce keen students to challenging mathematics can be found at [parallel.org.uk](http://parallel.org.uk).*

## Maximising Your Membership — The Verblunsky Members' Room



LMS members are warmly invited to visit De Morgan House in Russell Square, London and make use of the LMS Members' Room, or Verblunsky Room. Named after Samuel Verblunsky (1906–1996), a generous benefactor and long standing LMS member from 1929 to 1996, the LMS Members' Room offers a quiet and comfortable space to work and relax in the midst of the London hubbub.

With free wi-fi and complimentary tea and coffee in the nearby kitchen, members are welcome to use the room during office hours; Monday – Friday, 9.30–5.00pm. Please contact us by email on [membership@lms.ac.uk](mailto:membership@lms.ac.uk) to arrange a visit. Directions to De Morgan House can be found at: [demorganhouse.org.uk/our-location](http://demorganhouse.org.uk/our-location).

The Verblunsky Members' Room also houses two special collections: the Hardy Collection and the Philippa Fawcett Collection. The Hardy Collection, named after the Society's former President, contains over 300 volumes from G.H. Hardy's personal library of books, which were used by him at various points throughout his career. As such, one can get a glimpse of the authors who influenced his thinking or caught his attention. Many of these volumes contain Hardy's signature and in some

cases they also contain a dedication. For further information about the Hardy Collection, please visit: [lms.ac.uk/library/special-collections#hardy](http://lms.ac.uk/library/special-collections#hardy).

The Philippa Fawcett Collection, named after the first woman to come top in the Cambridge Mathematical Tripos exams and an early member of the LMS, is a wide-ranging library of books written by and about women who studied or worked in mathematical subjects in the nineteenth and first part of the twentieth century, or earlier. The Collection was donated to the London Mathematical Society by one of its members, A.E.L. Davis. These books are a useful resource to scholars of the history of women in mathematics as well as an inspiration to female mathematicians of the future. Please visit: [lms.ac.uk/library/special-collections#fawcett](http://lms.ac.uk/library/special-collections#fawcett) for further details.

Other treasures on display in the Verblunsky Members' Room include an Armada chess set, a collection of 90th birthday messages presented to Sir Christopher Zeeman, a facsimile of the LMS Members' Book from 1865-1990 and gifts from other Societies. If you have not yet signed the original Members' Book, look out for it at our Society Meetings: [lms.ac.uk/events/society-meetings](http://lms.ac.uk/events/society-meetings).

De Morgan House itself is situated on Russell Square within the creative heart of Bloomsbury and a short walk from the West End. With UCL only a 10 minute walk away, members can also take advantage of their complimentary use of the LMS Library that is housed at UCL. (For details on how to register/renew as a UCL Library user, visit: <https://www.lms.ac.uk/library/how-register>.)

We look forward to welcoming you at De Morgan House soon.

Valeriya Kolesnykova  
Accounts, Fellowships & Membership Assistant

## LMS Council Diary — A Personal View

Council met in person at De Morgan House on Friday 21 April. As usual the meeting started with President Ulrike Tillmann's report on her recent wide-ranging Society activities. Among these were several celebratory engagements, with the President representing the Society at the STEM for Britain early-career poster competition at the Houses of Parliament, presiding over the well-attended ceremony for the award of the 2020 and 2022 Christopher Zeeman Medals to Matt Parker and Simon Singh at the Royal Society, as well as Tim Browning's LMS Lecture at the British Mathematical Colloquium held in Bath. At the BMC the President was pleased to note the substantial number of members signing the Members' Book, which dates from 1865.

Council received an update from the Society's working group, led by Vice-President Iain Gordon, looking at relations with the developing Academy of Mathematical Sciences. The update included a clarification of routes for discussion with the Academy. Reports were also received from Publications Committee, on the Society's finances, and on operational plans and priorities for the Society's committees in the year 2023–24.

Council discussed a recent access audit of De Morgan House. Based on the audit's recommendations, Council agreed an action plan and timeline aimed

at improving access and mobility within De Morgan House.

A procedure committing the Society to advertising all its vacancies on committees from November 2023 was agreed by Council. The new procedure will enable all members to express their interest in joining committees and will further the Society's work through its excellent, committed, and diverse Members.

Council agreed to make an ad-hoc grant to support a summer school in algebra taking place in Turkey for students affected by the earthquake.

Council gave thought in recommending mathematicians for Honorary Membership of the Society. They will be announced at the Society General Meeting in June. Council also endorsed the recommendations of the Prizes Committee. These will also be announced at the General Meeting.

After finishing its business Council enjoyed a lunch break and then commenced a 2-day strategic retreat at De Morgan House where guidance was sought from Council for the development of the Society including its objectives for the next five years — but that is another story.

Robb McDonald  
General Secretary



De Morgan House offers a 40% discount on room hire to all mathematical charities and 20% to all not-for-profit organisations. Support the LMS by booking your next London event with us.

**Call us now on 0207 927 0800 or email [roombookings@demorganhouse.co.uk](mailto:roombookings@demorganhouse.co.uk) to check availability, receive a quote or arrange a visit to our venue.**

**#DeMorganHouse [www.demorganhouse.org](http://www.demorganhouse.org)**

## CONFERENCE FACILITIES



## REPORTS OF THE LMS

## Report: LMS Society Meeting at the BMC 2023



(left to right) The LMS stand; Participants enjoying a pre-dinner drink at the Roman Baths; Queuing to sign the Members' Book

It was my pleasure to attend my first LMS meeting at the British Mathematical Colloquium as a member of Council. This year the meeting was on 4 April at the University of Bath, and began with an address from the President, Ulrike Tillmann, who described many of the activities of the LMS, as well as the progress towards a national Academy for the Mathematical Sciences, and future ventures in which the LMS is involved. Members of the LMS were then invited to sign the Members' Book (if they had not yet done so) and it was great to see so many members take the opportunity to add their name to the illustrious list of members contained in the book. The President then introduced the speaker at the meeting, Tim Browning (Institute of Science and Technology, Austria), who is a very distinguished researcher working at the interface between analytic number theory and algebraic geometry.

Tim Browning has been recognised for his achievements through various awards, including LMS Whitehead and Phillip Leverhulme prizes. He gave a very engaging and fascinating talk entitled *Integer Points on Cubic Surfaces and a New Heuristic*. He described some new approaches he has developed with a collaborator to study the integer solutions to cubic equations using a novel version of the well-known circle method. It was interesting to see both the power of the method and its limitations, demonstrating that even though we have been studying such polynomial equations over the integers for several thousand years, they continue to hide many secrets.

Jason Lotay  
University of Oxford



The main concourse at the conference



LMS Education Secretary Kevin Houston

## Report from the 2020–21 Atiyah Fellow Maciej Dunajski

In April and September 2022 I made two Atiyah UK–Lebanon Fellowship visits to the Centre of Advanced Mathematical Sciences (CAMS) in Beirut. During the first visit I gave a mini-course on calculus of variations and geometry attended by the masters students, and some faculty, and a public colloquium ‘Four Facets of Geometry’. During my second visit I gave two research seminars. One at the physics department on masses of black holes, and one at the mathematics department on the metrizable problem in projective geometry. My original plan was to also visit other universities in Lebanon, but I was advised against it: the financial situation and electricity rationing meant that students and the faculty find it hard to travel within Lebanon or even greater Beirut, and a lot of activities have been moved online. All my seminars were therefore streamed on Zoom.

Sir Michael Atiyah was influential in establishing CAMS back in 1999 and has supported mathematics in Lebanon throughout his life. CAMS is based at the American University of Beirut (AUB) which was founded in 1866. The teaching there is based on the American liberal arts model, and the research profile is one of the highest in the Middle East. The funding and the governance of the AUB is linked to the United States. On the other hand, the student and faculty bodies contain the whole spectrum of Lebanese politics. This makes the AUB campus one of the safest places in the Middle East. I have found the rest of Beirut and Lebanon also safe (more so than the centre of Cambridge on a Friday night), and its people generous, helpful, and welcoming. While the University of Cambridge did not share my optimism and refused to insure me for one of the trips, I do not think that the safety concerns should deter any

of us from visiting and contributing to mathematics and higher education in Lebanon.

I have been regularly visiting Lebanon since 2009, mostly collaborating with Professor Wafic Sabra, the then director of CAMS. Lebanon, despite being a troubled country with complex and fragile politics, was fairly stable until the summer of 2020. AUB routinely hired world class academics from the West and organized international research meetings. The political situation changed dramatically in August 2020, after the explosion in the port of Beirut. The currency plummeted and has now been devalued. The electricity is rationed. Many academics have left the country or are contemplating to do it.

One weekend I decided to hike up the Qurnat as Sawda, the highest mountain in Lebanon and the Levant. I was surprised that the local mountain guide could not keep up with me on the ascent despite being half my age. He confessed that he suffered from Covid and could barely breath but had not had any Western tourism business for over two years, and desperately needed the income. This absence of international presence has of course been felt outside the world of mountain guides, and affects the ordinary population of Lebanon, as well as its institutions. It is because of this, that supporting collaborations between mathematics in the UK and Lebanon is now more important than ever. The people I interacted with in AUB, like the Director of CAMS Professor Jihad Touma, or the Head of Pure Mathematics Professor Florian Bertrand, show great resilience and commitment to keep the departments functioning and research active. It is great that the LMS is willing to help with its Atiyah Fellowships programme!

Maciej Dunajski  
Fellow of Clare College  
DAMTP, University of Cambridge

## LEVELLING UP

This column includes the latest updates about the Levelling Up: Maths scheme being developed by the LMS, made possible by a generous donation from Dr Tony Hill and Mr Simon Goodwin. The scheme seeks to widen participation of those who are under-represented in mathematics. It is part of a broader Levelling Up: STEM project which also covers Physics and Chemistry.

Levelling Up: Maths currently has 12 universities in the scheme, with another confirmed to join in the autumn. There are about 300 A-level students taking part this academic year with some continuing to next year. An evaluation framework for participating universities to use independently is being commissioned by the LMS and IMA. There was

a presentation on the Levelling Up: Maths scheme at the LMS Education Day in May. The presentation was very well received and was followed up by enquiries about how to get involved in the scheme.

Jennifer Gunn  
LMS Head of Society Business

### Records and Proceedings at LMS meetings Ordinary Meeting at the BMC 2023

held on Tuesday 4 April in the CB 1.10, Department of Mathematics, University of Bath, during the British Mathematical Colloquium. Over 100 members and guests were present for all or part of the meeting.

The meeting began at 12.00 noon with The President, Professor Ulrike Tillmann FRS, in the Chair.

There were no members elected to Membership at this Society Meeting.

11 members signed the Members' Book and were admitted to the Society.

As Professor Tillmann was relaying information about the activities and aims of the Society, she invited any attendees who wished to hear more about the Society to approach, after the lecture, either the present LMS staff or one of the many Council Members that were also present. At that point, Professor Tillmann invited all Members to raise their hand so that the attendees could later recognise them, which led to a truly touching and inspiring moment, with a distinct sense of LMS solidarity.

Following, Professor Tillmann introduced the LMS lecture given by Professor Tim Browning (Institute of Science and Technology Austria) on *Integer points on cubic surfaces and a new heuristic*.

Once the lecture concluded, Professor Tillmann thanked Professor Browning for his excellent lecture and then expressed the thanks of the Society to the organisers, Professor Kirill Cherednichenko (University of Bath) for a wonderful meeting and BMC.

The BMC reception and dinner was held later that day at the Roman Baths and Pump Room.

# Torus – sphere – Antisphere

## Compassing Mathematical Curvature through Musical Composition

EMILY HOWARD

Composer Emily Howard's music is frequently informed by mathematical concepts and constructs. Here she presents some of her thinking behind the development of a series of geometry-inspired musical compositions that includes the British Composer Award-winning large-scale orchestral work *Torus*, a 2016 BBC Proms Commission.

### Introduction

Over the past decade, the development of a series of geometry-inspired musical compositions has been a central preoccupation. For each of these works, an abstract mathematical shape was the imaginative cornerstone for the creative process; a filter through which myriad decisions about the piece were made. The series is ongoing and three existing orchestral works explore different curvatures: Euclidean geometry (*Torus*, 2016), elliptic geometry (*sphere*, 2017) and hyperbolic geometry (*Antisphere*, 2019). *Compass* (2022) is a chamber work partially set inside a Seifert-Weber space. Recordings of these four works are collated on the 2023 NMC Recordings portrait album **Torus**.<sup>1</sup>

In conjunction with the release of **Torus**, here I present some of the mathematically-inspired thinking that underpins these geometrically motivated compositions alongside the concurrent development of two very different chamber works: *Orbits: Études in Dynamical Systems* (2015) and *Four Musical Proofs and a Conjecture* (2017), both intended to be, in some sense, mathematical structures.

### Torus

As I constructed the large orchestral work *Torus*, I was primarily concerned with musical implications that could arise from considering the important mathematical result that there are two non-homotopic ways around a torus. The piece can be viewed in two parts in accordance with this result.

In the first part of *Torus* (roughly two-thirds of its c.24' duration) I consider repeated journeys around a torus in one direction only (the blue arrow in Figure

1). Musical parameters are torus-shaped by design. Pitch is structured as the expansion and contraction of two consonant chords (major 6th dyads) heard most prominently in the strings. I imagine being on the surface of a torus travelling round and round, and that with each rotation, the 'soundscape' is changing. While the strings are a more consistent presence, musical material for the orchestra's wind, brass and percussion sections differs wildly with each full rotation. This I find to be a very satisfying way to write, with a guiding principle providing a rock from which to deviate at liberty.

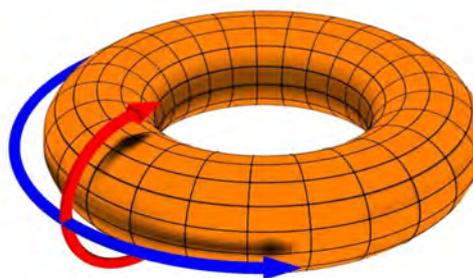


Figure 1. Torus

The rhythmic structure of these chords is also torus-shaped. The chord durations increase and decrease based loosely around timings derived from an exponential function, thus creating a series of predictable yet unpredictable chord rotations that then underpin further and more spontaneous decisions. These include instrumentation choices and sparsely or densely orchestrated sections. In the work itself, only the first torus-shaped rotation is experienced in full. This lasts around five minutes after which the rotations

<sup>1</sup>There is a special offer on the new album **Torus** from NMC Recordings for LMS Newsletter readers who will get 20% off by entering the code LMSTORUS20 at checkout — see NMC's advert on page 26.

are progressively hidden and/or cut in conjunction with changes to the tempo, towards its most clipped version encapsulated within a short solo for viola. Conjuring giant orchestral soundworlds that reduce down at times as though everything hangs in the balance upon solo instrumental threads - or silences - is a recurring love of mine. After the viola solo, there is a notable and abrupt shift in direction. In this second part of *Torus* (roughly the final third of the work), I imagine journeys around a torus in its second direction (the red arrow in Figure 1), in part manifesting as an exponentially timed sequence (fast to slow) of descending fast repetitions of a major 6th 'bounded' motif.

I began to write *Torus* towards the end of a very fruitful year as a Leverhulme Artist in Residence in the Department of Mathematics at the University of Liverpool. I was invited by Professor Lasse Rempe who works in dynamical systems, modelling systems which evolve in time, and my main focus was the exploration of aspects of his research through composition. Tangentially, I also spent a few hours each week chatting with other researchers in wide-ranging areas including topology, geometry, mathematical biology and theoretical physics. In this way, a conversation with Professor Anna Pratoussevich about Albrecht Dürer's polyhedron in *Melencolia I* strongly influenced my decision to traverse mathematical-musical shapes.

### Orbits: Études in Dynamical Systems

With Rempe, I explored dynamical systems in sound through the composition of two short works for saxophone and percussion - *Études in Dynamical Systems* - titled *Orbits* (2015). It was an intense period of mathematical focus when I learned about his research in detail with the intention of applying ideas within my music as 'rigorously' as I could. *Orbits* is a direct response to a paper by Rempe and van Strien entitled 'Density of hyperbolicity for classes of real transcendental entire functions and circle maps' [1]. I challenged Rempe's dynamics group to create sets of numeric data encapsulating key ideas from the work, which I then incorporated literally. For example, in *Orbit 1a*, I sent pitches from a Bach cello suite through a rhythmic dynamical system, where note durations were mapped on to the data. Conceptually, I found it helpful to think about the following translation of the paper's main theorem:

"No matter how much chaos there is, if you can make a very small change to the system, you can create stability". Lasse Rempe, 2015

And this found its way into my thinking for *Torus*: 'perturbation of the system' was a helpful way for me to view the abrupt disruption of the first part of *Torus* with the second. Conceptually, other ideas from dynamical systems were also influential: Edward Lorenz's evocative quote from chaos theory "The flap of a butterfly's wings in Brazil can set off a tornado in Texas" was ever-present, shaping the way I approached the entire form of *Torus*.

### Notes on Howard's works

Emily Howard's distinctive music is notable for its granular use of instrumental colour, powerful word-setting and inventive connections with mathematical shapes and processes.

Howard (b.1979) first won critical acclaim with *Magnetite*, commissioned for the Royal Liverpool Philharmonic Orchestra and Vasily Petrenko, to mark Liverpool's recognition as a European Capital of Culture in 2008. The work is the title track on her NMC Debut Disc, a collection that BBC Radio 3's Record Review noted 'brilliantly articulates' scientific ideas.

Howard's works include the 2016 BBC Proms commission and British Composer Award-winning *Torus* — described by The Times as 'visionary' and by The Guardian as 'one of this year's finest new works' — and chamber operas *To See The Invisible* (2018), commissioned by Aldeburgh Festival, and *Zátopek!*, commissioned as part of New Music 20x12 for the 2012 London Cultural Olympiad. *sphere* received its first performance by the Bamberger Symphoniker conducted by Alondra de la Parra in 2017, and *Antisphere* — continuing Howard's ongoing series of orchestral geometries — was commissioned by the Barbican for Sir Simon Rattle and the London Symphony Orchestra, and opened the 2019-20 season. The 2019 premiere of a new setting of mathematician Ada Lovelace's text *But then, what are these numbers?* also took place at the Barbican, as part of 'Ada Lovelace: Imagining the Analytical Engine', curated by Howard. In 2022, *Compass* was premiered by Birmingham Contemporary Music Group and conducted by Gabriella Teychenné as part of their Music and Maths Festival.

## Four Musical Proofs and a Conjecture

In 2017, I began a collaboration with Professor Marcus du Sautoy (University of Oxford) based around a musical exploration of mathematical proof. Du Sautoy chose proofs from Euclid's *The Elements* as the basis for our discussions. In response, I created a string quartet *Four Musical Proofs and a Conjecture* (2017) by considering the question:

"What if I approach writing music as though I am proceeding with the construction of a mathematical proof?"

This led me to a new way of working that involved making logical compositional decisions wherever possible. The piece is a collection of five short movements: 1) *Proof by Contradiction*; 2) *Geometric Proof*; 3) *Proof by Induction*; 4) *Proof by Algebraic Transformation* and 5) *Conjecture*. I chose to work with direct quotations from string quartets by Haydn, Beethoven and Schubert as a way to provide clear arrival and departure points within the music.

*Four Musical Proofs and a Conjecture* is the subject of a PRISM Blog that presents video introductions to each movement – conversations between myself and du Sautoy – alongside performances given by the Piatti String Quartet. Below is an outline of the approach I took for *Movement 1: Proof by Contradiction*.



Figure 2. Howard in conversation with Du Sautoy

I chose the opening bars of Haydn's first published string quartet to represent something 'axiomatic'. Haydn made his own journey through to the end of the movement i.e. the work itself. I then imagined what it might sound like if I attempted to prove this ending 'true', simulating a proof by contradiction. I asked myself 'Suppose the opposite is true', and this helped me with a number of creative decisions.

- Beginning with extremely high and low sounds (rather than those in middle register)
- A gradual transition from soft to loud (rather than loud and then abruptly soft)
- All four instruments playing at different times (rather than together)
- Always getting faster i.e. a gradual accelerando (rather than having a steady pulse).

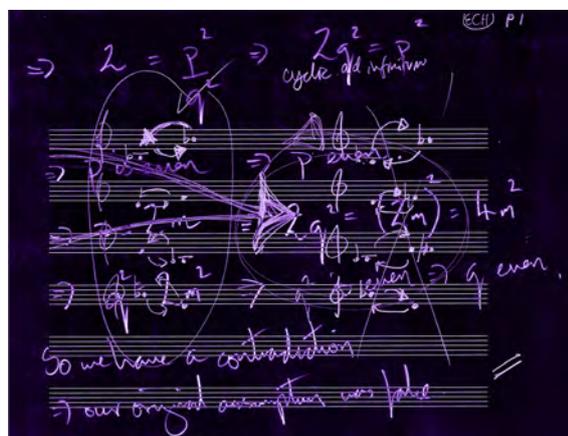


Figure 3. Music-maths sketching

I then used these ideas to create music with the aim that it led directly into a musical contradiction of Haydn's axiomatic opening bars; I have titled this "Reductio ad absurdum, repeat ad infinitum" in the score (Movt.1, bars 22-25). And with this game-like approach, I accept the Haydn ending as 'true'.

As with the *Études in Dynamical Systems*, these short studies in mathematical proof have enriched my musical language and scope, establishing enduring compositional principles and techniques. "Suppose the opposite is true" is a procedure I can now apply in very different compositional situations.

### sphere

As I worked on *Four Musical Proofs and a Conjecture*, I was having ongoing conversations with du Sautoy about travelling around shapes with different curvatures. Torus and its flat Euclidean geometry, but where next? We spoke about how aural imagination might be very flexible for thinking about higher dimensional shapes. Can local 'musical' space inform notions of higher dimensional global 'mathematical' space that are otherwise impossible to visualise? How can thinking about journeying around in higher dimensional musical space shape sound?

*sphere* (2017), a short work for chamber orchestra, is my mind's exploration of everything else during one complete loop across the convex surface of an imagined 2-sphere. In the PRISM Blog 'Orchestral Geometries', I discuss some of the extra-mathematical influences on this series.

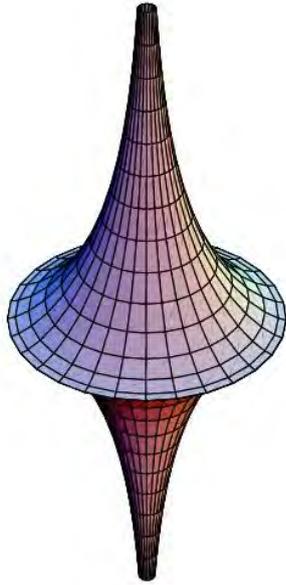


Figure 4. Antisphere

### Antisphere

'Non-Euclidean composition' was developed much further in the subsequent large-scale orchestral work *Antisphere* (2019). An 18' composition featuring an array of metallic percussion sounds, *Antisphere* interprets two different models of hyperbolic space – the Poincaré disc and the antisphere – simultaneously. The work also draws on the notion of a strange loop (D. Hofstadter (1979), Gödel, Escher, Bach) with chord sequences from an earlier 'Euclidean' work *Mesmerism* (2011) cycling incognito through *Antisphere's* negatively curved musical space in a musical exploration of the notion of infinity versus the infinitesimal.

Notions of negative curvature and shrinkage (for the angles of a triangle add up to less than 180° in hyperbolic space) have led to musical parameters being transformed as though through a saddle-shaped lens. Regular pulse within sections (along with the overarching structure itself) is estranged in this manner as are elements of traditional tonality, for example a circle of fifths becoming a circle of four and three-quarters through a language built using quarter tones.

### Compass

*Compass* (2022) marks my first venture into the realm of  $H^3$ . I am grateful to Marcus du Sautoy for the following description of a Seifert-Weber space:

"A Seifert-Weber space is constructed by gluing each face of a dodecahedron to its opposite in a way that produces a closed 3-manifold. There are three ways to do this gluing consistently. Opposite faces are misaligned by  $1/10$  of a turn, so to match them they must be rotated by  $1/10$ ,  $3/10$  or  $5/10$  turn: a rotation of  $3/10$  gives the Seifert-Weber space, an example of  $H^3$  hyperbolic geometry. Rotation of  $1/10$  gives a spherical geometry and rotation of  $5/10$  gives 3-dimensional real projective space."

This gave my imagination the rich possibility of a dodecahedral space where musical materials are circling, and changing – often in very subtle ways – as they travel through different curvatures. In *Compass*, a 22' chamber work scored for percussion and string septet, percussion sounds arise from a palette of metallic resonances, often extremely quiet, some stable and others less so, against which the strings oscillate between richly romantic gestures and their imagined counterparts, as though experienced from within differently curved musical spaces.

### Postscript

The transformation of mathematical notions into musical ideas has become a highly valued research methodology within my compositional practice. Whilst never a direct translation, it is precisely by attempting to carry out this impossible task that something is gained. I find that this approach often reveals new questions from unusual vantage points that result in unexpected ways to organise sound. I find it helpful to distinguish between compositions that are, in some sense, intended to be mathematical structures, and those that are motivated by mathematical structures. Both scenarios are creatively interesting to me and I strive to balance intense immersion in areas of mathematics, in collaboration with mathematicians, with periods when I allow my imagination to run wild.

### Acknowledgements

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# Hamilton's Discovery of the Quaternions and Creativity in Mathematics

NICHOLAS J. HIGHAM AND DENNIS SHERWOOD

Creativity is, arguably, mankind's—let alone a mathematician's— most important, and valuable, attribute. Yet it remains tantalisingly elusive. What, precisely, is it? Is it a 'natural gift'? Or is it a skill that can be learnt? Hamilton's discovery of quaternions is well-known as an example of a 'sudden flash of inspiration', but this is not the full story, for he had been thinking about the problem for a long time before the solution emerged. But what had he been thinking about? Fortunately, Hamilton left some important clues—clues that can be built upon to identify a process of creative discovery that can be generalized very broadly.

Sir William Rowan Hamilton's discovery of the quaternions is a famous example of a flash of inspiration. To quote his own words, as he was walking with his wife along the Royal Canal in Dublin on 16 October 1843, "an *electric* circuit seemed to *close*; and a spark flashed forth" [8], that spark being the quaternion  $a + ib + jc + kd$ , with the new items  $j$  and  $k$  defined such that  $i^2 = j^2 = k^2 = ijk = -1$ .

So excited was he that he carved these formulas on a stone of Brougham Bridge. And on the following day, Hamilton wrote a letter to John T. Graves [7], in which he offers some rare insights into the process of mathematical discovery, showing that although the moment at which everything fell into place was so sudden, the discovery of quaternions was by no means an instantaneous "spark flashing forth", but the result of much hard work, experimentation, and deep thinking.

Hamilton's end-result, in modern terminology, was the construction of a normed division algebra [4]. In Hamilton's day, the known dimensions of such algebras were 1 (real numbers) and 2 (complex numbers), and in his letter to Graves, Hamilton talks of having "long wished ... to possess a Theory of Triplets, analogous to my published Theory of Couplets". This clearly indicates the Hamilton had for some time been thinking about how to generalize the concept of complex numbers, for which the most obvious idea is to extend "couplets"  $a + ib$  into "triplets"  $a + ib + jc$  by introducing  $j$  as an analogue to  $i$ .

When, however, Hamilton multiplied two triplets,  $a + ib + jc$  and  $x + iy + jz$ , he ran into difficulty with the product  $ij$ : to quote Hamilton once more, "But what are we to do with  $ij$ ? ... This might tempt us to take  $ij = 1$  or  $ij = -1$ ; but with neither assumption shall we have the sum of the squares of the coefficients of 1,  $i$  and  $j$  = to the product of the corresponding sums of squares in the factors ... Behold me therefore tempted for a moment to fancy that  $ij = 0$ . But this seemed odd and

uncomfortable ... [so I assumed] what seemed to me to be less harsh, namely that  $ij = -ji$ . I made therefore  $ij = k$ ,  $ji = -k$ ."

Hamilton had not only discovered that  $i$  and  $j$  are non-commutative but had also stumbled on the possibility of an additional imaginary quantity  $k$ , as he described in the delightful sentence "And here there dawned on me the notion that we must admit, in some sense, of a *fourth dimension* ... or transferring the paradox to algebra, must admit a *third* distinct imaginary symbol  $k$ , not to be confounded with either  $i$  or  $j$ , but equal to the product of the first as multiplier, and the second as multiplicand; and therefore was led to introduce *quaternions*, such as  $a + ib + jc + kd$ ."

Hamilton's letter, and subsequent paper [6], present many fascinating personal details relating to his discovery, but three fundamental features stand out.

The first is curiosity, a desire to enquire—and, in Hamilton's case, with the luxury of having the space to enquire as an abstract exercise, rather than as driven by the urgency of a problem that demanded solution. Real numbers and complex numbers were well known and understood, and served their purpose well. No one was saying, "Hey, Hamilton! Complex numbers don't do [whatever]! Fix it, will you?". Rather, Hamilton was curious as to whether a "Theory of Triplets", analogous to the "Theory of Couplets" might exist, and what it might look like.

That curiosity led to the second fundamental feature, exploration, a well-directed search, in which he examined the algebra of triplets  $a + ib + jc$ , and discovered—no doubt to his initial disappointment and frustration—that things didn't work out, especially as regards the product  $ij$ .

And in considering different alternative possibilities, Hamilton discovered that if  $ij \neq ji$ , but rather

that  $ij = -ji$ , then everything worked. That must have been truly startling, for in 1843, non-commutative multiplication ‘broke all the rules’. The non-commutative properties of matrix multiplication were not to be described until the work of Arthur Cayley some 15 years later [2], and although it is possible that Hamilton was aware of non-commutation in the context of rotations, the commutative properties of numbers and algebraic expressions was so ingrained into everyone’s knowledge that the very possibility that the product  $ij$  might be non-commutative would, to most people, have been unthinkable—or, if thought, at once dismissed as ‘ridiculous’. But Hamilton both thought it and developed it, and in so doing had been willing to throw away the conventional wisdom about multiplication. Which makes ‘unlearning’ the third fundamental feature of the story. And as Hamilton describes, the recognition that  $i$  could not commute with  $j$  led him to introduce  $k$  as  $ij = k$ , and that triggered the possibility of, as he put it, a “fourth dimension”, leading him to explore the entity  $a + ib + jc + kd$ , at which point everything worked.

Those three features—curiosity, exploration, and a willingness to unlearn—are exemplified by Hamilton’s discovery of quaternions, but we propose here that they are in fact fundamental features of all creativity. Furthermore, they are unified when creativity is perceived not so much as the quest for the *new*, but as the discovery of the *different*.

That might appear to be both strange and counterintuitive. Yes, in 1843, quaternions were ‘new’; more importantly, however, they were different from their immediate antecedents, real numbers and complex numbers, but different only in two respects: four terms rather than one or two and, crucially, employing non-commutative multiplication. Furthermore, in discovering quaternions, Hamilton did not just ‘stare into blue space hoping that the lightning would strike’; on the contrary, he started from what he knew, the algebra of real and complex numbers, and undertook a thorough and diligent exploration of possibilities.

When seeking to be creative, the value of starting from what you already know is highlighted by this insight from Arthur Koestler’s book *The Act of Creation* [11]: “The creative act is not an act of creation in the sense of the Old Testament. It does not create something out of nothing; it uncovers, selects, re-shuffles, combines, synthesises already existing facts, ideas, faculties, skills. The more familiar the parts, the more striking the new whole.”

Koestler therefore presents creativity as a process of pattern formation, using components that already exist, but have not been combined in that particular way before. Accordingly, in so far as novelty is present, it is at the level of the pattern, rather than the components from which that pattern is formed. Beethoven, for example, did not invent the notes or the musical instruments that played them, but he did create some truly magical patterns. And each of Beethoven’s successive musical patterns was necessarily different from all previous patterns.

That suggests that a process for the discovery of ideas—creativity—might be to take an existing construct, and then ask ‘How might [this] be different?’ of any specific feature (that’s curiosity), the consequence of that question being to trigger exploration. And if, during that exploration, there is a willingness to unlearn, then perhaps ideas will emerge.

To test this proposal, here is a thought experiment that applies this process to the discovery of quaternions.

The starting point is Hamilton’s wish “to possess a Theory of Triplets, analogous to my published Theory of Couplets”. The “Theory of Couplets”—complex numbers—is therefore Hamilton’s starting point, of which these are the key features with which Hamilton would have been familiar:

- (1) A complex number  $a + ib$  has two parts: a real part,  $a$ , and an imaginary part,  $b$ .
- (2)  $|z_1 z_2| = |z_1| |z_2|$  for any complex numbers  $z_1$  and  $z_2$  (the law of moduli).
- (3)  $i^2 = -1$ .
- (4) Multiplication of complex numbers is associative.
- (5) Multiplication of complex numbers is commutative.

How might each of these be different, and where might that lead?

The simplest possible, and most obvious, difference is to add a third term, changing  $a + ib$  to  $a + ib + jc$ , with  $j^2 = -1$  just as  $i^2 = -1$ . But when Hamilton tested the other properties, he identified the problem with the product  $ij$ —which, in a wonderful example of unlearning, he resolved by throwing commutative multiplication away and hypothesizing that  $ij = -ji = k$ . This in turn triggered another ‘How might [this] be different?’ question, where, in the relevant context, [this] was ‘my assumption that there are three terms’. The most obvious answer is ‘suppose there are four

terms  $a + ib + jc + kd'$ , and when Hamilton tested that, it all worked.

Inspired by Hamilton's letter, John T. Graves discovered the 8-dimensional octonions in December 1843, as did Cayley, independently, in 1845. Like quaternions, octonions are non-commutative under multiplication, but unlike real numbers, complex numbers and quaternions, octonions are also non-associative under multiplication—this being a further example of asking 'How might [this] be different?', this time of the feature of the precedent quaternions that multiplication is associative.

It is of course most unlikely that Hamilton, Graves and Cayley actually discovered quaternions and octonions in this way. But they might have done so. Asking 'How might [this] be different?' of all the features of something that exists now is a hugely powerful way of discovering ideas, for the question gets to the heart of what creativity is: something different from, and hopefully better than, something that exists now.

Hamilton's discovery of the quaternions is just one of many examples of creativity in mathematics that fit the pattern of combining pre-existing components in different ways. Others include Andrew Wiles's proof of Fermat's Last Theorem, Henri Poincaré's discovery of automorphic forms, and Olga Taussky's work on determinantal conditions for matrix nonsingularity [9, Chap. 3]. Furthermore, seventy years of research on iterative refinement for the numerical solution of linear systems can be interpreted as arising from changing different features of the basic algorithm programmed by Wilkinson in 1948 for the Pilot ACE computer at the National Physical Laboratory [10].

In considering 'How might [this] be different?' in any context, we can think about a variety of attributes, such as:

- Size: can some quantity be bigger or smaller? In the quaternion example, we need 4 rather than 3 components. Often, thinking much bigger or much smaller (a quantity tending to infinity or to zero) will trigger useful ideas.
- Sequence: in a process involving a sequence of steps, do those steps have to be done in a certain order or can they be reordered or some steps even be merged?
- Established practice: if a property or condition is conventionally assumed, can it nevertheless be modified or dropped?

## Creativity Workshops

In 2010, the Engineering and Physical Sciences Research Council (EPSRC) started the Creativity@home initiative<sup>a</sup> and encouraged PIs of large grants to request funds to run workshops on creative problem solving for their teams. The authors met at such a workshop that year and have since collaborated on several creativity workshops. Our creativity workshops consist of up to 20 people working in groups of up to 8, usually meeting for two days at an off-site location. They invariably generate a large number of ideas for later evaluation. These workshops are an excellent way to train people in creativity and help build a team environment in which creativity flourishes [9]. We have run some creativity workshops virtually in the last couple of years, and while they were successful we think that face to face events work better for all concerned. Our experience is consistent with recent research that reports that "videoconferencing hampers idea generation because it focuses communicators on a screen, which prompts a narrower cognitive focus" [1].

<sup>a</sup>[bit.ly/ukri-creativity-at-home](http://bit.ly/ukri-creativity-at-home)

Asking 'How might [this] be different?' is not, of course, the only way to generate ideas. In mathematics, three sources predominate. George Pólya's classic, *How to Solve It*, originally published in 1945, identifies no fewer than 70 problem-solving heuristics [14]. *The Mathematician's Mind* by Jacques Hadamard, also first published in 1945, is perhaps more philosophical than operational [5]. And Henri Poincaré's masterful 1908 lecture, *Mathematical Creation*, provides a highly personal insight into how he discovered Fuchsian functions [13]. In a wider context, authors such as Edward de Bono [3], Alex Osborn (to whom 'brainstorming' is attributed) [12], and Arthur van Gundy [16] are among the most notable. Asking 'How might [this] be different?', however, is easy, simple, and pragmatic.

The process for generating ideas that we have outlined is the basis of a six-step process that one of us has developed over the last twenty years [9], [15]. It can be applied to any focus of attention for which one wants to generate ideas and is well suited to tackling mathematical problems.

While the process can be carried out by individuals, it is at its most powerful when used in small groups. Creativity is much richer when people speak to each other, ask each other questions, and share knowledge. A group can spot more features of the topic under consideration and can produce a wider selection of ways in which those features could be different than any individual is likely to do.

Mathematicians are creative people, as the history of our subject shows. The process of listing all the properties of the problem at hand and asking ‘How might [this] be different?’ for each one, whether carried out by individuals or groups, can generate ideas that might otherwise be missed—and it’s fun to apply it!

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Photo credit: Rob Whitrow

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### Dennis Sherwood

For the last 20 years, Dennis Sherwood has been running The Silver Bullet Machine Manufacturing Company Limited, specializing in creativity and innovation. Previously, he was a consulting partner for Deloitte Haskins + Sells

and Coopers & Lybrand, an Executive Director at Goldman Sachs, and Managing Director of the UK operations of SRI Consulting. He is the author of many articles and of 15 books.

## Notes of a Numerical Analyst

## Double Exponential Bump Functions

NICK TREFETHEN FRS

I want to tell you about my new favourite function,

$$\tau(x) = \tanh\left(\frac{\pi}{2} \sinh(3.2x)\right), \quad (1)$$

and its derivative rescaled to height 1,

$$d(x) = \cosh(3.2x) \operatorname{sech}^2\left(\frac{\pi}{2} \sinh(3.2x)\right). \quad (2)$$

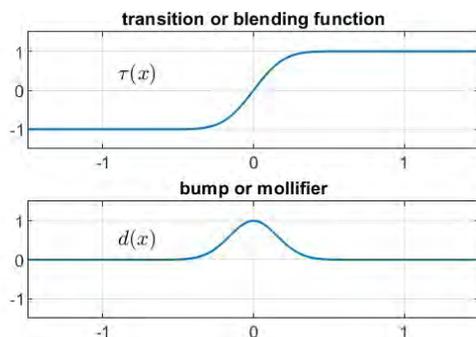


Figure 1. Double exponential blending and bump functions.

These are analytic functions, the first making a transition from  $-1$  to  $1$  and the second defining a bump or (if multiplied by  $0.8\pi$ ) mollifier. They approach their limiting values “double exponentially”—not just exponentially like  $\tanh(x)$  or exponentially-squared like  $\exp(-x^2)$ .

How could anyone’s favourite function have a decimal in it like  $3.2$ ? Well, this constant has been chosen so that  $\tau(x)$  rounds to exactly  $\pm 1$  for  $|x| \geq 1$  in 16-digit IEEE floating-point arithmetic. So the transition region of  $\tau$  has compact support in  $[-1, 1]$  in the standard arithmetic of computational science and engineering.

The uncertainty principle states that a nonzero function and its Fourier transform cannot both have compact support. This implies a trade-off between locality and smoothness, and the well-known balanced compromise is a Gaussian  $\exp(-x^2)$ , where both  $f$  and  $\hat{f}$  are entire functions with exponential-squared decay. The functions (1) and (2) make a different choice, prioritizing locality over

smoothness. The locality is spectacular (we have  $|d(x)| < 10^{-400}$  for  $|x| \geq 2$ ) yet the smoothness is still excellent, since the functions are analytic. (The constant  $\pi/2$  arises since smaller values give poorer localization and larger ones give a narrower strip of analyticity.) Figure 2 illustrates how  $\tau(x)$  can be used to blend one analytic function into another. I think of the transition region as a “fat branch cut”.

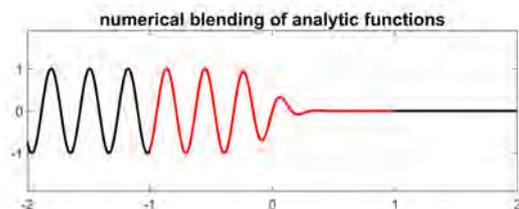


Figure 2. Numerical blending of  $\sin(20x)$  for  $x \leq -1$  and  $0$  for  $x \geq 1$ . The blend function is analytic and matches the pieces to machine precision.

Of course, for proving a theorem, one may need true compact support, and the standard choices are  $C^\infty$  functions composed of pieces, such as  $\exp(-1/(1-x^2))$  for  $|x| < 1$  and  $0$  for  $|x| \geq 1$ . When it comes to applying such ideas computationally, however, it is not obvious that  $C^\infty$  functions are the best starting point. The  $\tanh((\pi/2) \sinh(x))$  combination was introduced by Takahasi and Mori in 1974 for numerical quadrature of functions with singularities [1], but I am not aware that it has been proposed for other applications.

## FURTHER READING

[1] M. Mori, Discovery of the double exponential transformation and its developments, *Publ. Res. Inst. Math. Sci.*, 41 (2005), 897–935.



## Nick Trefethen

Nick is a Professor of Numerical Analysis and Head of the Numerical Analysis Group at the Mathematical Institute, University of Oxford.

## Mathematics News Flash

Jonathan Fraser reports on some recent breakthroughs in mathematics.

### An exponential improvement for diagonal Ramsey numbers

**AUTHORS:** Marcelo Campos, Simon Griffiths, Robert Morris and Julian Sahasrabudhe

**ACCESS:** <https://arxiv.org/abs/2303.09521>

The  $k$ th Ramsey number ( $k \geq 3$ ) is the smallest integer  $n$  such that any 2-colouring of the edges of a complete graph on  $n$  points must contain a monochromatic copy of the complete graph on  $k$  points. It is straightforward but instructive to show that  $R(3) = 6$  (a problem I recommend to our readers!) It is rather harder to see that  $R(4) = 18$  and it may shock some readers to learn that  $R(5)$  is not known precisely but lies somewhere between 43 and 48 inclusive.

Ramsey proved that  $R(k) < \infty$  for all  $k$  in 1930 and Erdős and Szekeres proved that  $R(k) \leq 4^k$  in 1935. Various attempts to improve this upper bound have been made over the years, but this remained the best exponential bound for a long time (that is, nobody could beat “4”).

This much celebrated paper, appearing on arXiv earlier this year, finally achieved an exponential improvement by showing that  $R(k) \leq (4 - \varepsilon)^k$  for some  $\varepsilon > 0$  and all sufficiently large  $k$ .

### A counterexample to the theorem of Laplace-Lagrange on the stability of semimajor axes

**AUTHORS:** Andrew Clarke, Jacques Fejoz and Marcel Guardia

**ACCESS:** <https://arxiv.org/abs/2303.05979>

Consider a model of the solar system where planets follow an elliptical orbit around the sun. If one only considers the gravitational interaction between the sun and each individual planet, then the situation is quite simple and well-understood. Of course, this is a simplification of the true situation because the planets also interact gravitationally with each other. Under this additional hypothesis, the situation is

much more complicated. Nevertheless, it was a long-held belief that the orbits were stable in the sense that small perturbations to initial conditions would not give rise to vastly different behaviour in the long run. This remarkable paper, appearing on the arXiv earlier this year, proves that this is not the case. In a model with only three planets they find orbits along which the semi-major axis of the outer planet exhibits instability.

### Convergence of uniform triangulations under the Cardy embedding

**AUTHORS:** Nina Holden and Xin Sun

**ACCESS:** <https://arxiv.org/abs/1905.13207>

A central topic in modern probability theory is to construct and describe random surfaces. One approach is to study the scaling limit of random planar maps (a random finite connected planar graph considered up to deformations that preserve the orientation). Another approach is via Liouville quantum gravity, where one studies the (random) surface with a volume measure defined by sampling from the Gaussian free field. This model has been popular in the physics literature for some time but the mathematical treatment of it began with Duplantier and Sheffield’s seminal paper from 2011. There is an important programme of research aimed at establishing when certain types of Liouville quantum gravity can be recovered as the scaling limit of a suitable family of random planar maps.

This paper, published in *Acta Mathematica* in 2023, proves that the scaling limit of a natural family of random planar maps on an equilateral triangle (the Cardy embedding) is the  $\sqrt{8/3}$ -Liouville quantum gravity disk.



**Jonathan Fraser** is a pure mathematician working at the University of St Andrews. He is pictured here out exploring with Reuben.

Microtheses and Nanotheses provide space in the Newsletter for current and recent research students to communicate their research findings with the community. We welcome submissions for this section from current and recent research students. See [newsletter.lms.ac.uk](http://newsletter.lms.ac.uk) for preparation and submission guidance.

## Microthesis: Asymptotics of Operator Semigroups and Applications

ABRAHAM CHI SHUN NG

A driving purpose behind mathematical modelling is the desire to predict long-term behaviour of the phenomena being modelled. This microthesis looks at some of the goals achieved in my DPhil: quantifying the rate at which the energy in various evolution systems decays to 0.

### Evolution systems

An *evolution* system is a system that evolves through time. Usually, these systems are governed by a partial differential equation (PDE) or a system of PDEs, in which one of the variables is time. A classic example of such a system is that of heat flowing throughout a particular homogeneous material. In this case, the distribution of heat is modelled by the heat equation:

$$u_t(x, t) = k \sum_{i=1}^3 u_{x_i x_i}(x, t), \quad x \in \mathbb{R}^3, t > 0,$$

where  $k > 0$  is a constant that captures the density, thermal conductivity and capacity of the material. Replacing 3 with  $n$ , we get a general heat equation in  $\mathbb{R}^n$ .

One of the most interesting questions we can ask about an evolution system is what happens in the long run. The technical term for this is *asymptotics*.

### Asymptotics of the heat and wave equation

The undergraduate method of separation of variables shows that the 1D heat equation ( $u_t = u_{xx}$ ) on the interval  $[0, 1]$  with Dirichlet boundary condition (i.e.  $u(0, t) = u(1, t) = 0$ ) has a solution of the form

$$u(x, t) = \sum_{n=1}^{\infty} A_n \sin(n\pi x) e^{-n^2 \pi^2 t}, \quad x \in [0, 1], t > 0.$$

Some simple analysis shows that there is some constant  $c > 0$  such that

$$\|u\|_{L^2(dx)}^2 \leq c \exp(-\pi^2 t) \rightarrow 0, \quad t \rightarrow \infty,$$

that is, the heat energy in a finite interval decays exponentially quickly.

On the other hand, the 1D wave equation (i.e.  $w_{tt} = w_{xx}$ ) on the interval  $[0, 1]$  with Dirichlet boundary condition has a solution of the form

$$w(x, t) = \sum_{n=1}^{\infty} \sin(n\pi x) (B_n \sin(n\pi t) + C_n \cos(n\pi t))$$

for  $x \in [0, 1], t > 0$ . In this case,  $w$  is now periodic in  $t$ , and an easy way to see this is to try the points  $t = 0, 2, 4, 6$  and so on. Hence,  $\|w\|_{L^2(dx)}^2$  does not decay at all, let alone exponentially.

### Coupled wave-heat systems

Given that the energy in the finite length 1D heat equation decays exponentially quickly and the energy in the finite length 1D wave equation does not decay at all, a natural question would be to ask what happens if we glued these two systems together. This can be done by insisting that we find functions  $u, w$  such that  $w$  satisfies the wave equation in  $[-1, 0]$ ,  $u$  satisfies the heat equation in  $[0, 1]$ , and  $u, w$  together satisfy a certain coupling condition at  $x = 0$ .

This *coupled* wave-heat system and others like it provide linear approximate models that shed light on more complicated phenomena such as structure-fluid interaction. Although simply described, such a system can no longer easily be directly solved and analysed, so the question of asymptotics becomes highly nontrivial. As such, we want a different approach to the problem.

## Playing the semigroup game

Given an evolution system, we play the semigroup game with the following steps:

- (1) Cast the evolution system as an Abstract Cauchy Problem,

$$(ACP) = \begin{cases} z'(t) = Az(t), \\ z(0) = x. \end{cases}$$

Here,  $A : D(A) \subseteq X \rightarrow X$  is an unbounded operator which captures all the data of the system and is defined in a domain within an appropriate function space, so that solving (ACP) ends up solving the original system.

### An Example of Step (1)

As an example, consider the coupled wave-heat system aforementioned. Ignoring the question of which space the following functions live in, let  $A$  be an operator that takes a triple of functions  $(u, v, w)$  to  $(u'', w'', v)$ , where the dashes denote derivatives in the  $x$ -direction.

Let  $z$  be a function of time with values that are triples of functions, so that we can write  $z(t) = (u(\cdot, t), v(\cdot, t), w(\cdot, t))$ .

Then requiring  $z'(t) = Az(t)$  would yield

$$\begin{aligned} (u_t(\cdot, t), v_t(\cdot, t), w_t(\cdot, t)) &= z'(t) \\ &= Az(t) = (u_{xx}(\cdot, t), w_{xx}(\cdot, t), v(\cdot, t)), \end{aligned}$$

and thus

$$w_{tt} = v_t = w_{xx}, \quad u_t = u_{xx},$$

solving the coupled wave-heat system (the coupling condition would be solved by insisting  $z(t) \in D(A)$ ).

- (2) Show that  $A$  generates a generalised exponential family

$$T(t) = e^{tA} \in \mathcal{B}(X), \quad t \geq 0,$$

of bounded linear operators called a *strongly continuous semigroup* (or *semigroup* for short). This typically involves so-called *generation* theorems.

- (3) If  $A$  generates a semigroup  $(T(t))_{t \geq 0}$ , then setting

$$z(t) = T(t)x, \quad t \geq 0,$$

will solve (ACP) and thus solve the original system.

### The generator $A$

The relation between  $A$  and the semigroup  $(T(t))_{t \geq 0}$  it generates is as follows:

$$Ax = \lim_{t \rightarrow 0^+} \frac{T(t)x - x}{t}, \quad x \in D(A).$$

The domain of  $A$  consists of exactly those  $x \in X$  for which the limit exists. If this looks suspiciously like a derivative, well, think about the derivative of  $e^{ta}$  for some  $a \in \mathbb{R} \dots$

If you know that

(a)  $T(t+s) = T(t)T(s)$  for  $t, s \geq 0$ , and

(b)  $T(t)x \rightarrow x$  as  $t \rightarrow 0$  for  $x \in X$ ,

can you see why the  $z(t) = T(t)x$  in Step (3) would solve the ACP?

Conditions (a) and (b) are in fact part of the axioms for a strongly continuous semigroup.

Recent theory (of which [2] is a prime example) has enabled us in certain cases to convert spectral conditions and resolvent estimates of the generator into descriptions of asymptotic behaviour of the semigroup, including the rate at which certain behaviour such as decay occurs. This allows us to extend the semigroup game provided appropriate conditions are met.

- (4) Check that  $\sup_{t>0} \|T(t)\| < \infty$  and the spectrum  $\sigma(A)$  of  $A$  does not touch the imaginary axis. This will (nontrivially) imply that  $T(t)x \rightarrow 0$  as  $t \rightarrow \infty$  for all  $x \in X$ .
- (5) Estimate the growth of the resolvent of  $A$ ,  $(\lambda \text{Id} - A)^{-1}$ , along the imaginary axis (i.e. for  $\lambda \in i\mathbb{R}$ ).
- (6) Using so-called *quantified Tauberian* theorems to convert the resolvent estimates into estimates of the rate at which  $\|T(t)A^{-1}\| \rightarrow 0$  as  $t \rightarrow \infty$ . The rate of  $T(t)x$  then depends on if  $x \in D(A)$ .

### Discrete analogues

We can also look at evolution systems with discrete time, modelling them with discrete semigroups.

Continuous	Discrete
$(T(t))_{t \geq 0} \subset \mathcal{B}(X)$	Given $S \in \mathcal{B}(X)$ , Consider $(S^n)_{n \in \mathbb{N}} \subset \mathcal{B}(X)$
$(T(t))_{t > 0}$ bounded, i.e. $\sup_{t > 0} \ T(t)\  < \infty$	$S$ power-bounded, i.e. $\sup_{n \in \mathbb{N}} \ S^n\  < \infty$
Generator $A$ (derivative)	$S - \text{Id}$ (difference)
$\sigma(A) \cap i\mathbb{R} = \emptyset$	$\sigma(T) \cap \mathbb{T} \subseteq \{1\}$
Estimate $\ (is \text{Id} - A)^{-1}\ $	Estimate $\ (e^{i\theta} \text{Id} - A)^{-1}\ $
Decay rate of $\ T(t)A^{-1}\ $	Decay rate of $\ S^n(S - \text{Id})\ $

Table 1. Discrete glossary.

Half of my thesis was dedicated to proving theoretical results for these discrete analogues (see [5]).

### Results

In Figure 1, we have three coupled systems made of different combinations of wave (W) and heat (H) parts. The first is the system we have already encountered with finite wave and finite heat. The second is like the first but with an extra wave part (the values of the actual interval end points are unimportant). The third replaces the heat equation on a finite interval with the heat equation on the half-line ( $H_\infty$ ).

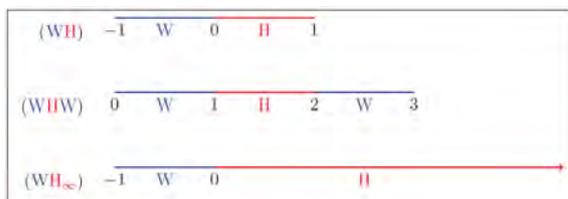


Figure 1. Coupled wave-heat systems.

In [6], the finite WH system was studied without semigroups in an impressive but highly technical and lengthy work. This same system was studied in [1] by playing the semigroup game which drastically cut short the necessary work and resulted in a paper a quarter of the length of the original study. Half of my thesis was dedicated to studying the WHW and  $WH_\infty$  systems, resulting in [3, 4] and completing Table 2.

System	Studied by	Energy decay (optimal)
W	Classical	None
H	Classical	Exponential
$H_\infty$	Classical	$t^{-2}$ Polynomial
WH	[6], [1]	None + Exp = $t^{-4}$ Poly
WHW	[3]	None + Exp + None = $t^{-4}$ Poly
$WH_\infty$	[4]	None + $t^{-2}$ Poly = $t^{-2}$ Poly

Table 2. Results.

### FURTHER READING

- [1] C. J. K. Batty, L. Paunonen and D. Seifert, *Optimal energy decay in a one-dimensional coupled wave-heat system*. J. Evol. Equ. **16** (2016), 649–664.
- [2] A. Borichev and Yu. Tomilov, *Optimal polynomial decay of functions and operator semigroups*. Math. Ann. **347** (2010), 455–479.
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- [4] A. C. S. Ng and D. Seifert, *Optimal energy decay in one-dimensional wave-heat system with infinite heat part*. J. Math. Anal. Appl. **482** (2020), 123563, 15pp.
- [5] A. C. S. Ng and D. Seifert, *Optimal rates of decay in the Katznelson–Tzafriri theorem for operators on Hilbert spaces*. J. Funct. Anal. **279** (2020), 108799, 21pp.
- [6] X. Zhang and E. Zuazua, *Polynomial decay and control of a 1-d hyperbolic-parabolic coupled system*. J. Differential Equations **204** (2004), 380–438.



### Abraham Chi Shun Ng

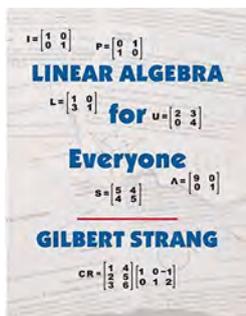
Abraham is an Australian postdoctoral researcher at the University of Wollongong working in functional analysis. Since completing his DPhil in operator semigroups at the University of Oxford,

he has expanded his interests in functional analysis to include  $C^*$ -algebras, for which there is an associated  $C^*$ -game you can also play. The  $C^*$ -at is Sima Qian (named for my favourite eunuch).

# Linear Algebra for Everyone

by Gilbert Strang, Wellesley–Cambridge Press, 2020, ISBN: 978-1-7331466-3-0

Review by Robert D. Gray



The author Professor Gilbert Strang has many years of experience teaching linear algebra at MIT. The main idea of this book is to present his new approach to teaching undergraduate linear algebra based on his extensive experience.

He calls this approach a “new start”. The “new start” refers to the way in which he now likes to start introducing the subject when he teaches linear algebra. So “new start” means a new way to start learning linear algebra. His “new start” was tested in MIT lecture courses that he delivered and are now freely available online as part of MIT’s OpenCourseWare; specifically the courses Math 18.06, Math 18.065, and Math 18.065C. In fact, as I shall explain below, I believe a student will get the most out of this book by reading it alongside watching the lectures from the above MIT OpenCourseWare linear algebra courses.

In the video “MIT A 2020 Vision of Linear Algebra, Spring 2020”, which also is available online, Professor Strang gives a helpful series of short lectures in which he explains his new approach, which is concerned with the recommended order of topics in teaching and learning linear algebra. The book under review begins, following Strang’s “new start”, with the observation that the column vector  $Ax$  is a combination of the columns of the matrix  $A$ . That leads to the notion of the column space of a matrix, the idea of independent columns, and then to the factorisation  $A = CR$  of a matrix where  $C$  is  $m$  by  $r$ , and  $R$  is  $r$  by  $m$ , and  $A$  has  $r$  independent columns. This then quickly brings us to the fact that the column space and the row space have the same rank. The sequence of ideas he then goes on to introduce in the book can each be connected with a particular factorisation from linear algebra. Specifically, solving linear equations  $Ax = b$  using elimination is discussed, inverses of matrices, Gauss–Jordan elimination for computing the inverse

of a matrix, and the “great factorisation”  $A = LU$  into a product of a lower triangular matrix  $L$  and an upper triangular matrix  $U$ . The next factorisation is  $A = QR$  where  $Q$  has orthonormal columns and  $R$  is upper triangular with positive diagonal. The author explains how this decomposition  $A = QR$  arises from the Gram–Schmidt process and its connection to the least squares method. The remaining topics in the book are also associated with factorisations, namely those arising from diagonalising a matrix and in particular diagonalising symmetric matrices, and later a factorisation given by the singular value decomposition of a matrix. This is a nice approach to teaching linear algebra that links all the topics of the book together using the common theme of finding interesting and useful factorisations of matrices.

The book is written with a pleasant conversational style. It is not a traditional definition-theorem-proof style mathematical textbook. For example, it does not contain proofs—theorems are stated as facts that are nicely displayed in boxes, often in the form of formulas. Some concepts such as the dot product are explained in words together with some concrete examples to demonstrate the idea, rather than by a formal notation-heavy definition. This fits with the overall style of the book which is example-focussed throughout and emphasises practical and computational aspects of the subject. Given this, it might be hard for a student with no knowledge of abstract linear algebra to learn the subject by just reading this book. However, this would be an excellent book to accompany a linear algebra course or to be read alongside a more traditional textbook. As mentioned above, this book may naturally be used as a companion text to the freely available MIT OpenCourseWare linear algebra lectures of the author.

The author’s passion for linear algebra comes across clearly throughout, and the book is written with an enthusiasm that makes it engaging and entertaining to read. There are many places where the author takes interesting digressions showing how the material links in with other topics e.g. connections with the theory of differential equations,

the Fast Fourier Transform, and Kirchhoff's current and voltage laws. This makes the text less dry than a traditional linear algebra textbook, and I am sure is something that will be appreciated by undergraduate readers.

Other things that make this book stand out from the crowd are the important and interesting applications of linear algebra to image compression and deep learning discussed in the final two chapters. These chapters will help students see the relevance of linear algebra to modern applications and also form an excellent foundation for students who want to go on and explore these topics in more detail. I especially liked the flag examples in the discussion of SVD for image compression, and also the discussion of AlphaGo in the deep learning chapter which is something I am sure will also interest students.

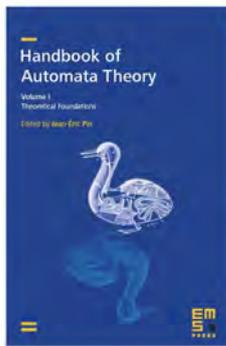
There are many exercises and worked examples throughout, with exercises helpfully divided into recommended and challenging problems. There are bullet point summaries of key ideas displayed clearly in boxes at the start of subsections that students will find useful. Throughout, the material is beautifully arranged and formatted making the book easy to navigate, which will help students looking back for a particular concept or result.

There are a lot of linear algebra books out there, which makes the task of writing an original one quite a challenge. But the author has succeeded in doing so with this book and, together with his freely available MIT OpenCourseWare courses, he has produced a text that gives a fresh and interesting approach to the subject that has a lot to offer for any student of linear algebra.

## Handbook of Automata Theory

Edited by Jean-Éric Pin, EMS press, 2021, hardback, vol.I and II £146, ISBN print 978-3-98547-006-8 ISBN digital 978-3-98547-506-3

Review by Catarina Carvalho



Automata: from Mathematics to Applications (AutoMathA) was a research networking programme from the European Science Foundation, which ran from 2005 to 2010. It involved 15 institutions as contributing member organisations, each

from different countries, and held 36 workshops throughout the 5 years. This book had its origins in this programme, and provides a comprehensive overview of the current research in automata theory.

At present, Automata Theory is not commonly taught as part of an undergraduate mathematics degree, but usually is in computer science. The mathematical connections and applications presented in this book made me wonder if we should review this. Automata also provide us with tools to explain to

non-mathematicians some mathematical concepts, such as the following.

Imagine a vending machine. For it to give you a packet of crisps you need to feed it coins that add up to 55p. You insert coins and the machine reads them, and it outputs a packet of crisps when the coins add up to 55p or more. The machine is a finite state automaton, and the language it reads (the possible combination of coins that add up to the correct amount) is a semigroup. This is the example I often give to non-mathematicians when they ask me what a semigroup is. As a bonus they also get told a rough definition of an automaton.

This book does not aim at being an introduction to automata — for that I recommend [1] — but instead is, as the title suggests, a handbook of current practice in automata theory. It reads like a quest to the world of automata, where no stone is left unturned.

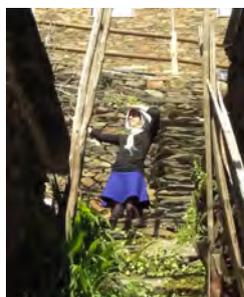
The first volume sets up the theoretical foundations of automata theory, including looking at different types of automata, looking at complexity issues such as enumeration, minimisation, and Cerný's conjecture, and finishing with algebraic and topological theory. In total it has 22 self-contained chapters by 42 contributors.

The second volume is set up in two parts: the first considers automata in mathematics, including group theory, number theory, finite model theory, and symbolic dynamics; the second looks at selected applications, including message-passing systems, data structures, synthesis, embedded systems, Markov chains, temporal logics, and quantum computing. It has 17 self-contained chapters by 30 contributors.

An added bonus in the applications section is that each chapter contains an introduction to the area where automata are being applied, with nice short examples and plenty of references for further reading, so this can also be considered a great book to learn about new topics.

Automata theory is probably one of the longest established theories in computer science, appearing from the need to study the possibilities and limits of the electronic digital computer [2]. Early works used abstract algebra to describe formal languages and the machines that describe them. To understand the basics of automata one needs only knowledge of elementary graph theory, abstract algebra (when also looking at formal languages), and combinatorics, which was what was required for the work done in the early 60s in this area. The area then expanded and new tools from non-commutative algebra (semigroups and semirings), logic, probability

theory, and symbolic dynamics were introduced. The most recent introductions have been topology and geometry. Since the area has grown in so many directions, it is good to finally have all them all, as well as many results, brought together in this handbook. The editor also thought of the recent advances and progresses that might be achieved since the book was published, and promises updates in <https://ems.press/books/standalone/172>. For a researcher in the area this book is a must-have; for mathematics lecturers it is a good source of ideas for interdisciplinary undergraduate research projects.



### Catarina Carvalho

Catarina is a senior lecturer in mathematics at the University of Hertfordshire. Her main research interests are in modern algebra and applications to theoretical computer

science, but she also has a keen interest in graph theory. Catarina likes the beach, yoga and fiction books, and when not doing any of this she is most likely having conversations about diversifying academia.

### FURTHER READING

- [1] Howie, John M., *Automata and Languages*, 1996 Oxford University Press, Inc.
- [2] Mahoney, Michael S. *The Structures of Computation and the Mathematical Structure of Nature*. The Rutherford Journal.

## Obituaries of Members

### Vicky Neale: 1984–2023



Dr Vicky Neale, who was elected a member of the London Mathematical Society on 9 June 2008, died on 3 May 2023, aged 39. Dr Neale was a

member of LMS Education Committee, a member of the Newsletter Editorial Board, LMS Popular Lecturer in 2013 and the Holgate Session Leader from 2015 to 2017.

*Kevin Houston writes:* Vicky will be remembered as an inspirational mathematician and as an exceptional teacher and public communicator of mathematics.

After receiving her BA in mathematics from Trinity College, Cambridge, Vicky stayed on to study for a PhD in analytic number theory and additive combinatorics supervised by Ben Green. This was awarded in 2011 for her thesis entitled *Bracket Quadratics as Asymptotic Bases for the Natural Numbers*.

After her PhD she was employed as a lecturer by Department of Pure Mathematics and Mathematical Statistics at the Cambridge. In 2014, she moved to the University of Oxford, where she held the position of Whitehead Lecturer at the Mathematical Institute and Balliol College, and was a Supernumerary Fellow at Balliol.

Vicky will be remembered by many as an excellent teacher; many of the tributes paid to her are from her students who describe her patient, inspirational teaching that brought life to the subject. She received a prize for achieving the highest grade in cohort for her portfolio in her Postgraduate Certificate (PGCert) in Teaching and Learning in Higher Education.

Vicky was very active in the LMS. She was a member of the Newsletter Editorial Board and was a regular contributor to the LMS Newsletter. She served on the Education Committee for 6 years, where she could be relied upon to produce sensible and workable solutions to the problems faced. She was a Holgate Session Leader from 2015 to 2017, and in 2013 was an LMS Popular Lecturer. Her talk, *Addictive Number*

*Theory*, which was, of course, on her love of prime numbers, is the 4th most popular video on the LMS's YouTube channel. Another of her talks, an introductory lecture on complex numbers for Oxford University, currently has over 2.3 million views.

Ever energetic, Vicky was involved many other organisations. She dedicated much of her time to those that encouraged young people, particularly girls, to enjoy and experience mathematics. She was Executive Director of PROMYS Europe, a scheme to bring secondary school students from around Europe to sample mathematics at Oxford. She was a volunteer for the United Kingdom Mathematics Trust (UKMT) and served on its council for many years; was a trustee of the recently founded Mathematics Education for Social Mobility and Excellence (MESME) and, perhaps most poignantly of all, was to be President of the Mathematical Association in 2024–2025.

Vicky was very much involved in public engagement of mathematics. After finishing her PhD she regularly appeared on radio and occasionally on television. She was a guest on a variety of Radio 4 programmes and hosted 'A Mathematician's Guide to Beauty'. Her first book, *Closing the Gap: The Quest to Understand Prime Numbers*, a book on recent astounding progress in the Twin Primes Conjecture, was aimed squarely at sharing her love of Number Theory with the general public. Her mathematical interests went beyond such abstract topics to include arts and crafts. She was keen on crafting, particularly knitting and crocheting, and created mathematically patterned Christmas cards.

In 2020, as part of her mission to encourage young people to study mathematics she published *Why Study Mathematics?* This book shows that mathematics is an exciting, lively (and even useful!) subject that is ideal for studying at university.

Vicky's final major project was the podcast Maths+Cancer. Having been diagnosed with a rare form of cancer, she recorded a series of podcasts interviewing mathematicians and scientists to explore how mathematics was being used in cancer research.

The series is a testament to her love of mathematics and service to a community that will miss her deeply.

## Yuri I. Manin: 1937–2023



Professor Yuri I. Manin, who was elected an Honorary Member of the London Mathematical Society on 1 July 2011, died on 7 January 2023, aged 85. Professor Manin

was the LMS Hardy Lecturer in 2006.

*Alexei Skorobogatov writes:* Yuri Manin, the only child of Ivan Gavrilovich Manin and Revekka Zinovievna Miller, was born on the 16 February 1937 in Simferopol (Crimea). His father, a lecturer in geography, was a son of an illiterate Russian peasant. His mother, a postgraduate student of Russian literature, was a daughter of a playwright and journalist of Jewish origin from Hughesovka (modern Donetsk in the Ukraine).

Yuri Manin's early personal history is an intense family tragedy caused by the catastrophe of the Second World War. In the fifth year of his life, in the face of the Wehrmacht's rapid advance, the Manin family together with the grandparents Zinovy and Hannah Miller fled to the North Caucasus and then further still, across the Caspian Sea, to Chardzhou in the Soviet Central Asia (modern Turkmenistan).

Refugees in their own country, they had no place to stay and no means of existence. During the year of 1942, Yuri's grandmother fell ill and died, soon thereafter his grandfather committed suicide. Later in the year Yuri's father volunteered to fight in the Red Army and perished in the war. After the liberation of Crimea, Yuri and his mother returned to Simferopol only to find the apartment where Yuri lived before the war occupied by another family. Yuri and Revekka became effectively homeless, though they were allowed to move in to Yuri's aunt's communal apartment. The difficult post-war childhood was further complicated when Yuri's mother lost her job in the Soviet state antisemitic campaign of 1948.

While at school, Yuri read I.M. Vinogradov's book *Elements of Number Theory* and sent the author a letter with a generalisation of the formula for the number of integer points in a circle. Yuri's life radically changed when he became an undergraduate in the Department of Mechanics and Mathematics of the Moscow State University in 1953, the year of Stalin's death and the inauguration of the grandiose Main Building of the University. Igor Shafarevich approached Yuri and offered to become

his supervisor, and — already in 1956 — Yuri's first published paper 'On cubic congruences to a prime modulus' appeared in *Izvestia*, a leading Soviet mathematical journal. The paper gives an elementary proof of Hasse's bound for the number of points on an elliptic curve over the finite field with  $p$  elements. Manin's precocious talent manifested itself in a series of papers on algebraic curves, many of which are nothing short of miraculous. The high point of this period is Manin's proof of the functional analogue of Mordell's conjecture in characteristic zero (1963). This ingenious proof is based on the discovery of a major new method involving the algebraisation of the theory of Picard–Fuchs differential equations and what Grothendieck later called the Gauss–Manin connection, by now a standard tool when dealing with cohomology of families of varieties over a base. In the words of Robert Coleman, "this work is testimony to the power and depth of Manin's intuition". Another foundational result of this period is Manin's classification of commutative formal groups over fields of finite characteristic. For these achievements he was awarded the Lenin prize in 1967, one of the most prestigious awards of the Soviet Union usually given to composers, ballet dancers and rocket scientists.

In 1960, Yuri Manin became a researcher at the Steklov Mathematical Institute of the Academy of Sciences of the USSR. He took part in the study group organised by Shafarevich with the aim of giving a modern treatment to the results and methods of the Italian school of algebraic geometry. The outcome was the celebrated volume *Algebraic Surfaces*. Manin's contribution to the project concerned rational and ruled surfaces; thus began his long series of papers on the geometry, combinatorics, and arithmetic of geometrically rational surfaces summarised in his book *Cubic Forms* (1972). A closely related work is Manin's joint paper with Vasily Iskovskikh which proves birational rigidity of quartic threefolds, thus giving a negative answer to Lüroth's problem.

In May and June of 1967, Yuri Manin participated in Alexandre Grothendieck's Séminaire de Géométrie Algébrique (SGA 6) at IHES, Bures-sur-Yvette. A result of this visit, Manin's paper on motives was the first ever publication on this subject. Grothendieck liked it and recommended it to David Mumford as "a nice foundational paper".

When I became Manin's student in 1980, he was not allowed to do any undergraduate teaching, presumably because that would have brought him too close to the 'ideological frontline'. This may

have been a blessing in disguise. Besides, he was not restricted in the choice of his 'special courses', seminars and study groups aimed at a smaller circle of his own students. The weekly Manin seminar had something of a cult status. The seminar was about much more than talks: it was a meeting place where mathematical ideas were passed around. In the breaks, participants walked in pairs in circular corridors of the Main Building of the university discussing mathematics. Many joint papers started in this way.

Manin's graduate students include a large number of highly successful mathematicians. He has had about 60 PhD graduates, and has influenced the development of a great many more young mathematicians, including people who became famous in their own right, such as the Fields medallists Drinfeld and Kontsevich, and many others.

Perhaps the most characteristic feature of Manin's mathematical genius was his uncanny ability to see a far-reaching theoretical potential in mathematical facts and observations that could be perceived by others as specific and isolated, or merely as part of a familiar picture. Manin generously shared his insights; his papers are scattered with open questions and suggestions. Two major examples of his amazing insight are the Brauer–Manin obstruction (1970) and the Manin–Batyrev conjectures (1989–91) that essentially created the area of arithmetic geometry concerned with rational points on higher-dimensional varieties. Many other of Manin's research interests have generated flourishing schools of contemporary mathematics. He made fundamental contributions to instantons, Yang–Mills fields, supergeometry, quantum groups, quantum cohomology, operads, and much more.

Manin's interests took him beyond mathematics: to the human sciences, the literary and cultural milieu. He was a close friend of Vladimir Vysotsky, the singer/songwriter and actor known to every single person in the USSR, and the brothers Arkady and Boris Strugatsky, the best-known science fiction writers of the country.

Perhaps Manin's trademark was his unique ability to see mathematics as a whole, and to form a profound vision of mathematics and its connections with other areas of science, language and the arts. This combination of cultural breadth with a supreme mastery of research in a way that is seldom seen in contemporary mathematics contributed to his immense popularity among students.

In 1992–93 Manin spent a year in MIT. In 1993 he moved to the Max Planck Institute for Mathematics in Bonn to take up the position of a director. From 2002 to 2011 he was also a professor at Northwestern University. He became director emeritus in 2005, without any slowing down in his research.

Yuri Manin was a foreign member of several national academies, and has received a long string of prizes and honours: the Brouwer Medal in 1987, the Frederic Esser Nemmers Prize in 1994, the Schock prize of the Swedish Academy in 1999, the King Faisal International prize in 2002, the Georg Cantor medal of the German Mathematical Society in 2002, Order Pour le Mérite for Science and Art in 2007, the Great Cross of Merit with Star in 2008, and the János Bolyai International Mathematical prize of the Hungarian Academy of Sciences in 2010. He was an invited speaker to the International Congress of Mathematicians on no fewer than five occasions.

I would like to finish with a quote from T.S. Eliot's *Four Quartets* that Manin used to describe his attitude to mathematics and, perhaps, to life itself: "For us, there is only the trying. The rest is not our business".

## David B. Singmaster: 1938–2023



Professor David Singmaster, who was elected a member of the London Mathematical Society on 15 October 1970, died on 13 February 2023, aged 84. Professor Singmaster was LMS Meetings & Membership Secretary 1976–79.

*Snezana Lawrence writes:* David Breyer Singmaster was born in Fergusson, Missouri, in December 1938. He went to Caltech, a beginning of his academic career that, in his own words, was 'a bit checkered'. At the end of his third year there his study days were discontinued, but considering that he became something of a mathematics celebrity and well known throughout the world for his mathematical work, his academic ability doesn't seem to have been lacking.

From Caltech, David went to work for a year and then went to Berkeley where he got married. He went on to work in Beirut and then lived for a while in Cyprus, until he found his way to London. There he found job as a Lecturer in the Mathematics Department of the Polytechnic of the South Bank, now South Bank University, and married for the second time to his wife Deborah. He remained at South Bank until

he retired in 1996, when he became an honorary research fellow at UCL, and emeritus professor at South Bank in 2000.

At first, David had wanted to be a civil engineer, then an organic chemist, but while at Berkeley he found he didn't like labs, and learned of a computer programme on puzzles which he enjoyed learning about. He started reading on number theory and taking mathematics and physics courses, and at some point came up with a question of 'Why are people hiding these interesting ideas from everyone?' At this point, he said, he became a mathematician.

In a class on number theory by D.H. Lehmer (1905–1991) he became enchanted with puzzles, and in this way got into a graduate school after completing his degree. By the time he finished his PhD in 1966 Singmaster had written three papers. One of these he remembered fondly to the end of his days, with the title *On Round Pegs in Square Holes and Square Pegs in Round Holes* (Singmaster, 1964). This fascinating little paper begins with these words:

"Some time ago, the following problem occurred to me: which fits better, a round peg in a square hole or a square peg in a round hole? This can easily be solved once one arrives at the following mathematical formulation of the problem. Which is larger: the ratio of the area of a circle to the area of the circumscribed square, or the ratio of the area of a square to the area of the circumscribed circle?"

And so it went — all the way to the dimension 7 and beyond. Here is perhaps already the mould that would become a recognisable one in David's future works: he would start from a seemingly simple question, and lead on to very nice, and new, mathematics. David's musings of this sort, on the simple things we meet in life that could bring some interesting and very insightful new knowledge about mathematics itself, were his way of being in the world.

A friend of a friend told him at some point about some mathematical puzzles and old problems. Through these problems David also became interested in the history of mathematics. He realised that many of the older problems have solutions, but authors don't go far enough in asking whether that is the only solution, so he would try to find all the solutions, both from history and through his own problem-solving. These two aspects of 'attacking' a puzzle became interchangeable for him — working out the puzzle and researching its history. In England he found a stable and good home and was able to

dedicate himself more to this work; he was for many years an enthusiast of history and attended meetings of the British Society for the History of Mathematics.

But it was the late 1970s that brought David to a great prominence of a kind. In 1978 he went to the ICM, held in Helsinki, as Secretary of the LMS (he was joint Membership and Meetings Secretary 1976–1979), where he spoke to various people: some Hungarians, he said, and his English colleagues John Conway (1937–2020), and Roger Penrose (1931–). They were all trying to work out this new puzzle the Hungarians brought with them. It was of course the Rubik's cube, invented by Hungarian sculptor and architect Ern Rubik (1944–). At that time it was called only the Hungarian Magic Cube. The cube was first made in 1974 and by 1977 it was produced in some numbers in Budapest. It was so popular that Hungarian colleagues told David they used it as a hard currency (at that time Hungarian currency was not worth much in the West).

After playing with this cube for about two weeks David worked out how to solve the puzzle. He realised this was one interesting puzzle indeed, and thought that many people would enjoy playing with it too. He asked the Hungarian embassy in London to help him get in touch with the author of the puzzle and worked out how to make it. Meanwhile he got a call from a Hungarian importer in London who told him he could get him some directly from a Hungarian factory — and David obtained the first supply: first sixty, but within some months they had sold hundreds, mainly among academics in England! David became in effect the facilitator of bringing this cube to the wider, Western, world. He played with it day and night and even set up a shop in South London to sell it and related puzzles. He introduced notation to describe how to move and solve the puzzle, something that remains to this day in Rubik-related publications.

In 1979 David began to write a book on group theory related to Rubik's cube (Singmaster, 1981). He started a publication *The Cubic Circular*, published between 1981–85. Later David's book and articles were adopted to teach group theory, and so now Rubik's cube is probably one of the most common artefacts in mathematics departments.

There is a conjecture named after David in combinatorial number theory stating that there is a finite upper bound of the number of times a number other than 1 can appear in Pascal's triangle. Erdős doubted this but didn't disprove it. One of David's contributions to the history of

mathematics was finding a lost manuscript by Luca Pacioli (1447–1517), *De Viribus Quantitatis* [*On the Powers of Numbers*], written sometime between 1496 and 1508 (Hirth, 2015) — an amazing achievement for a historian of mathematics that never called himself a historian. David was a common feature at the various conferences and meetings and particularly those dedicated to puzzles, like Gathering4Gardner.

David was an extremely nice and generous person, a large, friendly, smiley figure, with great capacity to think and play mathematically. He was one of a kind and will be sorely missed by all who knew him.

### FURTHER READING

[1] Hirth, Tiago (2015). Luca Pacioli and his 1500 book *De Viribus Quantitatis*, PhD thesis submitted to the University of Lisbon. (accessed 25th April 2023).

[2] Singmaster, David (1964). On Round Pegs in Square Holes and Square Pegs in Round Holes, *Mathematics Magazine*, 37 (5), 335-337.

[3] Singmaster, David (1981, 5th ed). *Notes on Rubik's 'Magic Cube'*. Southbank University, London.

## Death Notices

We regret to announce the following deaths:

- Dr Donald James Collins, formerly of Queen Mary, University of London, who was elected an LMS member on 21 November 1968, LMS Meetings & Membership Secretary 1992-95, died on 21 February 2023, aged 81.
- Professor Eduardo L. Ortiz, formerly of Imperial College London, who was elected an LMS member on 19 February 1982, died on 29 December 2021, aged 90.
- Dr Hans Peter Rogosinski, who was elected an LMS member on 20 June 1963, died 10 August 2022, aged 89.

## Dynamics, Bifurcations and Numerics

Location: University of Surrey, Room 39AA04  
 Date: 4-7 July 2023  
 Website: [tinyurl.com/mt4yd6c8](https://tinyurl.com/mt4yd6c8)

The international workshop is dedicated to the memory of internationally recognised Surrey mathematician Claudia Wulff. Its aim is to bring together some topics of classical dynamics, numerical analysis and analysis of infinite dimensional systems generated by PDEs focusing on: Dynamical systems and bifurcation theory, Analysis of PDEs and applications to hydrodynamics, Numerical analysis and computation. This meeting is supported by an LMS Conference Grant.

## The Role of Wandering Domains in Holomorphic Dynamics

Location: The Open University, Milton Keynes  
 Date: 19 July 2023  
 Website: [tinyurl.com/3kdemfz9](https://tinyurl.com/3kdemfz9)

This meeting will focus on new developments in Holomorphic Dynamics, and particularly on wandering domains, a topic on which remarkable progress has been achieved in recent years. Talks will be given by Vasiliki Evdoridou (The Open University), Nuria Fagella (University of Barcelona), Leticia Pardo-Simón (University of Manchester) and Phil Rippon (The Open University).

## Early Career Researchers in Mathematics (ECRM) 2023

Location: Durham University  
 Date: 11-14 July 2023  
 Website: [ecrm2023.com](https://ecrm2023.com)

ECRM will bring together PhD students from all areas of mathematics to meet, discuss mathematics and listen to talks from both participants and a range of academics. The aim is to give all participants the opportunity to give a talk on their own area of research. There will also be a panel discussion on PhD and career advice from academics at a variety of career stages.

## DANGER: Data, Numbers and Geometry

Location: LIMS and Zoom  
 Date: 24-25 August 2023  
 Website: [tinyurl.com/DANGER2023](https://tinyurl.com/DANGER2023)

DANGER is a two-day hybrid workshop, to be held at the London Institute of Mathematical Sciences and on Zoom, focused on interactions between data science and pure mathematics. The talks feature topics in algebraic geometry, number theory, and representation theory. Participants can expect to learn more about data scientific methods, especially their role in pure mathematical research.

## LMS Meeting

# LMS Invited Lecture Series 2023

17-21 July 2023, University of Durham

Website: [bit.ly/3MOX9ZG](https://bit.ly/3MOX9ZG)

The Invited Lecturer in 2023 is Filippo Santambrogio (Université Lyon 1, France), whose talk will be titled *Optimal Transport and its Applications*. There will be accompanying lectures by David Bourne (Heriot-Watt University), Sara Farinelli (Lagrange Centre, Paris, France), Hugo Lavenant (Bocconi University, Milan, Italy), Emanuela Radici (University of L'Aquila, Italy) and Matthew Thorpe (University of Manchester).

Funds are available for partial support to attend. Requests with an estimate of expenses should

be addressed to the organiser, Dr Alpár Mészáros ([alpar.r.meszáros@durham.ac.uk](mailto:alpar.r.meszáros@durham.ac.uk)).

The annual Invited Lecturers Series aim to bring a distinguished overseas mathematician to the UK to present a small course of about ten lectures spread over a week. Each course of Invited Lectures is on a major field of current mathematical research, and is instructional in nature, being directed both at graduate students beginning research and at established mathematicians who wish to learn about a field outside their own research specialism.

### 36th British Topology Meeting

Location: University of Sheffield  
 Date: 6-8 September 2023  
 Website: [tinyurl.com/yck2j73a](https://tinyurl.com/yck2j73a)

The British Topology Meeting is open to everyone interested in algebraic or geometric topology, related areas, applications, computations, or foundations. The speakers will cover a wide area of topology, and the organisers also invite attendees to propose contributed talks. The meeting will be preceded by an Early Careers Researchers' Conference, which shares the website.

### British Logic Colloquium 2023

Location: University of Bristol  
 Date: 7-8 September 2023  
 Website: [tinyurl.com/ms8nkj6](https://tinyurl.com/ms8nkj6)

The British Logic Colloquium will hold its annual conference at Bristol. Invited speakers include: Sandra Müller (TU Wien), Paul Shafer (Leeds), Juliette Kennedy (Helsinki), Elaine Pimentel (UCL), Juan Aguilera (TU Wien & Ghent); Carlo Nicolai (KCL); Vincenzo Mantova (Leeds). The meeting is co-located and contiguous with the Workshop Working on Truth organised by Johannes Stern on Saturday 9 and Sunday 10 September. The meeting is supported by an LMS Conference Grant.

### Clay Research Conference and Workshops

Location: Mathematical Institute, Oxford  
 Date: 25-29 September 2023  
 Website: [claymath.org](https://claymath.org)

Four workshops, *About Entropy in Large Classical Particle Systems*; *K-stability and Birational Geometry*; *New Perspectives in the Analytic Theory of Automorphic Forms*; *Symplectic Geometry, Low Dimensional Topology*, and *Quantum Fields*, will be held throughout the week with the conference held on 27 September. Conference speakers: Valentin Blomer, John Pardon, Laure Saint Raymond, Chenyang Xu. To register for the Conference and to register interest in a workshop, email Naomi Kraker at [admin@claymath.org](mailto:admin@claymath.org).

### Heilbronn Annual Conference

Location: University of Bristol  
 Date: 7-8 September 2023  
 Website: [tinyurl.com/2w3aajuv](https://tinyurl.com/2w3aajuv)

The Heilbronn Annual Conference is the institute's flagship event. It takes place over two days and it covers a broad range of mathematics, including algebra, combinatorics, data science, geometry, number theory, probability, quantum information. It brings together members of the Institute, distinguished visiting speakers, and other members of the UK mathematical community. This year we welcome eight distinguished speakers, to deliver lectures intended to be accessible to a general audience of mathematicians.

### London Mathematical Biology Conference

Location: University College London  
 Date: 18-19 September 2023  
 Website: [tinyurl.com/LondonMathBioConf](https://tinyurl.com/LondonMathBioConf)

This conference will focus on recent research results in Mathematical Biology and its applications. Everyone is welcome, whether based in London or otherwise. There is an exciting list of invited speakers, and we welcome contributed abstracts from early career researchers (deadline 15 July). Please visit the website for registration (deadline 5 September). We thank the London Mathematical Society and the Institute of Mathematics and its Applications for funding.

### Cryptography and Coding Conference

Location: Royal Holloway, London  
 Date: 12-14 December 2023  
 Website: [tinyurl.com/2mxu4v9t](https://tinyurl.com/2mxu4v9t)

The mathematical theory and practice of both cryptography and coding underpins the provision of effective security and reliability for data communication, processing and storage. This 19th international conference in an established and successful IMA series on the theme of Cryptography and Coding solicits original research papers on any cryptographic or coding-theoretic topic including, but not limited to: Foundational theory and mathematics; Design, proposal, and analysis of cryptographic or coding primitives and protocols; Secure implementation and optimisation; Applied aspects of cryptography and coding.

# Society Meetings and Events

## July

- 11-14 Early Career Researchers in Mathematics (ECRM) 2023
- 16-28 LMS Undergraduate Summer School, Sheffield
- 17-21 LMS Invited Lecture Series 2023, University of Durham
- 17-21 LMS Research School Machine Learning in Mathematics and Theoretical Physics, Oxford
- 24-28 LMS Research School Algebraic Groups and their Representations, Birmingham
- 24-4 Aug LMS-Bath Mathematical Symposium Operators, Asymptotics, Waves, hosted at Bath

## August

- 1-11 LMS-Bath Mathematical Symposium Categorical and Geometric Representation Theory, Bath

## September

- 4-6 LMS Northern Regional Meeting & Workshop, University of York

## October

- 3-4 Black Heroes of Mathematics, online
- 13 Joint Meeting with the IMA

# Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website ([www.lms.ac.uk/content/calendar](http://www.lms.ac.uk/content/calendar)). Please send updates and corrections to [calendar@lms.ac.uk](mailto:calendar@lms.ac.uk).

## July

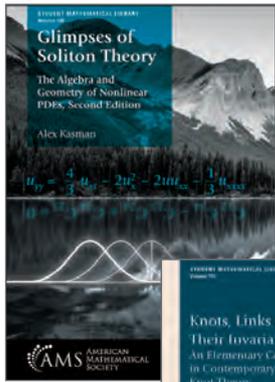
- 4-7 Dynamics, Bifurcations and Numerics Workshop, University of Surrey (507)
- 6-7 Queer and Trans Mathematicians in Combinatorics, Queen Mary University of London (506)
- 6-7 Numerical Methods for Mean Field Games and Related PDE Workshop, University College London (506)
- 10-14 Stability and Dynamics in Fluid Mechanics and Kinetic Theory, Imperial College London (506)
- 11-14 Early Career Researchers in Mathematics (ECRM) 2023, Durham University (507)
- 17-21 European School of Information Theory 2023, University of Bristol (506)
- 19 The Role of Wandering Domains in Holomorphic Dynamics, The Open University (507)
- 26-27 Classical, Un-Classical and Semi-Classical Problems in Operator Theory, Cardiff University (506)

## August

- 24-25 DANGER: Data, Numbers and Geometry, London Institute for Mathematical Sciences (507)

## September

- 7 Mathematics in Defence and Security IMA Conference, Imperial College, London (506)
- 6-8 36th British Topology Meeting, University of Sheffield (507)
- 7-8 British Logic Colloquium 2023, University of Bristol (507)
- 7-8 Heilbronn Annual Conference, University of Bristol (507)
- 18-19 London Mathematical Biology Conference, University College London (507)
- 25-29 Clay Research Conference and Workshops, Mathematical Institute, Oxford (507)



### GLIMPSES OF SOLITON THEORY

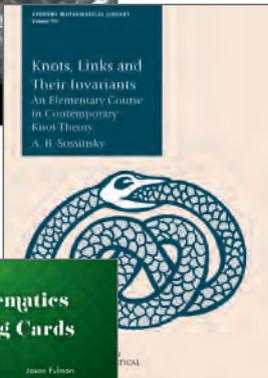
#### The Algebra and Geometry of Nonlinear PDEs, Second Edition

Alex Kasman, *College of Charleston*

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Reveals the hidden connections discovered over the last half-century that explain the existence of these mysterious mathematical objects. This book aims to convince the reader that the underlying algebro-geometric structure of soliton equations provides an elegant explanation of something seemingly miraculous.

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### KNOTS, LINKS AND THEIR INVARIANTS

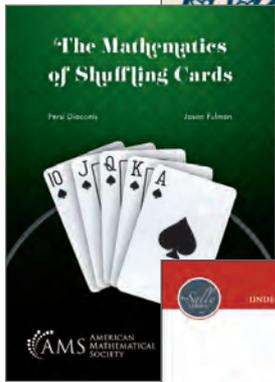
#### An Elementary Course in Contemporary Knot Theory

A. B. Sossinsky, *Independent University of Moscow and Poncelete Laboratory IUM-CNR*

*Student Mathematical Library, Vol. 101*

An elementary introduction to knot theory. Unlike many other books on knot theory, this book has practically no prerequisites; it requires only basic plane and spatial Euclidean geometry but no knowledge of topology or group theory. It contains the first elementary proof of the existence of the Alexander polynomial of a knot or a link based on the Conway axioms, particularly the Conway skein relation.

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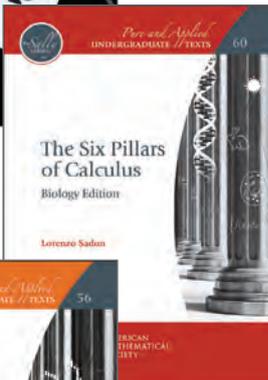


### THE MATHEMATICS OF SHUFFLING CARDS

Persi Diaconis, *Stanford University* & Jason Fulman, *University of Southern California*

Provides a lively development of the mathematics needed to answer the question, 'How many times should a deck of cards be shuffled to mix it up?' The shuffles studied are the usual ones that real people use: riffle, overhand, and smooching cards around on the table.

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Lorenzo Sadun, *University of Texas at Austin*

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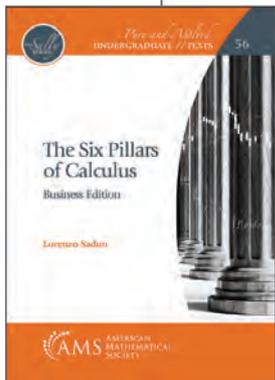
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